SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF CHEMICAL AND FOOD TECHNOLOGY
INSTITUTE OF INFORMATION ENGINEERING, AUTOMATION AND MATHEMATICS

MPC-Based Reference Governors

Diploma thesis

FCHPT-5414-50920

Bratislava 2013 **Bc. Martin Klaučo**

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Study program:	Automation and information engineering in chemical and food technologies.
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Abstract

Model predictive control (MPC) strategy is very well known for more then three decades. Despite obvious advantages in implementing such strategy, PID controllers are still widely used in industry. Main obstacle of implementing MPC is its high demand on computational resources, when a quadratic optimization problem with constraints must be calculated during specified sample time. This thesis deals mainly with MPC situated as governor of set points for PID controller. Such strategy arises from industry requirement, when they are reluctant to remove well known PID controllers. Formulation of optimization problem and simulation results are discussed in this project.

Abstrakt

Prediktívne riadenie (MPC) je známou stratégiou riadenia viac než tri desaťročia. Napriek zjavným prínosom pre riadenie procesu ako takého, PID regulátory sú stále dominantnými regulátormi v priemysle. Najväčšou prekážkou nasadzovanie MPC regulátorov je jeho výpočtová náročnosť, ktorá spočíva v riešení kvadratickej optimalizačnej účelovej funkcie s ohraničeniami každú vzorkovaciu periódu. Táto diplomová práca sa zaoberá návrhom MPC regulátora, ktorý bude riadiť žiadanú hodnotu pre vnútorný PID regulátor. Formulácia optimalizačnej úlohy ako aj simulácie sú v tejto práci ukázané.

I would like to express my sincere thanks to my supervisor Assoc. Prof. Ing. Michal Kvasnica PhD. for his guidance and valuable input during preparation of this master thesis and during my study at this university.

Martin Klaučo

Contents

Chapter 1

Introduction

Despite the fact that MPC has proven to be more efficient than simple PID controllers, industry is reluctant to employ such control strategies as a primary controllers. In this project is proposed a combination of inner closed loop with PID controller and outer closed loop with MPC governor Fig. 1.1. MPC governor will be responsible for calculating set point values for inner loop controller, while obeying technological and safety related constraints.

First and foremost the benefit of implementing such strategy is to improve overall performance of the plant. Second advantage is, that MPC can be easily switched off without dangering the operation of the plant, because PID controller is still present and active in the inner loop. One of the first control strategy with reference governors has been proposed by Bemporad (1998). In this article are also discussed some properties of primal controller, where it is suggested smaller time constant then in reference governor. It has been also proposed to compute reference profle for inner loop controller in advance to alleviate the computational burden during online operation.

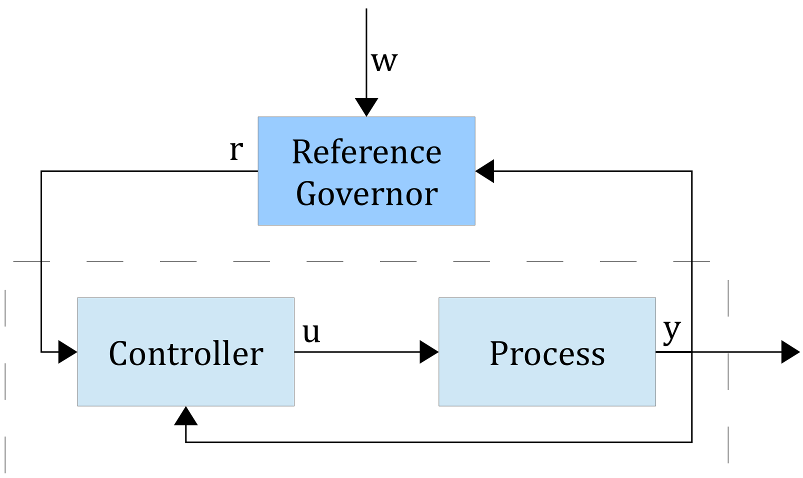

Figure 1.1: Reference Governor

Reference governor is outer loop controller, which main purpose is to maintain the constraints and filter the reference for inner loop controller. In this work is assumed that inner controller, mostly PID controller, can achieve stable reference tracking Borrelli et al. (2009). More elaborate and complex command governors designed on non-linear systems have been presented by Gilbert and Kolmanovsky (2002). There was also published a paper by Kvasnica et al. (2012), where a simple MPC with prediction horizon equal to 1 was used to correct a control action of arbitrary inner loop controller.

1.1 Boiler System

Technological process which is used in this thesis is boiler-turbine system. Boiler-turbine unit is a complex non-linear system. Main functionality of this technological process is to heat water inside the boiler in order to provide power.

A 3$^{\text{rd}}$ order non-linear mathematical model developed by Åström and Eklund (1972) was used. In Dimeo and Lee (1995) is presented PI controller based on same mathematical model developed by Åström and Eklund. This PI controller is used in design of reference governor. For further expansion of this research more complex dynamics of boiler-turbine system will be necessary to include. More detailed mathematical description of such system can be found in e.g. Åström and Bell (2000); Flynn and O'Malley (1999).

1.2 Model Predictive Control Overview

MPC belongs to a family of optimal MIMO state feedback control strategies. The core principle is to calculate an optimal control inputs sequence over the prediction horizon. To the plant is then applied only the first control input from the sequence. Current state of the plant is used as a initial condition for the optimization.

In research related to MPC, there are several milestones worth of mentioning. MPC originates in Dynamic Matrix Control (DMC) first presented by Cutler and Ramaker in 1980. MPC formulation with linear model was discussed in detail in Muske and Rawlings (1993). In this article stability of such control strategy was discussed. Offset free tracking was also proposed using optimal linear observer. Similar approach to remove offset in reference tracking was proposed by Pannocchia and Rawlings (2003). Disturbance modelling was proposed to estimate unmeasured disturbances, which can be then compensated by the controller.

Very extensive survey of MPC advantages and implementation has been written by Mayne et al. (2000). This paper discuss mathematical formulations, stability and optimality of various MPC policies. Robust MPC implementation is presented as well.

Model predictive control strategy offers great deal of advantages in control, it is recognized by the industry and several software packages are developed in order to implement MPC in industry. Summary of industrial applications can be found in Qin and Badgwell (2003). Worth mentioning are companies like Invensys Systems, Inc. and Honeywell Hi-Spec Solutions. These companies have working knowledge of MPC based controllers implementation on several plant all over the world.

1.3 Thesis Overview

In order to demonstrate proposed implementation simple third order linear model of boiler turbine system taken from Dimeo and Lee (1995) is used. This paper also introduce PI controller for this system. Before case study of boiler system is shown, background and theory is presented (chapter 2).

This chapter discuss and derive state space model of closed loop system with PID controllers (Section 2.1) and presents formulation of model predictive control strategy (Section 2.2). Two formulations of MPC are considered in this project. First, standard approach, where controlled process is the boiler-turbine system (Section 2.2.1). Second formulation is related to governing the closed loop system, where actual control inputs to the process becomes an output from state space model, thus MPC formulation must be changed accordingly (Section 2.2.2).

Chapter 3 is devoted to presenting boiler-turbine system (Section 3.1) and control design (Section 3.2). In this chapter is also discussed discretization and tuning of model predictive control strategies. Finally in second to last chapter are shown simulations where three control strategies are compared - PID controller taken from paper, MPC on boiler-turbine system and MPC governor employed on closed loop system. In last chapter are discussed conclusions and possibilities of future expansion of this research.

Chapter 2

Model Predictive Control of Closed Loop System

In this chapter theory explaining reference governing and model predictive control will be presented. In first section state space model of closed loop system will be presented. Upon this state space MPC strategy will operate. In second section overview of model predictive control in general will be given. Since standard formulations of MPC strategy could not be implemented in this case, modifications to the formulations will be presented as well.

2.1 State Space Model of Closed Loop

Closed loop in general consists of process and a controller. In this case given controller is PID regulator. Typical set up of such feedback control is shown on figure (2.1). One of the objectives of this project is to derive state space model of such feedback control.

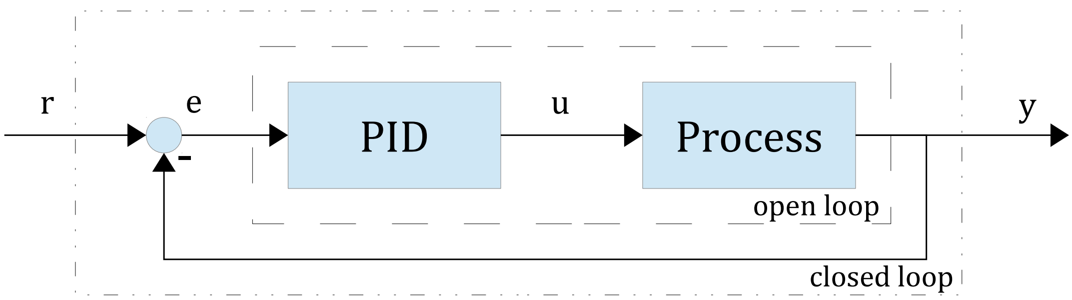

Figure 2.1: Standard feedback control

Very well known formulation of state space model in continuous time for the process is chosen (2.1).

$$\dot{x} = Ax + Bu \tag{2.1a}$$

$$y = Cx + Du \tag{2.1b}$$

In this model, x represents state vector, u is control input vector and y is the output vector. Matrix representing dynamics of the system is A, B is control matrix, output matrix is denoted as C and D is direct term matrix.

In control theory, PID controllers are widely reffered to in a transfer function representation (2.2). However for the purpose of this project, state space model of PID controllers in neccesary (2.3). Such state space model can be formulated also for systems with multiple feedback loops. This is also the case for used boiler-turbine system.

$$G_R(s) = \frac{U(s)}{E(s)} = Z_R + \frac{1}{T_I s} + T_D s \tag{2.2}$$

$$\dot{x}_r = A_r x_r + B_r e \tag{2.3a}$$
$$u = C_r x_r + D_r e \tag{2.3b}$$

In this state space model x_r is denoting the states od the PID controller, e is control error vector and u is control action vector, which is input to the actual process.

Since output from state space model representing PID controller is control action "u", formula (2.4) can be written. This can be combined to one state space model representation shown in (2.5).

$$\dot{x}_r = A_r x_r + B_r e \tag{2.4a}$$
$$\dot{x} = Ax + B\left(C_r x_r + D_r e\right) \tag{2.4b}$$
$$y = Cx + D\left(C_r x_r + D_r e\right) \tag{2.4c}$$

$$\begin{bmatrix} \dot{x}_r \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A_r & 0 \\ BC_r & A \end{bmatrix} \begin{bmatrix} x_r \\ x \end{bmatrix} + \begin{bmatrix} B_r \\ BD_r \end{bmatrix} e \tag{2.5a}$$
$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} DC_r & C \\ C_r & 0 \end{bmatrix} \begin{bmatrix} x_r \\ x \end{bmatrix} + \begin{bmatrix} DD_r \\ D_r \end{bmatrix} e \tag{2.5b}$$

$$\dot{\mathbf{x}} = A_{OL}\mathbf{x} + B_{OL}e \tag{2.6a}$$
$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} C_{OL,y} \\ C_{OL,u} \end{bmatrix} \mathbf{x} + \begin{bmatrix} D_{OL,y} \\ D_{OL,u} \end{bmatrix} e \tag{2.6b}$$

State model in (2.5) is simplyfied and new matrix definitions are made in (2.6). In this model boldface state vector $\mathbf{x}$ denoted comination of controller and process states. Index OL in matrix notations denotes state space representation of open loop system. Note, that output equation is split into to parts. Reason for this is, that only output y is used in feedback.

