SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF CHEMICAL AND FOOD TECHNOLOGY

Modelling and Controlling of the Laboratory Distillation Column

Diploma thesis

FCHPT-5414-58728

Bratislava 2014 Bc. Matej Štefánik

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Study program: Automation and information engineering in chemical and food technologies
Study field: 5.2.14 Automation
Workplace: Faculty of Chemical and Food Technology
Supervisor: Prof. Ing. Miroslav Fikar, DrSc.
Consultant: Ing. MSc. Martin Klaučo

Bratislava 2014 **Bc. Matej Štefánik**

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DIPLOMA THESIS TOPIC

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Topic: **Modelling and Controlling of the Laboratory Distallation Column**

Specification of Assignment:

Separation of binary mixture methanol-water is performed in laboratory column UOP3CC. The aim of the work is to design control of temperature at column head with manipulated variable either reflux ration of reboiler heat. Control will be implemented in MATLAB/Simulink.

Tasks:
1 – modelling of the column
2 – identification of the column and estimation of parameters
3 – selection of manipulated variable
4 – design and optimisation of PID controller parameters
5 – design of model predictive control
6 – implementation and verification on the laboratory process

Length of thesis: 40

Selected bibliography:

1. Mikleš, J. – Fikar, M. *Process Modelling, Identification, and Control.* Berlin Heidelberg: Springer Berlin Heidelberg New York, 2007. 480 s. ISBN 978-3-540-71969-4.
2. Skogestad, S. – Postlethwaite, I. *Multivariable Feedback Control : Analysis and Design.* Chichester: John Wiley & Sons, 2005. 574 s. ISBN 0-470-01168-8.

Abstrakt

Práca sa zaoberá modelovaním a riadením laboratórnej rektifikačnej kolóny. Identifikáciou pomocou metódy rekurzívnych najmenších štvorcov boli určené parametre prenosovej funkcie medzi teplotou na hlave kolóny a refluxným pomerom a parametre prenosovej funkcie poruchovej veličiny, ktorou je teplota suroviny na nástrekovej etáži. Správnosť identifikovaných prenosov modelov bola overená na reálnom zariadení. Pomocou optimálneho pozorovača, ktorým je kalmanov filter, sa odhadli stavové veličiny kolóny. Identifikované modely slúžia na návrh PI a MPC regulátorov.

Abstract

The project deals with modelling and controlling of the laboratory distillation column. Using recursive least squares method for identification was identified parameters of transfer function between temperature on the top of the column and reflux rate and parameters of transfer function of disturbance, which is temperature of the feed on the feed tray. Model verification was been performed on laboratory device. Using Kalman filter as a optimal state observer have been estimated all of the state variables of the device. Identified models have been used to design PI and MPC controllers.

I would like to express my sincere thanks to my supervisor Prof. Ing. Miroslav Fikar, DrSc. for his guidance and valuable input during preparation of this diploma thesis and the same thanks belongs to my consultant Ing. MSc. Martin Klaučo for his advises and time, which he spent with me during preparation of this thesis.

Matej Štefánik

Contents

Chapter 1

Introduction

Distillation is one of the most important industrial processes for separating the different components of liquid mixture.

Distillation is defined as a process in which a liquid or vapour mixture of two or more substances is separated into its component fractions of desired purity, by the application and removal of heat.

Distillation is based on the fact that the vapour of a boiling mixture will be richer in the components that have lower boiling points.

Distillation is possible for a methanol-water mixture because their vapour and liquid concentrations vary with temperature, as shown in a T-x-y diagram is shown in Fig. 1.2. Using this property of the methanol-water mixture, it is possible produce high purity methanol. X-Y diagram of the methanol-water mixture is shown in Fig. 1.1.

Therefore, when this vapour is cooled and condensed, the condensate will contain more volatile components. At the same time, the original mixture will contain more of the less volatile material.

Distillation columns are designed to achieve this separation efficiently:

- distillation is the most common separation technique
- it consumes enormous amounts of energy, both in terms of cooling and heating requirements
- it can contribute to more than 50% of plant operating costs

The best way to reduce operating costs of existing units, is to improve their efficiency and operation via process optimisation and control. To achieve this improvement, a thorough understanding of distillation principles and how distillation systems are designed is essential.