State space model of closed loop shown in (2.10) is derived from (2.7). Output equation of state space model (2.7b) is reformulated as stated in (2.8). Result is then inserted into (2.7a) yielding (2.9).

$$\dot{\mathbf{x}} = A_{OL}\mathbf{x} + B_{OL}(r - y) \tag{2.7a}$$
$$y = C_{OL,y}\mathbf{x} + D_{OL,y}(r - y) \tag{2.7b}$$

$$y = \frac{C_{OL,y}}{I + D_{OL,y}}\mathbf{x} + \frac{D_{OL,y}}{I + D_{OL,y}}r \tag{2.8}$$

$$\dot{\mathbf{x}} = A_{OL}\mathbf{x} + B_{OL}\left(r - \frac{C_{OL,y}}{I + D_{OL,y}}\mathbf{x} - \frac{D_{OL,y}}{I + D_{OL,y}}r\right) \tag{2.9}$$

Final form of state space model of closed loop in continuous time is given in (2.10). As it was already mentioned model predictive control strategy will be based on this model. For further simplification of notation, index "CL" will be used in similar fashion as in (2.6)

$$\dot{\mathbf{x}} = \left(A_{OL} - B_{OL}\frac{C_{OL,y}}{I + D_{OL,y}}\right)\mathbf{x} + \left(B_{OL} - B_{OL}\frac{D_{OL,y}}{I + D_{OL,y}}\right)r \tag{2.10a}$$
$$y = \frac{C_{OL,y}}{I + D_{OL,y}}\mathbf{x} + \frac{D_{OL,y}}{I + D_{OL,y}}r \tag{2.10b}$$
$$u = \left(C_{OL,u} - D_{OL,u}\frac{C_{OL,y}}{I + D_{OL,y}}\right)\mathbf{x} + \left(D_{OL,u} - D_{OL,u}\frac{D_{OL,y}}{I + D_{OL,y}}\right)r \tag{2.10c}$$

2.2 Model Predictive Control Strategy

Model predictive control theory will be briefly presented in this section. The core principle of such control lies in solving optimization problem consisting of quadratic cost function with linear constraints (2.11a). In this formulation N is the prediction horizon, matrices A, B, C, D are state space model matrices in discrete time Camacho and Bordons (2007); Prasath et al. (2010). $\mathbb{X}$, $\mathbb{Y}$ and $\mathbb{U}$ are constraints imposed on states and control actions.

$$\Phi = \min \frac{1}{2}\sum_{k=0}^{N} ||r_k - y_k||_Q^2 + \frac{1}{2}\sum_{k=0}^{N-1} ||u_k||_R^2 \tag{2.11a}$$
$$s.t. \quad x_{k+1} = Ax_k + Bu_k \tag{2.11b}$$
$$y_k = Cx_k + Du_k \tag{2.11c}$$
$$x_k \in \mathbb{X} \quad k = 1 \dots N-1 \tag{2.11d}$$
$$y_k \in \mathbb{Y} \quad k = 1 \dots N-1 \tag{2.11e}$$
$$u_k \in \mathbb{U} \quad k = 1 \dots N-1 \tag{2.11f}$$

Also for the purpose of solving such opimization problem, (2.11a) needs to be rewritten into QP form Boyd and Vandenberghe (2009). Quadratic programming in general is expressed in (2.12a). Theory explaining solving such mathematical problem can be found in e.g. Nocedal and Wright (1999).

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}v^T H v + g^T v + r && (2.12a)\\ \text{s. t.} \quad & Cv \preceq d && (2.12b)\\ & Av = b && (2.12c) \end{aligned}$$

Cost function (2.11a) in case of MPC formulation consists of penalization of control inputs moves u and difference between reference r and output y at each sample time. In order to alleviates difficulties with substracting steady state values in MATLAB implementation, instead of u_k will be penalized $\triangle u_k$ (2.13). In must be noted, that there are several approaches of MPC formulations which are presented in detail in Maciejowski (2002) or in Mancuso and Kerrigan (2011).

$$\triangle u_k = u_k - u_{k-1} \tag{2.13}$$

As it is suggested in Prasath and Jørgensen (2009) cost function (2.11a) is converted into (2.12a), where actual control input u will become an optimization variable. In order to reformulate the objective function, equality constraints are incorporated into the objective function. Mentioned equality constraints is the state space model in discrete time (2.15), upon which the MPC is based. Using stacked vector notation cost function (2.11a) can be expressed as a weighted least square problem (2.14)

$$U^\star = \min \ \frac{1}{2}||Y - R||_Q^2 + \frac{1}{2}||\triangle U||_R^2 \tag{2.14}$$

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \quad k = 0 \ldots N && (2.15a)\\ y_k &= Cx_k + Du_k \quad k = 0 \ldots N && (2.15b) \end{aligned}$$

State space model can be formulated for each sample time for the prediction horizon as it is suggested in (2.16) and (2.17).

$$\begin{aligned} x_{k+2} &= Ax_{k+1} + Bu_{k+1}\\ &= A(Ax_k + Bu_k) + Bu_{k+1}\\ &= A^2x_k + ABu_k + Bu_{k+1} \end{aligned} \tag{2.16}$$

$$\begin{aligned} y_{k+1} &= Cx_{k+1} + Du_{k+1}\\ &= C(Ax_k + Bu_k) + Du_{k+1}\\ &= CAx_k + CBu_k + Du_{k+1} \end{aligned} \tag{2.17}$$

Sequence yielding from state space model formulations can be combined into matrix formulation (2.18).

$$Y = \Psi x_k + \Gamma U \tag{2.18}$$

in which:

$$U = \begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ u_{k+3} \\ \vdots \\ u_{k+N-1} \end{bmatrix} \qquad Y = \begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ \vdots \\ y_{k+N-1} \end{bmatrix} \tag{2.19}$$

$$\Psi = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N-1} \end{bmatrix} \tag{2.20}$$

$$\Gamma = \begin{bmatrix} D & 0 & 0 & 0 & \cdots & 0 \\ CB & D & 0 & 0 & \cdots & 0 \\ CAB & CB & D & 0 & \cdots & 0 \\ CA^2B & CAB & CB & D & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ CA^{N-2}B & CA^{N-3}B & CA^{N-4}B & \cdots & CAB & D \end{bmatrix} \tag{2.21}$$

Since the optimization variable is actual control input u, relation between control moves $\triangle u$ and u must be found. Set of equations from (2.22) to (2.24) explain the relation. Note, that matrix I_u is identity matrix of proper size.

$$\begin{bmatrix} \triangle u_k \\ \triangle u_{k+1} \\ \triangle u_{k+2} \\ \vdots \\ \triangle u_{k+N-1} \end{bmatrix} = \begin{bmatrix} u_k - u_{k-1} \\ u_{k+1} - u_k \\ u_{k+2} - u_{k+1} \\ \vdots \\ u_{k+N-1} - u_{k+N-2} \end{bmatrix} \tag{2.22}$$

$$\begin{bmatrix} \triangle u_k \\ \triangle u_{k+1} \\ \triangle u_{k+2} \\ \vdots \\ \triangle u_{k+N-1} \end{bmatrix} = \begin{bmatrix} I_u & 0 & 0 & 0 & 0 \\ -I_u & I_u & 0 & 0 & 0 \\ 0 & -I_u & I_u & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & -I_u & I_u \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ \vdots \\ u_{k+N-1} \end{bmatrix} - \begin{bmatrix} I_u \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_{k-1} \tag{2.23}$$

$$\triangle U = \Lambda U - I_{1,u} u_{k-1} \tag{2.24}$$

Let us define part of objective function (2.14) as follows in (2.25). Least square problem can be then elaborated as is shown in (2.26) through (2.28).

$$U^{\star} = \min \ \left(\frac{1}{2} ||R - Y||_Q^2 + \frac{1}{2} ||\triangle U||_R^2 \right) = \min \ \ (\Phi_Y + \Phi_U) \tag{2.25}$$

$$Y - R = (\Gamma U + \Phi x_k) - R = \Gamma U - (R - \Phi x_k) = \Gamma U - c \tag{2.26}$$

$$\Phi_Y = \frac{1}{2} U^T \Gamma^T Q \Gamma U - (\Gamma^T Q c)^T U + \frac{1}{2} c^T c \tag{2.27}$$

$$\Phi_U = \frac{1}{2} U^T \Lambda^T R \Lambda U - (\Lambda^T R I_{1,u} u_{k-1})^T U \tag{2.28}$$

When combining above presented expression, curvature matrix H and vector of first order coefficients g is obtained as shown in (2.29) and (2.30).

$$H = \Gamma^T Q \Gamma + \Lambda^T R \Lambda \tag{2.29}$$

$$\begin{aligned} g &= -\Gamma^T Q c - \Lambda^T R I_{1,u} u_{k-1} = \\ &= \Gamma^T Q R + \Gamma^T \Phi x_k - \Lambda^T R I_{1,u} u_{k-1} \end{aligned} \tag{2.30}$$

Unconstrained case of model predictive control strategy is now formulated. This corresponds to objective function from (2.12a). In next subsections formulation of inequality constraints will be presented.

2.2.1 Input Constraints

Including constraints in the MPC design is the main advantage. In this subsection matrices inequality constraints are formulated. These constraints are related to control inputs. Two sets of constraints are considered. First constraints for actual control inputs are defined, next bounds on control inputs are included as well. In (2.31) constraints are presented for each sample instance k.

$$\begin{array}{ccccc} u_{min} & \leq & u_k & \leq & u_{max} & \qquad k = 0 \dots N-1 \\ \triangle u_{min} & \leq & \triangle u_k & \leq & \triangle u_{max} & \qquad k = 0 \dots N-1 \end{array} \tag{2.31}$$

Since many QP solvers have arguments for "lower" and "upper" bound, constraints on actual control action can be simply stacked into vector as shown in (2.32).

$$U_{\min} = \begin{bmatrix} u_{\min} \\ u_{\min} \\ \vdots \\ u_{\min} \end{bmatrix} \qquad U_{\max} = \begin{bmatrix} u_{\max} \\ u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix} \tag{2.32}$$

In order to incorporate into MPC desing constraints on control moves, formulation resembling inequality constraints from (2.12b) is required. By referring to (2.24) matrix formulation of control move constraints can be expressed (2.33). Solvers often require formulation as shown in (2.34).

$$\triangle U_{\min} + I_{1,u}u_{k-1} \leq \Lambda U \leq \triangle U_{\max} + I_{1,u}u_{k-1} \tag{2.33}$$

$$\begin{bmatrix} \Lambda \\ -\Lambda \end{bmatrix} U \leq \begin{bmatrix} \triangle U_{\max} + I_{1,u}u_{k-1} \\ -\triangle U_{\min} + I_{1,u}u_{k-1} \end{bmatrix} \tag{2.34}$$

2.2.2 Output Constraints

Following equations will show how to set up and MPC with output constraints. Two types of constraints are considered in this project. In (2.42) are shown constraints on actual output at given sample time y_k and output rate $\triangle y_k$. As it was mentioned in previous section, stacked vector notation and matrix formulation are necessary to express in order to solve the QP problem with constraints in e.g. MATLAB . For actual output Y is the matrix notation known from (2.18).

$$\begin{array}{ccccc} y_{\min} & \leq & y_k & \leq & y_{\max} \\ \triangle y_{k,\min} & \leq & y_k - y_{k-1} & \leq & \triangle y_{k,\max} \end{array} \tag{2.35}$$

Next set of equations will show how to formulate matrices for QP optimization with constraints on output difference $\triangle y_k$. In (2.42) through (2.38) is expressed evolution at given sample instances.

$$\triangle y_k = y_k - y_{k-1} = (Cx_k + Du_k) - (Cx_{k-1} + Du_{k-1}) \tag{2.36}$$

$$\begin{aligned} \triangle y_{k+1} = {} & y_{k+1} - y_k = \\ & = (Cx_{k+1} + Du_{k+1}) - (Cx_k + Du_k) = \\ & = CAx_k + CBu_k + Du_{k+1} - Cx_k - Du_k = \\ & = (CA - C)x_k + (CB - D)u_k + Du_{k+1} \end{aligned} \tag{2.37}$$

$$\begin{aligned} \triangle y_{k+2} = {} & y_{k+2} - y_{k+1} = \\ & = (Cx_{k+2} + Du_{k+2}) - (Cx_{k+1} + Du_{k+1}) = \\ & = C\left(A^2x_k + CABu_k + CBu_{k+1}\right) + Du_{k+1} - C(Ax_k + Bu_k) - Du_{k+1} \\ & = (CA^2 - CA)x_k + (CAB - CB)u_k + (CB - D)u_{k+1} + Du_{k+1} \end{aligned} \tag{2.38}$$

Based on presented evolution, matrix formulation can be derived as shown in (2.39) and (2.40). Note that these matrices resemble prediction equation from (2.18).

$$\Psi_Y = \begin{bmatrix} C \\ CA - C \\ CA^2 - CA \\ CA^3 - CA^2 \\ \vdots \\ CA^{N-1} - CA^{N-2} \end{bmatrix} \tag{2.39}$$

$$\Gamma_Y = \begin{bmatrix} D & 0 & 0 & 0 & \cdots & 0 \\ CB - D & D & 0 & 0 & \cdots & 0 \\ CAB - CB & CB - D & D & 0 & \cdots & 0 \\ CA^2B - CAB & CAB - CB & CB - D & D & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ CA^{N-2}B - CA^{N-1}B & CA^{N-3}B - CA^{N-2}B & CA^{N-4}B - CA^{N-3}B & \cdots & CB - D & D \end{bmatrix} \tag{2.40}$$

Final equation expressing relation between optimization variable U and stacked output rate vector $\triangle Y$ is shown in (2.41). When previously derived matrix equations are inserted into stacked vector formulation of output constraints in (2.37), inequality constraints shown in (2.43) are expressed.

$$\triangle Y = \Psi_Y x_k + \Gamma_Y U - I_{1,y}\left(C x_{k-1} + D u_{k-1}\right) \tag{2.41}$$

$$\begin{array}{ccccc} Y_{\min} & \leq & Y & \leq & Y_{\max} \\ \triangle Y_{\min} & \leq & \triangle Y & \leq & \triangle Y_{\max} \end{array} \tag{2.42}$$

$$\begin{bmatrix} \Gamma \\ -\Gamma \\ \Gamma_Y \\ -\Gamma_Y \end{bmatrix} U \leq \begin{bmatrix} Y_{max} - \Psi x_k \\ -\left(Y_{min} - \Psi x_k\right) \\ \triangle Y_{max} - \Psi_Y x_k + I_{1,y}\left(C x_{k-1} + D u_{k-1}\right) \\ -\left(\triangle Y_{min} - \Psi_Y x_k + I_{1,y}\left(C x_{k-1} + D u_{k-1}\right)\right) \end{bmatrix} \tag{2.43}$$

Note, that (2.43) is consistent with definition of QP problem with constraints from (2.12b). Reader can also realize, that formulating output rate constraints matrices is similar to formulation of control move constraints in previous section.

Chapter 3

Case Study of Boiler-Turbine System

3.1 Boiler-Turbine System

Model of the boiler-turbine unit (Fig. 3.1) considered in this project consist of three control inputs, three states and three outputs.

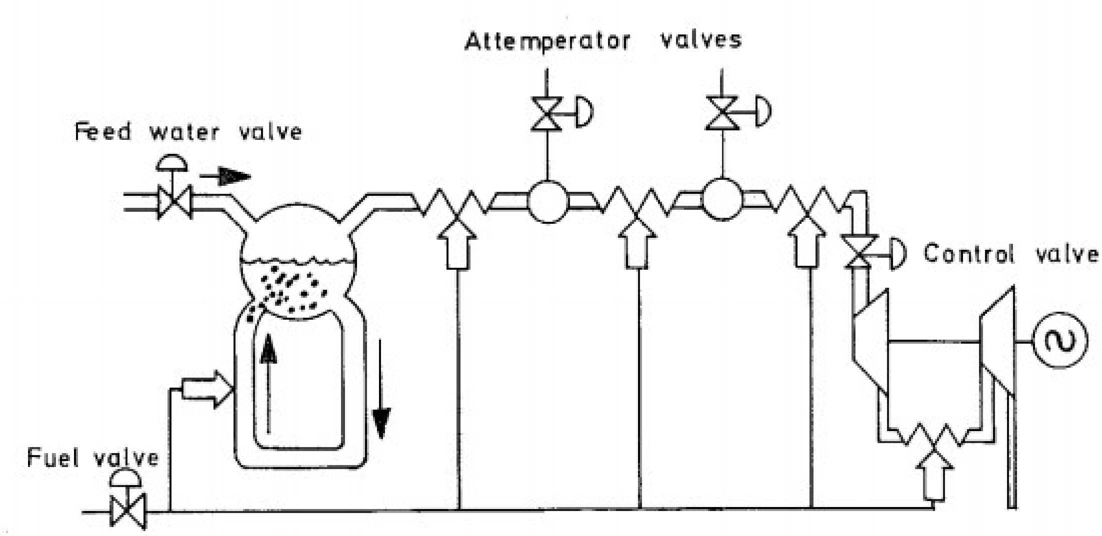

Figure 3.1: Diagram of boiler-turbine system

Control inputs to the system are positions of three valves. Fuel flow valve position u_F, steam control valve position u_S and feed water valve position u_W. State variables are, drum pressure p, power output P_0 and fluid density inside the boiler ρ_f. Outputs from this model are drum pressure p, power output P_0 and difference in level of water inside the boiler X_w. Values presented in Tab. 3.1 are related to 100% of nominal power of th boiler-turbine unit.

Table 3.1: Steady state value of parameters

Parameter	Value	Unit
u_F^s	0.34	%
u_S^s	0.69	%
u_W^s	0.435	%
x_1^s	10.59	MPa
x_2^s	66.65	MW
x_3^s	428	$kg.m^{-3}$
X_w^s	0	m

3.1.1 Mathematical Model of Boiler System

As it was previously mentioned mathematical model used in this project is taken from Åström and Bell (2000) and Åström and Eklund (1972). A 3rd order non-linear model of boiler-turbine unit is considered as shown in (3.1).

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -0.0018 u_2 p^{\frac{9}{8}} + 0.9 u_1 - 0.15 u_3 \tag{3.1a}$$
$$\frac{\mathrm{d}P_0}{\mathrm{d}t} = (0.073 u_2 - 0.016) p^{\frac{9}{8}} - 0.1 P_0 \tag{3.1b}$$
$$\frac{\mathrm{d}\rho_f}{\mathrm{d}t} = \frac{141 u_3 - (1.1 u_2 - 0.19)\, p}{85} \tag{3.1c}$$

Using stationary values from Tab. 3.1 linearization using first order Taylor expansion takes place. In (3.2) and (3.3) is shown state space model in continuous time of boiler-turbine system. Reader can notice in system matrix, that elements on diagonal are negative except the last element. This suggest that step responses will be stabilize in first two states, but the last one will not stabilize. This must be taken into consideration while designing the MPC governor.

$$A = \begin{bmatrix} -2.509 \cdot 10^{-3} & 0 & 0 \\ 6.94 \cdot 10^{-2} & -0.1 & 0 \\ -6.69 \cdot 10^{-3} & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.9 & -0.349 & -0.15 \\ 0 & 14.155 & 0 \\ 0 & -1.389 & 1.659 \end{bmatrix} \tag{3.2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6.34 \cdot 10^{-3} & 0 & 4.71 \cdot 10^{-3} \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.253 & 0.512 & -0.014 \end{bmatrix} \tag{3.3}$$

Naturally as on many technological process, on this one are also imposed constraints. Constraints (3.4a) are related to valves opening/closing speed and position. Later in this thesis will be shown, that PI controllers suggested by Dimeo and Lee (1995) are not able to satisfy these constraints. In chapter 4 will be shown, that if saturation and rate limiters are introduced in the decoupled

PI controller, it will destabilize entire closed loop.

$$-0.007\ \%/s \leq \frac{\mathrm{d}u_F}{\mathrm{d}t} \leq 0.007\ \%/s \tag{3.4a}$$

$$-2\ \%/s \leq \frac{\mathrm{d}u_S}{\mathrm{d}t} \leq 0.02\ \%/s \tag{3.4b}$$

$$-0.05\ \%/s \leq \frac{\mathrm{d}u_W}{\mathrm{d}t} \leq 0.05\ \%/s \tag{3.4c}$$

3.1.2 Step Responses

Behaviour of the system can be inspected by looking at step responses. Figures 3.2, 3.3 and 3.4 show responses to respective inputs u_F, u_S and u_W. If step change is made in one input, other remains unchanged. To keep the step responses close to reality, 10% is added into each respective control input at time 500. From these step responses reader can understood the behaviour of the system, which states are stable, how the outputs reacts when positive step change is made, and how fast the system is. Based on these step responses controller can be properly tuned.

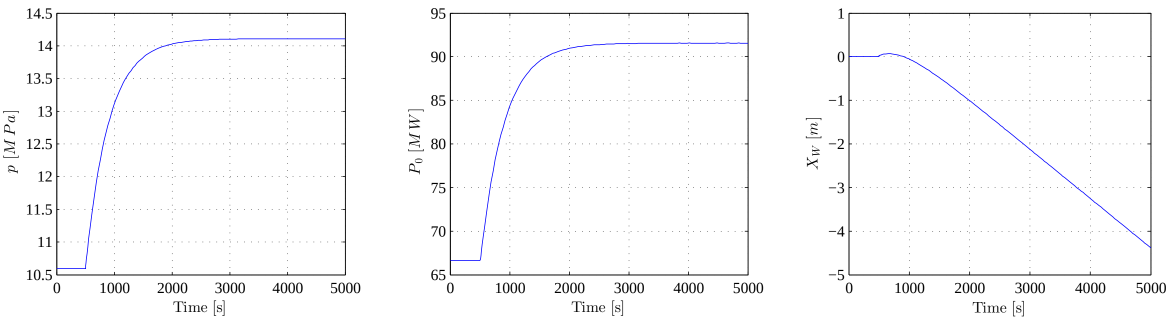

Figure 3.2: Step responses to 10% change in u_F

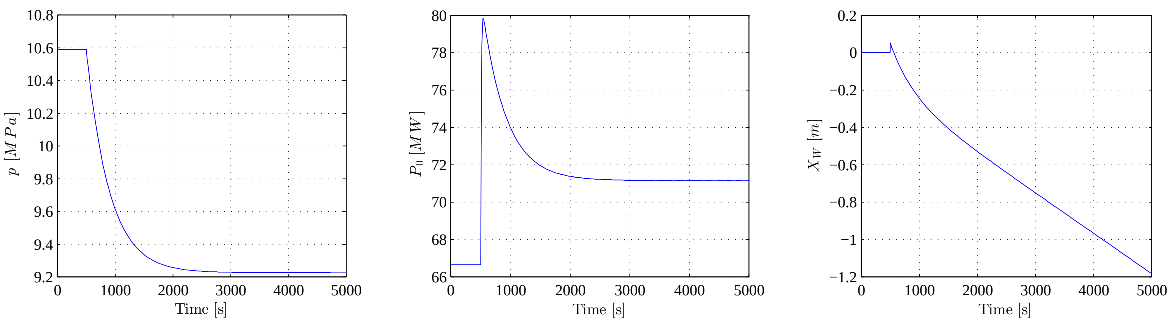

Figure 3.3: Step responses to 10% change in u_S

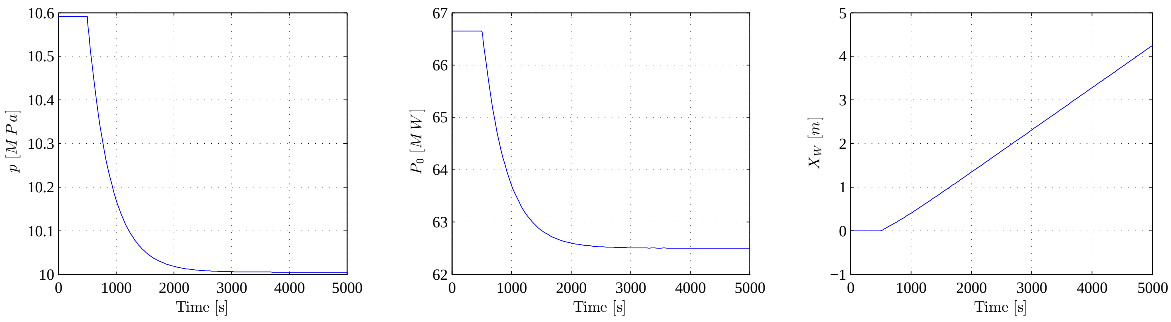

Figure 3.4: Step responses to 10% change in u_W

3.2 Controller design

3.2.1 State Space Model of Closed Loop System

Main objective of this project was to investigate and design model predictive control strategy on system, which includes closed loop controller. In this section will be presented state space model of boiler-turbine system, which includes three decoupled PI controllers. This setup is taken from Dimeo and Lee (1995).