(Tham, 1997)

Distillation columns becomes a favourite subject in the process systems engineering field, including the areas of process synthesis, process dynamics and process control. The reason is that distillation columns are themselves a system; a distillation columns may be viewed as a set of integrated,

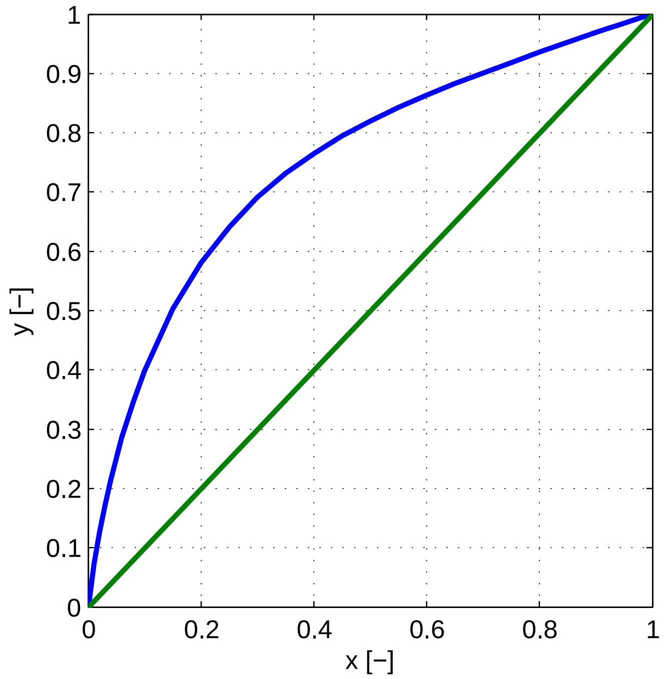


Figure 1.1: X-Y Diagram for mixture Methanol-Water

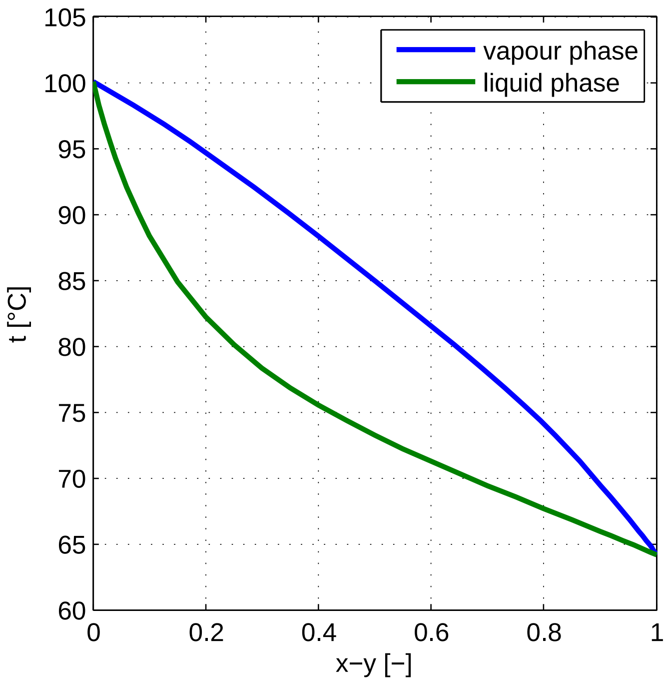


Figure 1.2: t-xy Diagram for mixture Methanol-Water

mostly cascaded, flash tanks. However, this integration gives rise to a complex and non-intuitive behaviour, and it is difficult to understand the system (the column) based on the knowledge about the behaviour of the individual pieces (the flash tanks). (Skogestad, 1997)

Predictive control is now one of the most widely used advanced control methods in industry, especially in the control of processes that are constrained, multivariable and uncertain. The cornerstone of MPC is the model. MPC uses models in 2 ways: using a reliable model to predict effect of current control input on future outputs, and using the same model to compute the optimal control action. (Ahmad and Wahid, 2007)

Chapter 2

Description of the Laboratory Device

The UOP3CC Continuous Distillation Column is a self-contained distillation facility consisting of two interconnected units: (UOP3CC, 2010)

1. a floor standing process unit
2. benchmounted control console

Schema of distillation column is shown in Fig. 2.2 and the laboratory device of distillation column is in Fig. 2.1.