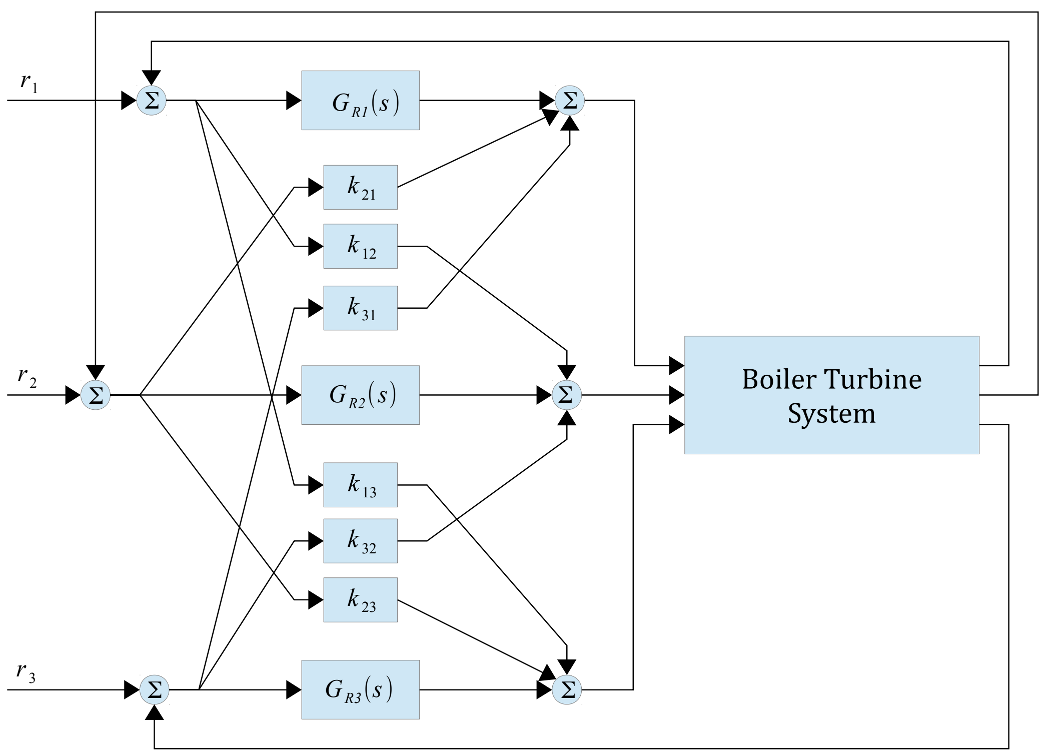

Figure 3.5: Feedback control of Boiler-turbine system with decoupled PI controllers

State Space Model of PI Controller

Closed loop system with PI controllers is shown on Fig. 3.5. PI controller is often represented by transfer functions, as shown in (3.5). In order to obtain the state space model of the controller as suggested in section 2.1, these transfer functions are converted into state space model and then merged together. Complete state space model of the controller is shown in (3.8) and (3.9)

Since PI controllers itself are not dependent on each other, respective state space models of transfer function can be easily merged into one state space model, where matrices A, B, C will be diagonal matrices. On the other hand, direct matrix D will have non zero element on sub-diagonal, due to the direct connections between control errors and control inputs to the system. On the Fig. 3.5 it is denoted by gains k_{21}, k_{12} etc. Reader can notice, that system matrix A is a zero matrix. This is caused by free integrators in PI controllers.

$$G_{R1}(s) = \frac{11.1185s + 0.0033}{s} \tag{3.5}$$

$$G_{R2}(s) = \frac{0.004s + 0.0093}{s} \tag{3.6}$$

$$G_{R3}(s) = \frac{1.1631s + 0.0186}{s} \tag{3.7}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.8}$$

$$C = \begin{bmatrix} 0.0033 & 0 & 0 \\ 0 & 0.0093 & 0 \\ 0 & 0 & 0.0186 \end{bmatrix} \quad D = \begin{bmatrix} 11.1185 & 0.0468 & 0.0842 \\ 0.0292 & 0.0040 & 0.0842 \\ 0.1344 & 0.0875 & 1.1631 \end{bmatrix} \tag{3.9}$$

State Space Model of Closed Loop System

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0030 & -0.0032 & -0.0028 & -0.0025 & 0 & 0 \\ 0 & 0.1316 & 0 & 0.0694 & -0.1000 & 0 \\ 0 & -0.0129 & 0.0292 & -0.0067 & 0 & 0 \end{bmatrix} \tag{3.10}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 9.9763 & 0.0276 & -0.1281 \\ 0.4133 & 0.0566 & 1.1919 \\ 0.1703 & 0.1317 & 1.7080 \end{bmatrix} \tag{3.11}$$

$$C = \begin{bmatrix} 0.0033 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0093 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0186 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.0008 & 0.0048 & -0.0003 & 0.0063 & 0 & 0.0047 \end{bmatrix} \tag{3.12}$$

$$D = \begin{bmatrix} 11.1185 & 0.0468 & 0.0842 \\ 0.0292 & 0.0040 & 0.0842 \\ 0.1344 & 0.0875 & 1.1631 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2.8260 & 0.0127 & 0.0481 \end{bmatrix} \tag{3.13}$$

3.2.2 Discretization

Discretization of the continuous time model is essential part of control design. Sample time effects all levels of control design as well as the performance of the controlled system. On process side, sample time must be chosen in that fashion, that important dynamics of the system can be captured in discrete time model. This means, that as highest as possible sampling frequency should be chosen.

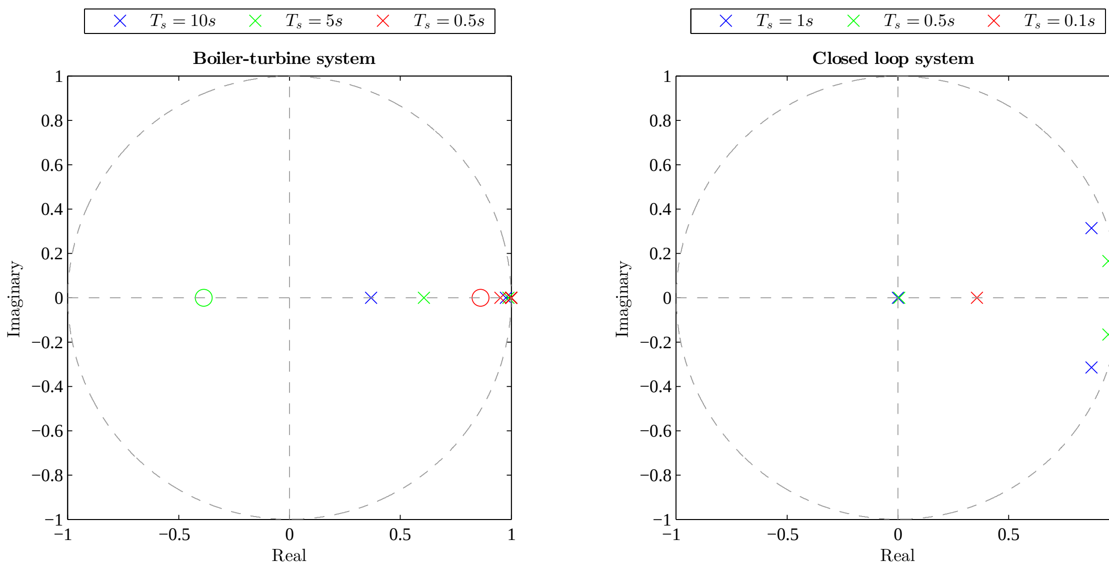

Figure 3.6: Pole-Zero placement of discrete time system

On the other hand, increasing sampling frequency cause tuning issues in MPC design. Sampling time is affecting pole and zero placement in unit disc, which needs to be considered in tuning of the controller. Increasing sampling time moves poles and zeros closer to instability region. On Fig. 3.6 are shown poles and zeros for original boiler-turbine system and closed loop system with

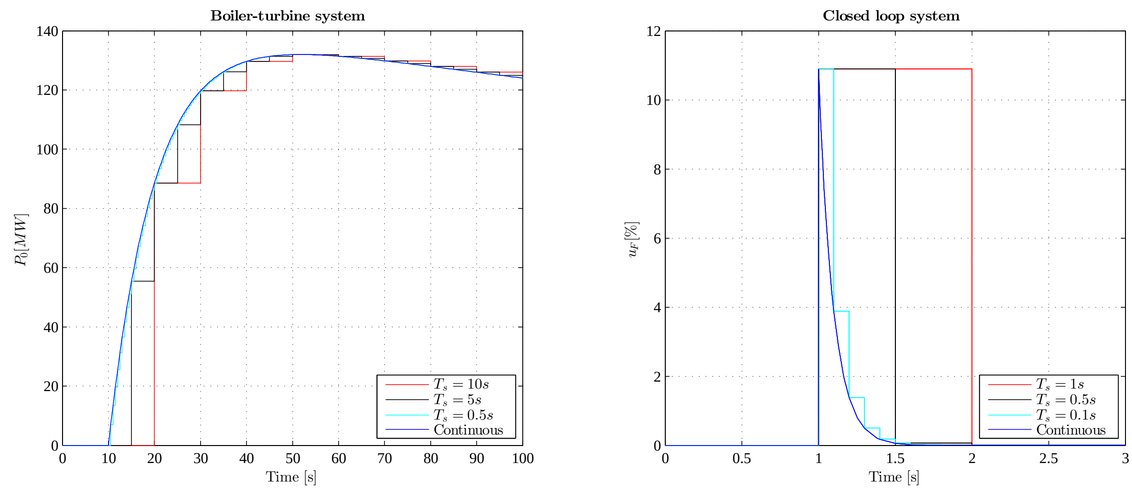

Figure 3.7: Fastest step responses - comparison between C-Time and D-Time models

different sampling times. Since closed loop system contains PI controllers, different sampling frequencies must be considered in order to capture dynamics of the continuous time process. It also must be noted that low sampling time makes implementing MPC difficult, since QP optimization is very demanding on computation resources.

It is also necessary to realize, that prediction horizon must be chosen in accordance with sampling time. If low sampling time is chosen, high prediction horizon must be set, so same performance of the control system can be achieved. Increasing prediction horizon have significant impact on scale of QP optimization which needs to be performed every sample.

Good way to choose sample time is to evaluate fastest step response in the controlled process. On Fig. 3.7 are shown step responses of discrete systems. Based on this step responses sampling time of 5 seconds has been chosen for original boiler-turbine model discretization. For closed loop system discretization, sample time of 0.5s is chosen. It seem better to choose even lover sample time, but then formulated optimization problem will become too complex to solve using available solvers in MATLAB in reasonable time. On the other hand, satisfactory performance has been achieved with this sampling frequency. All other transfer function relating inputs are actual outputs (p, P_o, X_W) have significantly lower time constants.

3.2.3 Tuning of MPC Regulator

Tuning of model predictive controller lies in specifying matrices Q and R from (2.11a), and setting the prediction horizon. Since model of the boiler-turbine system is in open loop case unstable, rather long horizon needs to be chosen. Attention must be also paid to the fact, that when prediction horizon is increased, complexity of QP problem rises, which affects the time of calculation of control inputs. In this project, prediction horizon of 20 second has proven to be enough to achieve satisfactory performance.

Many applications of MPC requires to introduce some elaborate scaling of the state space model and control inputs however, linear model considered in this work has already scaled inputs. This has significantly reduce the tuning issues. When simulations has been performed with identity weighting matrices reasonable performance has been achieved.

Tuning of MPC for original boiler-turbine system

$$u = \begin{bmatrix} u_F & u_S & u_W \end{bmatrix}^T \quad y = \begin{bmatrix} p & P_0 & X_W \end{bmatrix}^T \tag{3.14}$$

$$Q = \operatorname{diag}\left([200,\ 0.01,\ 0.05]\right) \tag{3.15}$$

$$R = \operatorname{diag}\left([0.01,\ 0.1,\ 0.01]\right) \tag{3.16}$$

Tuning of MPC governor for closed loop system

$$u = \begin{bmatrix} r_P & r_{P_0} & r_{X_W} \end{bmatrix}^T \quad y = \begin{bmatrix} u_F & u_S & u_W & p & P_0 & X_W \end{bmatrix}^T \tag{3.17}$$

$$Q = \operatorname{diag}\left([0,\ 0,\ 0,\ 100,\ 0.1,\ 1]\right) \tag{3.18}$$

$$R = \operatorname{diag}\left([1,\ 0.01,\ 1]]\right) \tag{3.19}$$

Chapter 4

Simulations and Comparisons

Application of presented theory is demonstrated on two simulation scenarios. First case is with no future reference profile. Such simulation will shown how MPC operates similarly to PI controller. Second scenario is when full proficiency of MPC is used. This means that future reference profile is provided to the model predictive controller. At last simulation with constrained PI controllers will be shown. In all cases step change is realized on power output reference, while this seem most reasonable to test the effects of the controller.

For each simulation scenario are presented figures showing outputs, control inputs, control inputs rates, control effort and reference supplied to PI controllers. Three cases of simulations are compared. Simulation with unconstrained PI controllers, simulation where PI controllers are replaced by MPC regulator and simulation with MPC governor. In all cases comparison with supplied PI controller is made. Reader can then notice, that either strategy with employed MPC achieve better performance than PI controller.

Most distinctive figure, which will shown advantages of each tested control setting in the control effort figure. Control effort explained as how much the controller must change the input to the system to achieve desired setpoint. In general control effort can be defined as shown in (4.1). However for the purpose of visualizing the results, cumulative sum can be used to achieve this goal. Quality of control is also validated using standard IAE criterion (4.2).

$$E = \sum_{k=1}^{k_f} |\triangle u_k| \tag{4.1}$$

$$\text{IAE} = \int_0^T |(r(t) - y(t))|\, dt \Rightarrow \text{IAE} = \sum_{k=1}^{k_f} |(r_k - y_k)| \tag{4.2}$$

where k_f represents final sample in given simulation.

Figures with simulations results are organized as follows: first are shown outputs, control inputs, control moves, control effort and reference for PI controllers. **Black** colour is reserved for simulation result with PI controller, **blue** is for PI replaced by MPC and **magenta** is for results with employed MPC governor. Reference is highlighted by **green**.

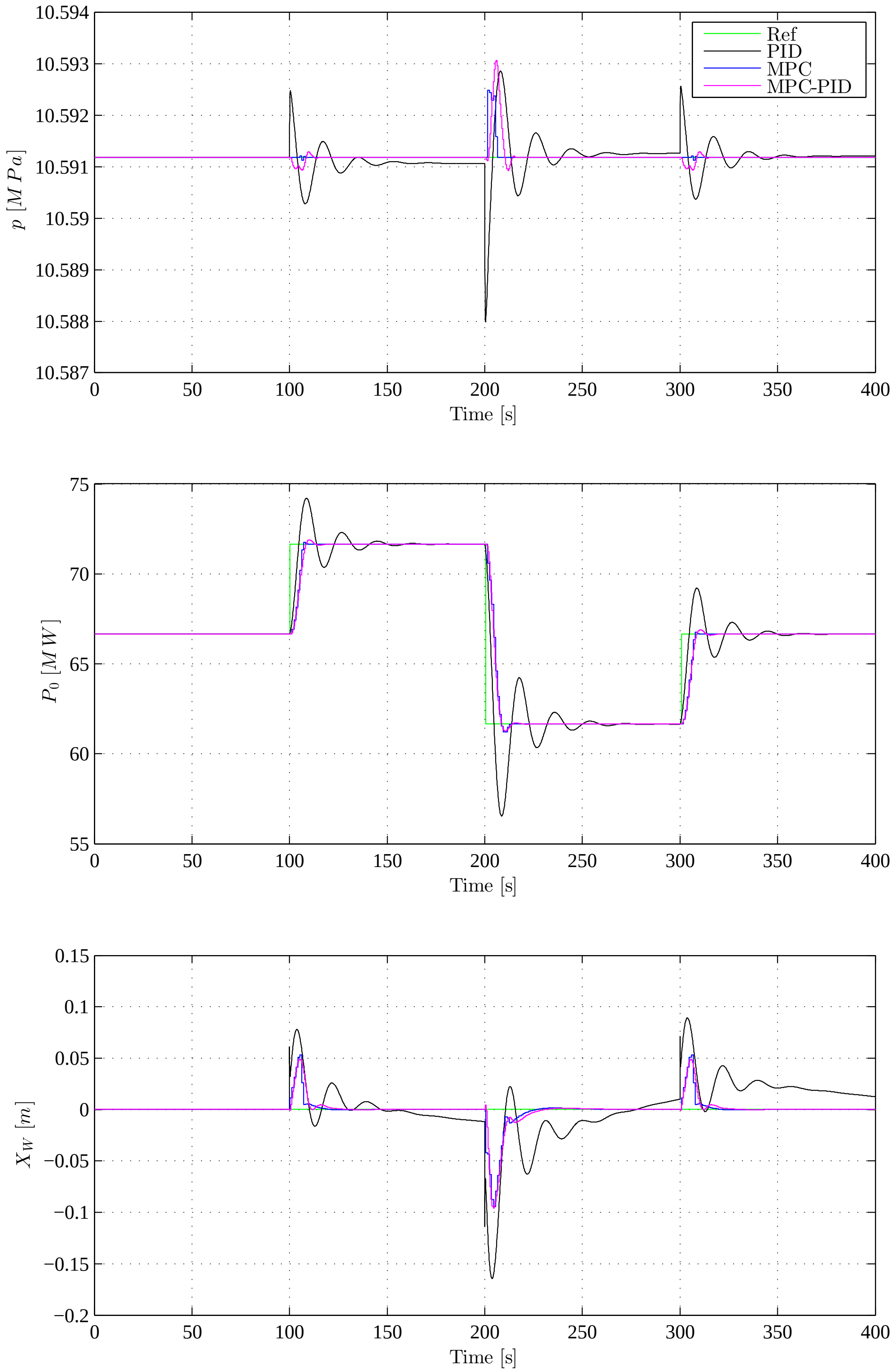

Figure 4.1: Outputs (with no reference profile)

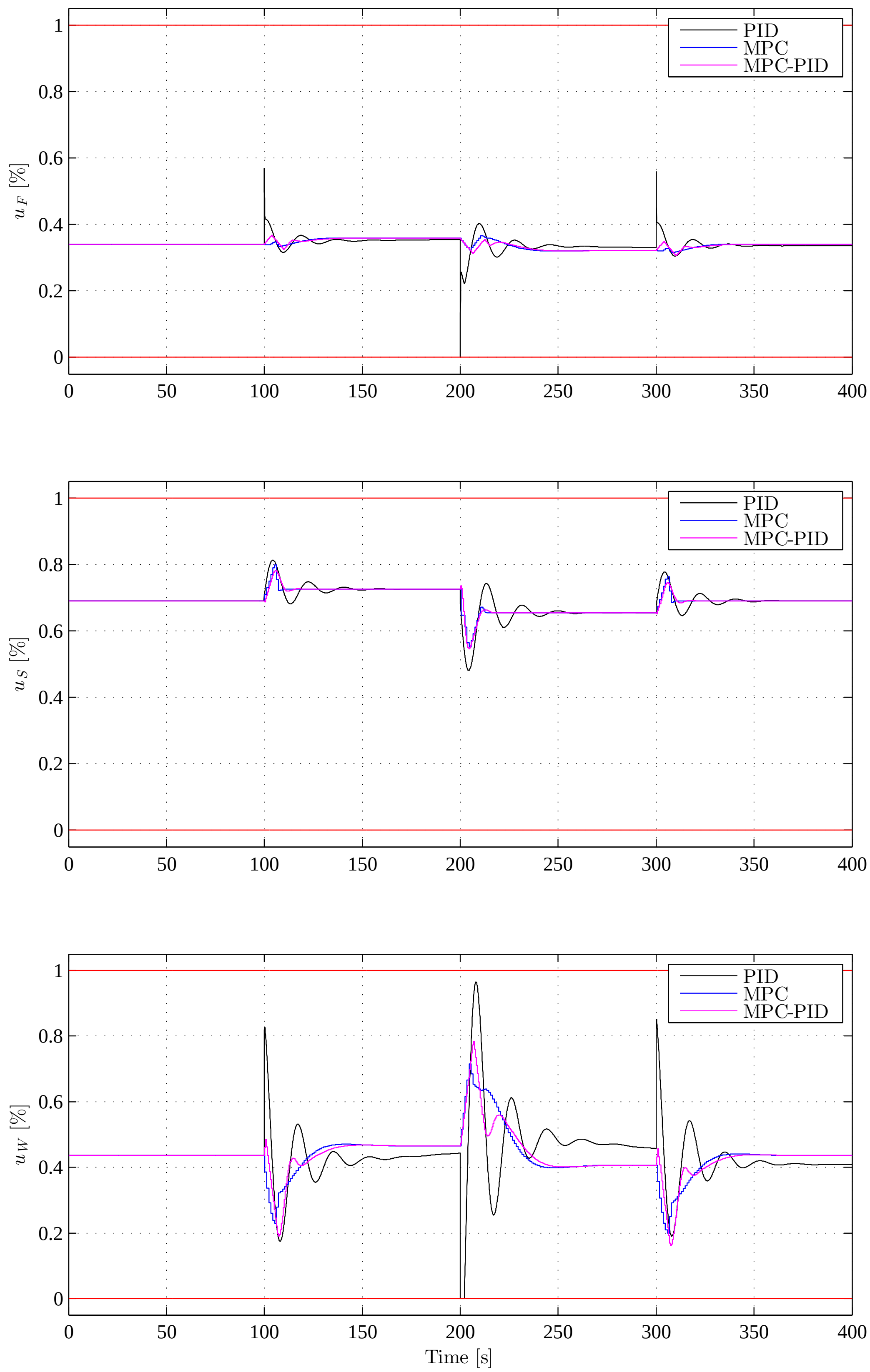

Figure 4.2: Inputs (with no reference profile)

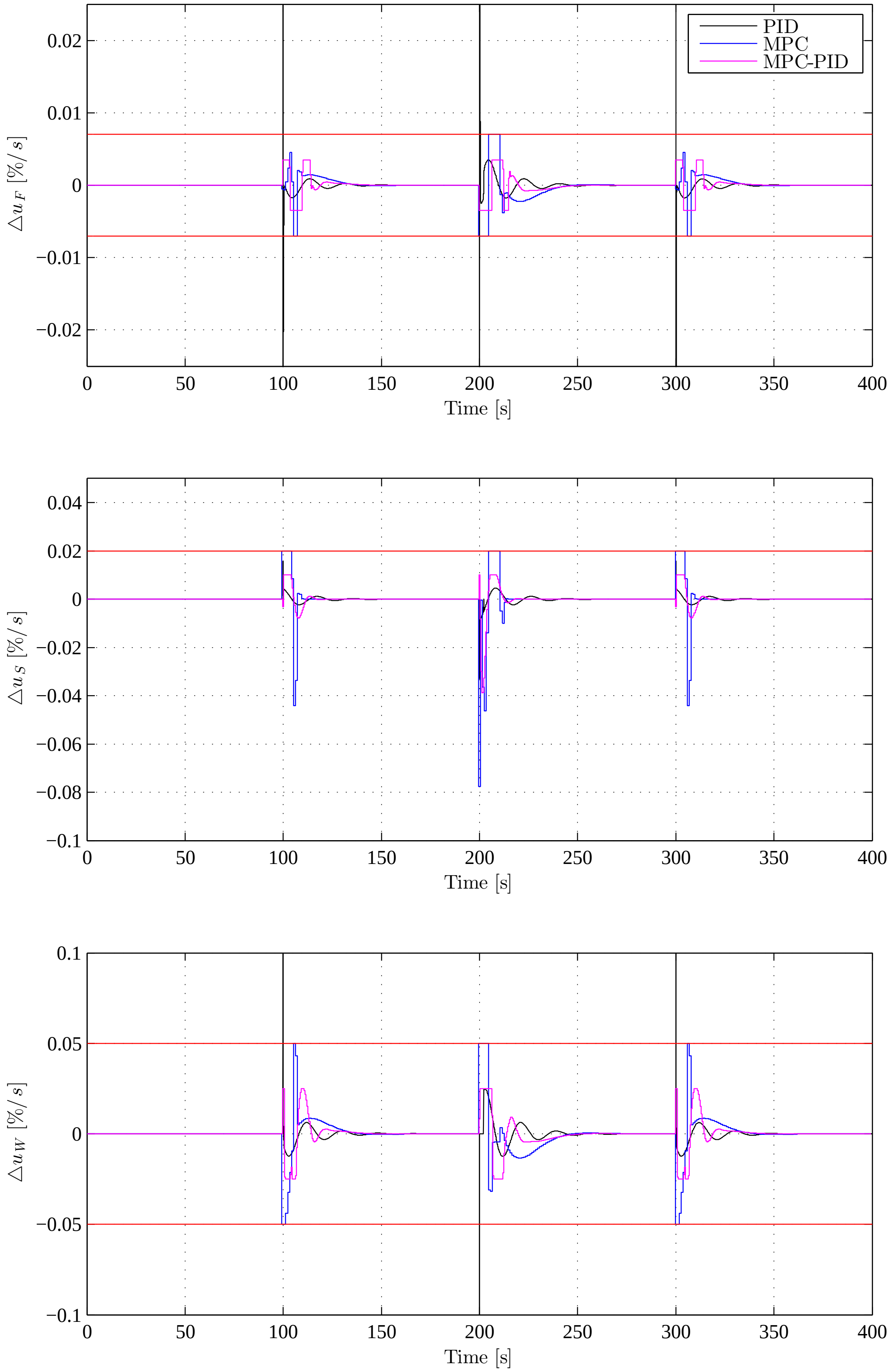

Figure 4.3: Control moves (with no reference profile)

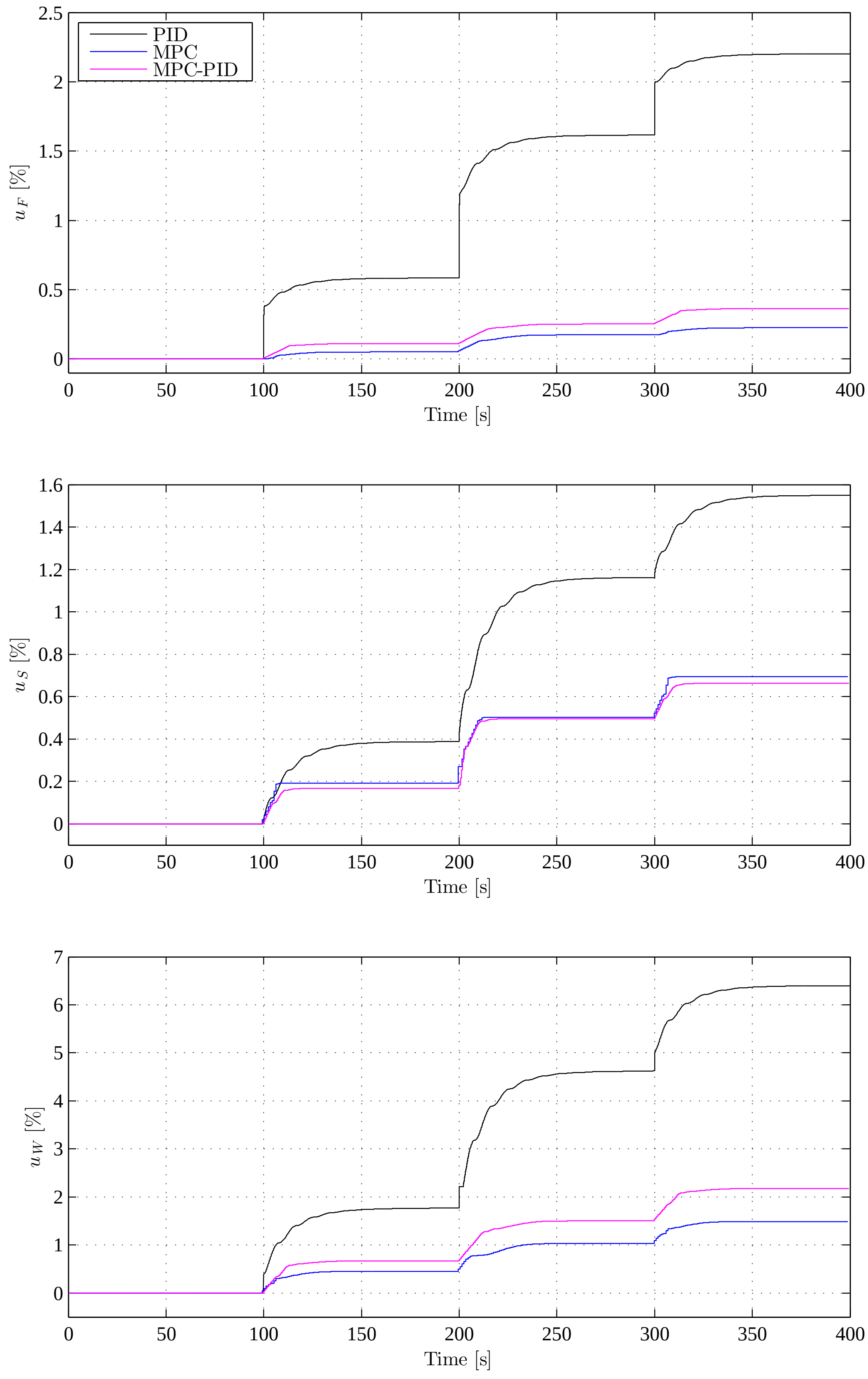

Figure 4.4: Control effort (with no reference profile)

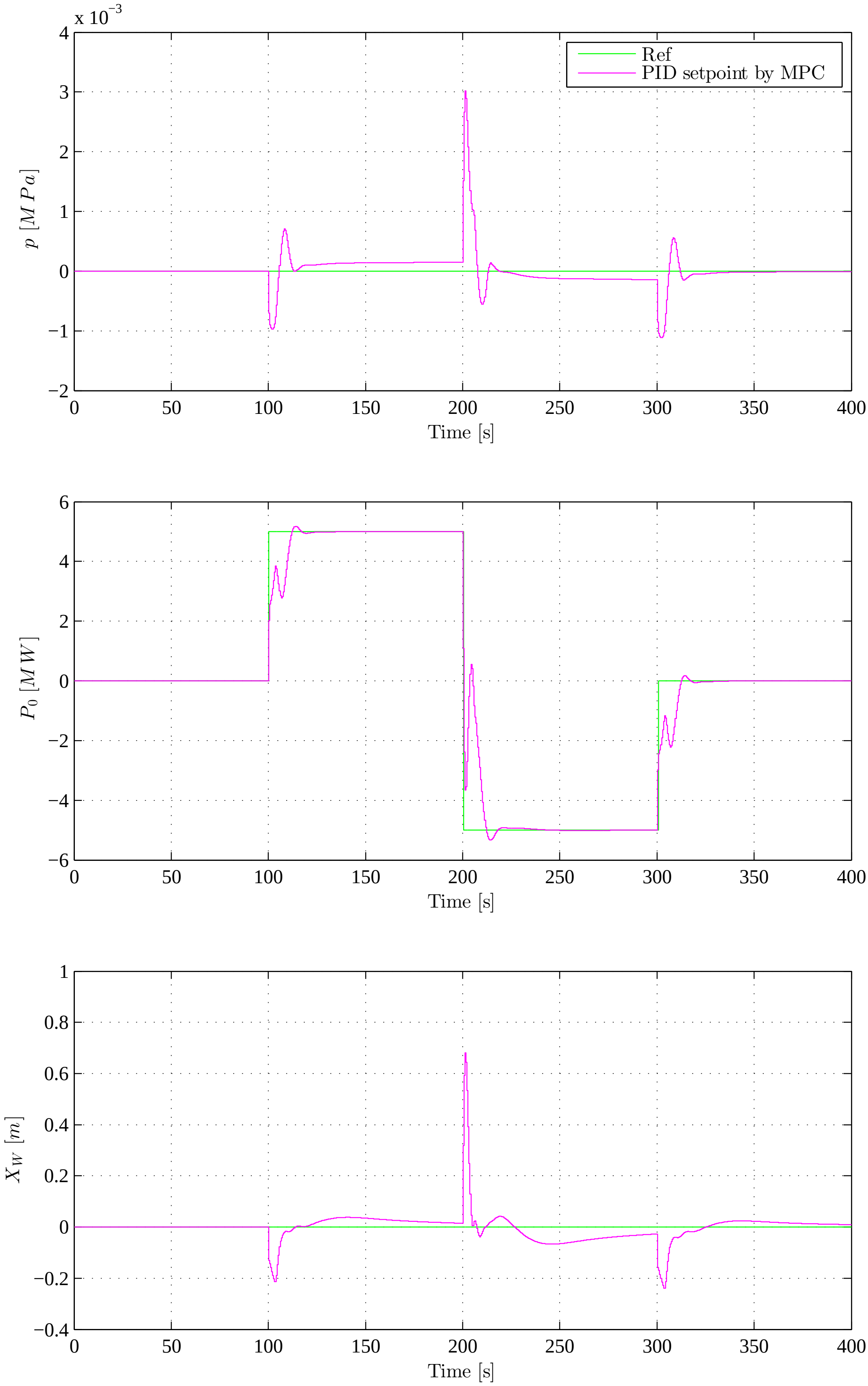

Figure 4.5: Reference profile for PI controller (with no reference profile)

Even if MPC is limited by not supplying future reference profile to it, there can be seen significant improvement in every aspect of control. Outputs (Fig. 4.1) are tracking reference values with great precision, while all constraints are obeyed (Fig. 4.2 and 4.3). On Fig. 4.4 reader can see, that if MPC is used controller has to "move" much less then if unconstrained PI controllers are implemented. Filtered reference for PI controllers in case of MPC governor is shown on Fig. 4.5.

Following tables show evaluated control quality criteria. Reader should focus on "scaled values" part of the table, where it can clearly be seen how much model predictive strategy improves efficiency of the process. Scaling has been done in such fashion, that results using PI controllers has been treated as 100%.

Table 4.1: IAE - quality criterion

		Absolute values			Scaled values		
		p $[MPa.s]$	P_0 $[MW.s]$	X_w $[kg.m^{-3}.s]$	p [%]	P_0 [%]	X_w [%]
	PID	0.6328	1841.8619	61.6829	100.00	100.00	100.00
No Ref	MPC	0.0055	88.3714	1.2628	0.87	4.79	2.04
	MPC-PID	0.0254	194.7641	4.9055	4.01	10.57	7.95
Ref	MPC	0.0010	30.6516	1.3650	0.16	1.66	2.21
	MPC-PID	0.0094	75.0605	3.2818	1.48	4.07	5.32

Table 4.2: E - Control effort summary

		Absolute values			Scaled values		
		u_F[%]	u_S[%]	u_W[%]	u_F[%]	u_F[%]	u_W[%]
	PID	2.2081	1.5494	6.4496	100.00	100.00	100.00
No Ref	MPC	0.2253	0.6931	1.4907	10.23	44.73	23.30
	MPC-PID	0.3532	0.6747	2.0834	16.05	43.54	32.57
Ref	MPC	0.5718	0.7197	2.7464	25.97	46.45	42.93
	MPC-PID	0.5718	0.7197	2.7464	25.97	46.45	42.93

Next set of figures shows performance of controlled boiler-turbine system, when future reference profile is supplied to MPC controllers. Reader will notice, that this will increase the control effort (shown on Fig. 4.9), but advantage of such strategy is even further decreased difference between reference and output (Fig. 4.6). The effect of future reference profile can be clearly seen from Fig. 4.10, where is shown filtered reference for PI controllers.

Naturally, when MPC is used on closed loop system, more control effort needs to be put in place, in order to satisfy reasonable tracking. Main difference in control effort is due to the oscillations introduced by PI controllers. Damping of these oscillations can be seen on filtered reference for PI controllers.

Final set of figures shows simulation when rate limiter and saturation is applied on PI controllers. This performance is unstable, thus unfit to implement in industry application. In order to avoid instability, much more conservative PI controllers must be implemented. Even by lowering

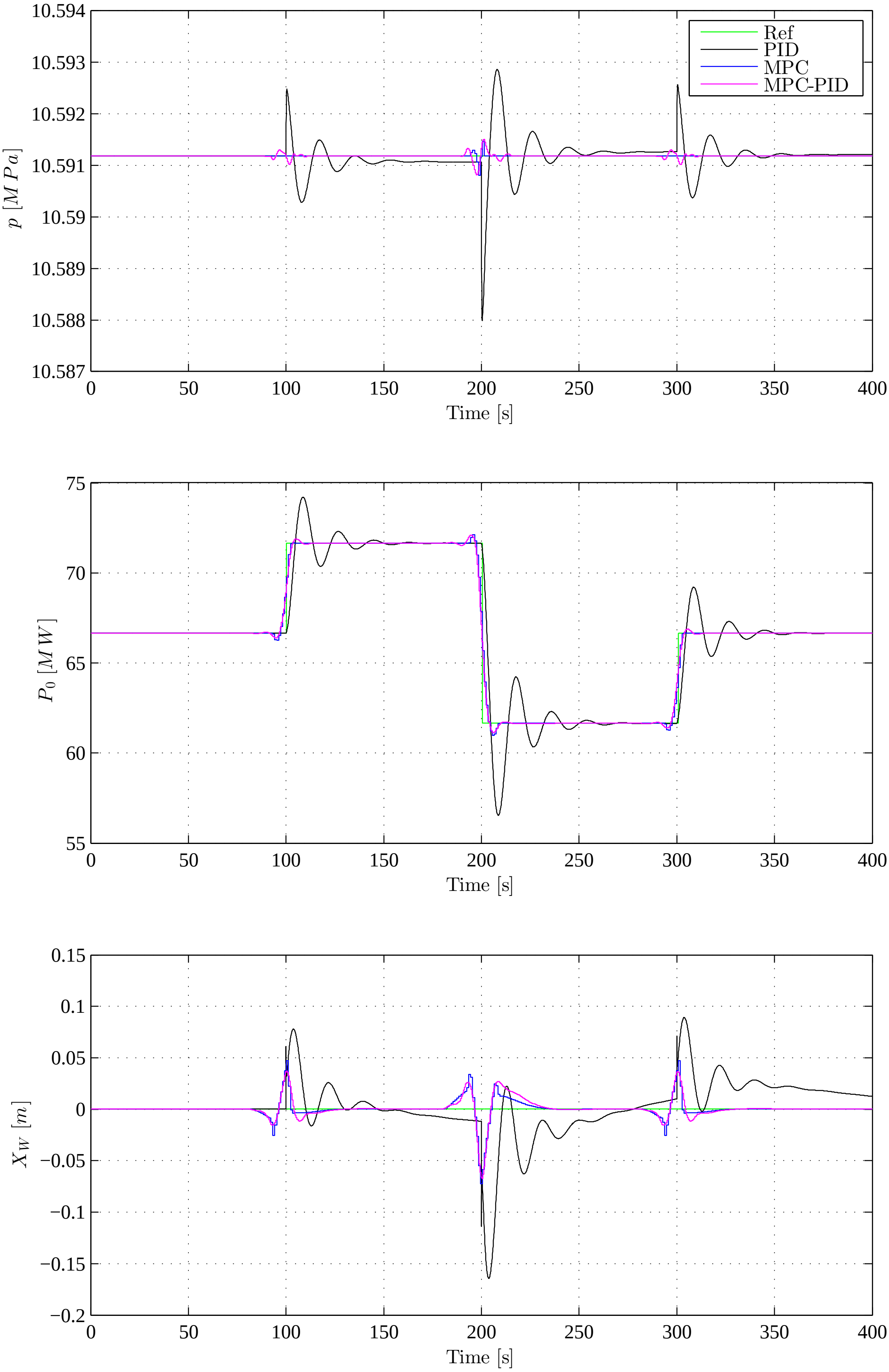

Figure 4.6: Outputs (with future reference profile)

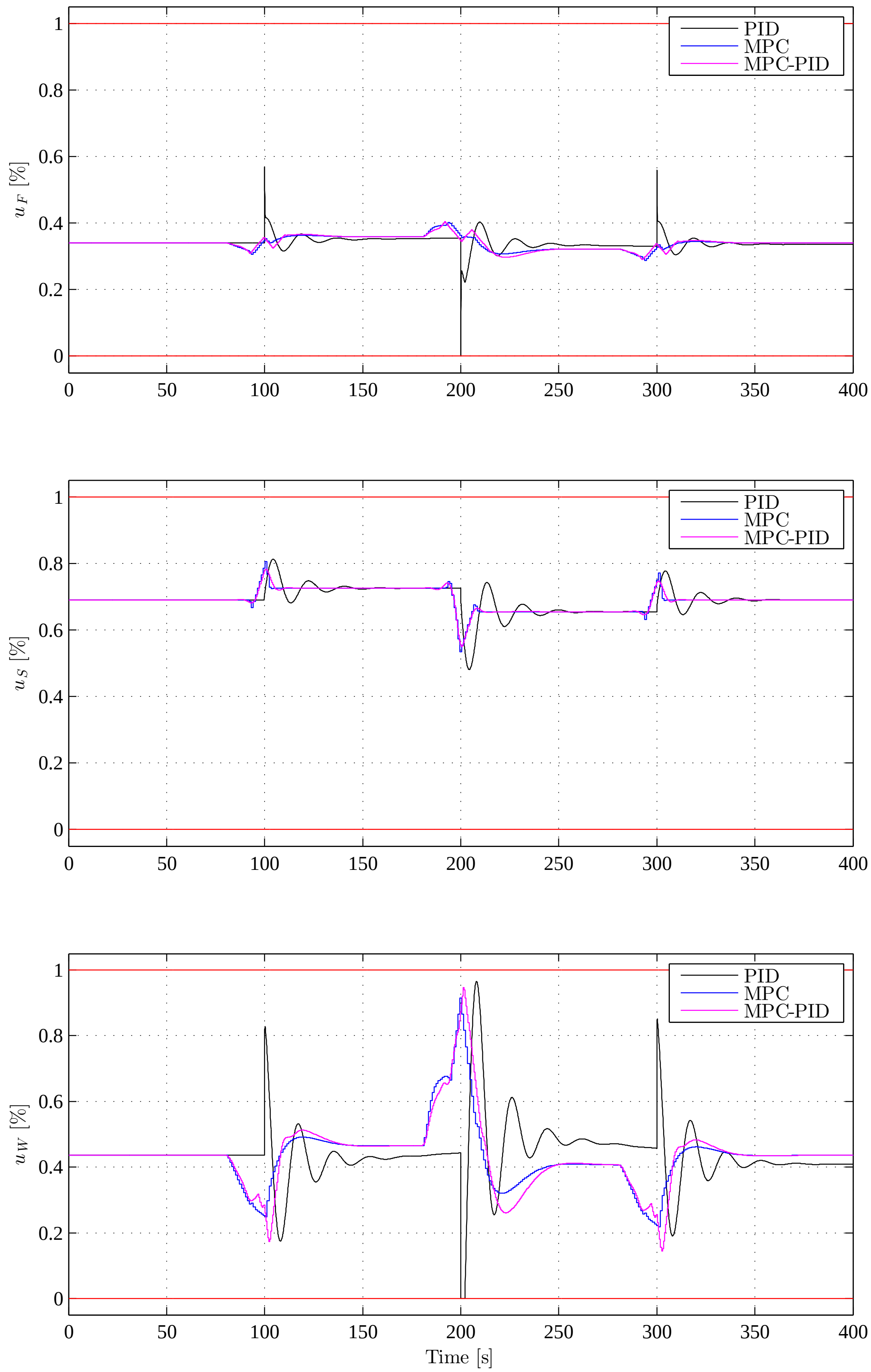

Figure 4.7: Inputs (with future reference profile)

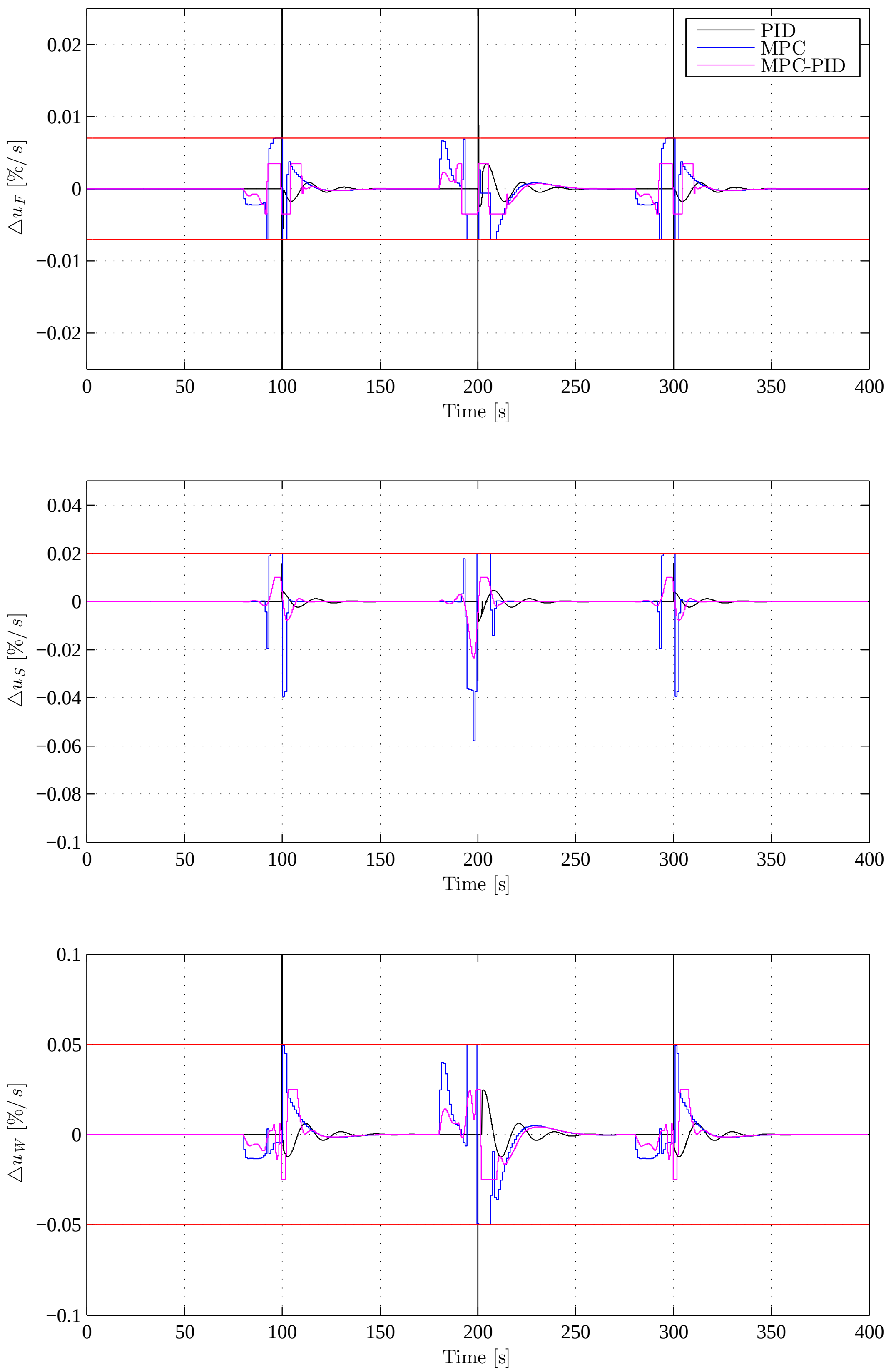

Figure 4.8: Control moves (with future reference profile)

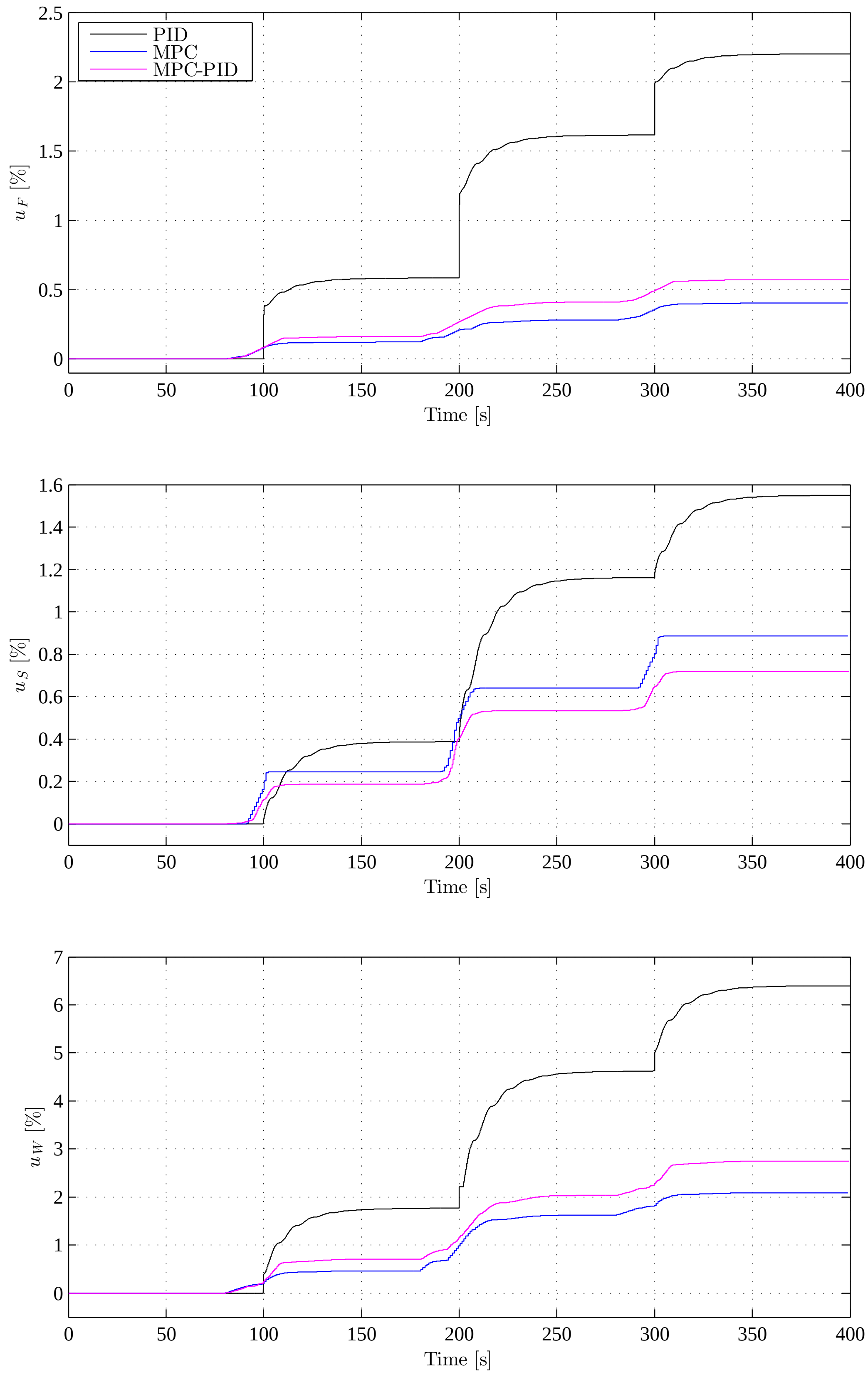

Figure 4.9: Control effort (with future reference profile)

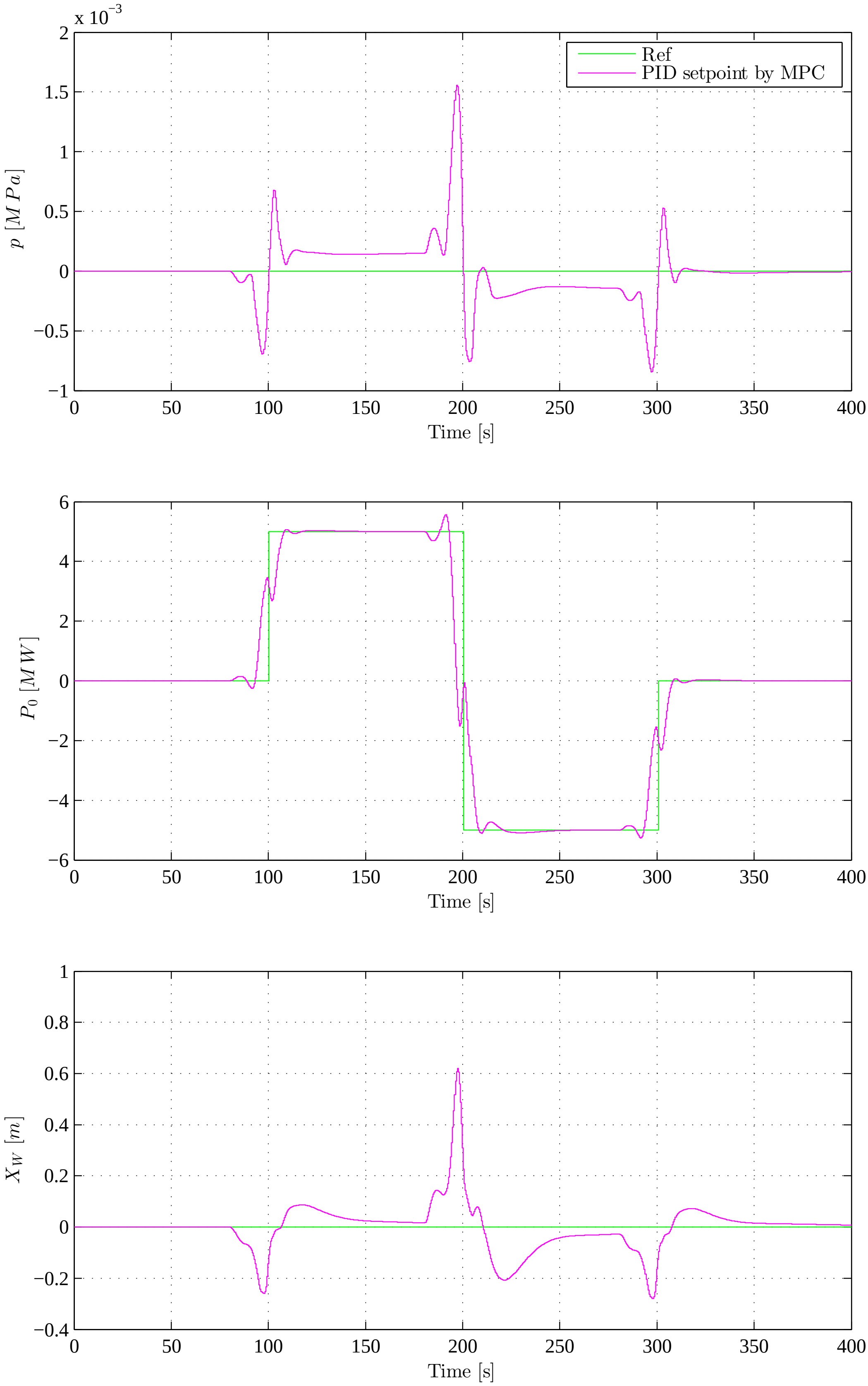

Figure 4.10: Reference profile for PI controller (with future reference profile)

the gains in the PI controllers, the performance of constrained feedback controller has been unsatisfactory.

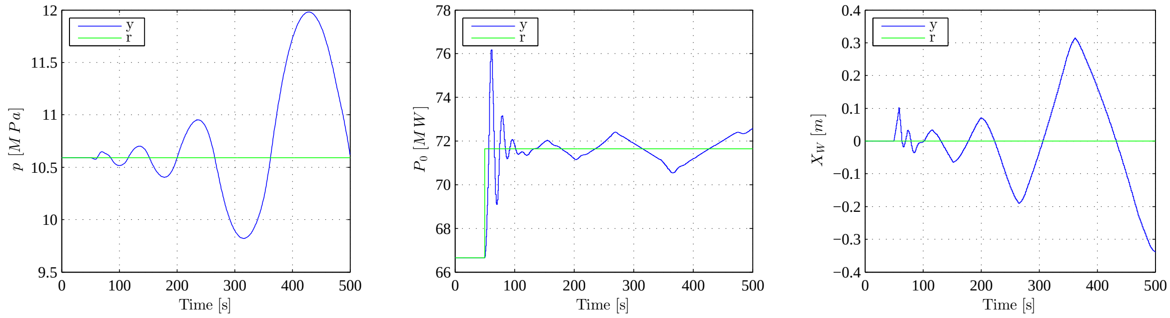

Figure 4.11: Outputs - constrained PI controllers

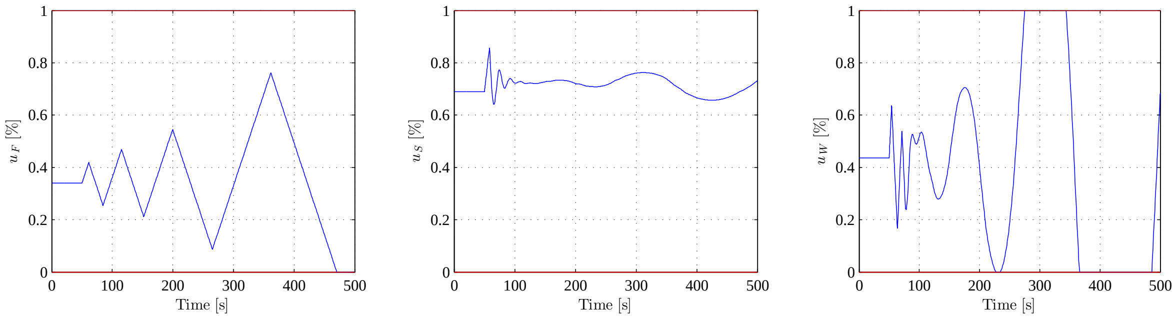

Figure 4.12: Inputs - Constrained PI controllers

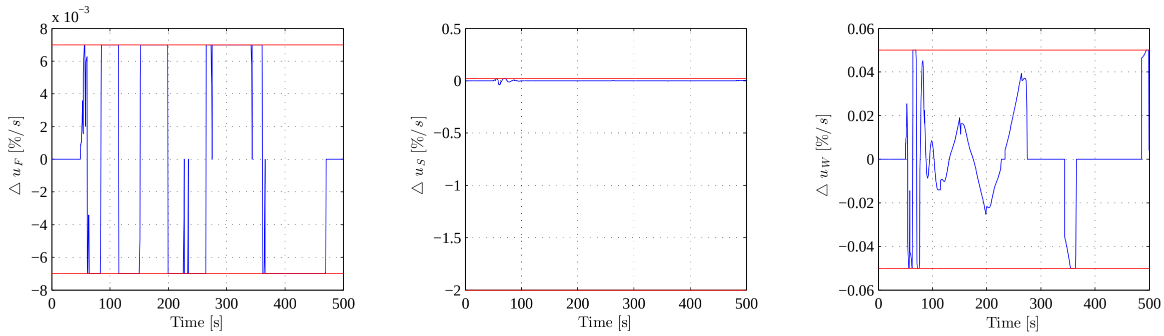

Figure 4.13: Control rate - Constrained PI controllers

Chapter 5

Conclusions

This project has been dealing with implementation of reference governor in form of model predictive controller on boiler-turbine unit. Based on the fact, that standard PID controllers are unable to fulfil technological constraints in general, exploring such control strategy can be beneficial for industry. As it was mentioned in the introduction, combination of MPC and PID controllers give operators enough freedom to switch off control governor if necessary. However, when MPC reference governor is active, all technological constraints are obeyed, and optimal control actions are set on the inputs to the system. As it was shown in simulations, this MPC governor has significantly improve tracking of reference, and has decreased control effort.

For the purpose of designing reference governor, state space model of closed loop system had to be obtained. Upon this model the MPC governor has been formulated. Speciality of this MPC formulation was, that actual control inputs have been treated as outputs from the closed loop state space model. Constraints in form of rate limiter and saturation have been applied in QP optimization.

Main implementation issue of MPC controller is the demand on computational power. However formulation proposed in this project has been easily handled by available solvers like `quadprog()`. It must be noted, that rather simple linear model has been used in this project. Realizing this, we come to a conclusion, that if such MPC governor will be implemented in industry, more complex model have to be considered. This will result in increased complexity of the QP formulation, which will affect the calculation time. However, state of the art optimization software should be able to handle such problem.

This project can be in future expanded mainly in two possible ways. First researcher can have a closer look on the technological process, to increase the complexity of the model, thus getting closer to reality. On the other hand, exploring the possibilities of reference governors, especially in connection with model predictive control, can have more beneficial impact on research in control field. Since in real application we will do not have linear models, state estimators and observers in connection with MPC governors should be considered.

Chapter 6

Resumé

Diplomová práca s názvom MPC-Based Reference Governors, pojednáva o návrhu prediktívneho regulátora, ktorého primárnou úlohou je riadiť referenciu pre vnútorný PID regulátor. Nasadenie takejto formy regulátora vychádza z požiadaviek priemyselnej praxe, kde nie je vôľa opustiť súčasné overené riešenia regulácie najme vo forme PID regulátorov. Výhoda prezentovaného riešenia spočíva v tom, že pri návrhu prediktívneho regulátora budeme uvažovať nielen riadený proces ale súčasne aj jednotlivé PID regulátory. Navrhované riešenie v sebe zahŕňa niekoľko významných výhod, ktoré aj priemysel v poslednom čase rozoznáva. Tou najdôležitejšou výhodou je zavedenie optimálnych akčných zásahov a dodržiavanie bezpečnostných a technologických podmienok, ohraničení a požiadaviek, ktoré štandardné PID regulátory nie sú schopné zabezpečiť.

Pri návrhu prediktívnych regulátorov však treba mať na pamäti ich dve nevýhody. V prvom rade sú potrebné presné matematické modely riadených procesov, a v druhom rade je to výpočtová náročnosť optimalizácie. Optimalizačná úloha, ktorá reprezentuje MPC regulátor má formu kvadratickej účelovej funkcie s lineárnymi ohraničeniami vo forme rovnosti a nerovnosti. Táto optimalizačná úloha, často označovaná ako QP problém, musí byť vyriešená v každej perióde vzorkovania.

Návrh MPC regulátora je založený na stavovom opise riadeného procesu. Ako prvé bolo potrebné určiť stavovú reprezentáciu PID regulátorov a následne odvodiť stavový model pre uzavretú spätno-väzbovú slučku. Prediktívny regulátor bol navrhnutý na základe matematického modelu systému kotol-turbína. Tento proces je charakterizovaný stavovým modelom tretieho rádu, s troma vstupmi a troma výstupmi, ktorý je primárne riadený troma nezávislými PI regulátormi.

Na otestovanie navrhovaného riešenia boli zvolené dva simulačné scenáre. V každom z nich sa porovnávajú tri možnosti riadenia systému kotol-turbína – použitie iba PI regulátorov, použitie iba MPC regulátora a použitie MPC regulátora, ktorý riadi referenciu PI regulátorov. Tak ako grafické aj numerické porovnanie je uvedené. Na grafoch môžeme vidieť, že v prípade použitia akejkoľvek stratégie riadenia s MPC, kvalita riadenia výrazne stúpla, znížili sa prekmity a takisto sa znížil aj suma akčných zásahov v niektorých prípadoch až o 70%. Pre porovnanie je uvedená aj simulácia s obmedzeniami zakomponovanými priamo v PID regulátore. Na tejto simulácii je vidno, že takáto forma riadenia destabilizuje riadený proces.

Na výsledkoch simulácie sme dokázali, že takáto forma riadenia zabezpečí dodržanie technologických a bezpečnostných podmienok a zároveň umožňuje naďalej používať overené PID regulátory. V tejto diplomovej práci bol regulátor testovaný na lineárnych systémoch, čo však nieje prípad praxe. Z tohto dôvodu je možné túto stratégiu riadenia rozšíriť o pozorovač stavov, čo nám umožní nasadiť takéto riešenie aj pre nelineárne systémy.

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