Distillation Column

The 50mm diameter sieve plate column is made up of two glass sections each containing four sieve plates. The columns are separated by a central feed section and arranged vertically for counter-current vapour/liquid flow. The column is insulated to minimise heat loss.

The glass column incorporates a total of eight sieve plates in two sections each containing four plates. Each plate is located by the central support rod and incorporates a weir and downcomer to create a liquid seal between successive states. The liquid seal on the final plate in each section is achieved by U-tube.

Reboiler

The reboiler is situated at the base of the column. In continuous operation, valve is open and bottom product flows from the reboiler through the bottom product cooler to the bottom product tank. It is possible to preheat the feed to the column by directing the feed through a spiral coin in the bottom product cooler where the heat is transferred from product leaving the reboiler at the boiling point.

Condenser

Vapour from the top of the column passes to a water-cooled, coil-in-shell condenser, which may be fitted with an insulating jacket to allow heat balances to be carried out. The shell of the

Figure 2.1: Laboratory Distillation Column

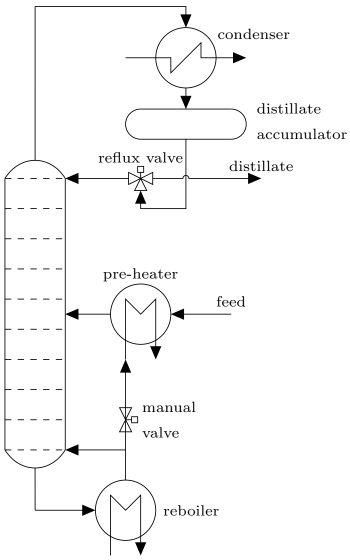


Figure 2.2: Scheme of Distillation Column

condenser incorporates a pressure relief valve to protect system in the event of a blocked vent and cooling water failure. Cooling water enters the condenser at a regulated rate through a variable area flow meter and the flow rate is controlled by diaphragm valve.

Decanter

Condensate is located in a glass decanter (phase separator) which is bypassed for normal distillation experiments by operating valve which is 3-way solenoid operated valve. Depending on the setting of the reflux timers, condensate is directed by the reflux valve either back to the top of the column or to the top product collecting vessel. When directed to the column, the reflux passes through a U-seal where a valve can be used for measuring boil-up rate.

Thermocouples

Temperatures within the system are monitored by fourteen thermocouple sensors located at the strategic positions in the system. T1 to T8 except T6 are located in the column and measure the temperature of the liquid on each plate. T10 measures the temperature of the vapour on the top of the column. T9 measures the temperature of the mixture in the reboiler. T6 measures the temperature of the feed on exit of the reboiler cooler. T12 measures the temperature of the water on exit of condenser.

Chapter 3

Identification

3.1 State-space Model

Model predictive control considered in this project, is based on state space model. For this purpose, state space model of distillation column can be derived. In order to obtain mathematical model, first several assumptions must be made (Minh and Rani, 2009):

- Binary mixture is separated and it has ideal properties, so it means, that compounds of binary mixture are ideally mixed.
- Mass transfer is only on trays and in reboiler.
- Efficiency of trays isn't equal to 1. Efficiency can be defined:

$$\eta_i = \frac{y_i - y_{i+1}}{y_i^* - y_{i+1}} \tag{3.1}$$

  where y_i is actual concentration of vapour phase on i-th tray and y_i^* is ideal concentration of vapour phase on i-th tray, it can be enumerated from t-xy diagram 1.2.

- Actual composition in liquid phase is equal to ideal composition in liquid phase $x = x^*$.
- In distillation column is always constant atmospheric pressure.

Variables of distillation column are as follows:

- x_i is concentration of methanol in liquid phase on i-th tray
- $x_i(0)$ is initial condition of concentration of methanol in liquid phase on i-th tray at time $t = 0$
- y_i is actual concentration of methanol in vapour phase on i-th tray
- y_i^* is ideal concentration of methanol in vapour phase on i-th tray
- Z_i is hold-up on i-th tray.

- $\dot{n}_L$ is molar flow of liquid along distillation column
- $\dot{n}_G$ is molar flow of vapour along distillation column
- $\dot{n}_F$ is molar flow of feed with concentration x_F
- $\dot{n}_D$ is molar flow of distillate and $\dot{n}_W$ is molar flow of bottom product from reboiler

Formulation of nonlinear state-space model was taken from (Fikar and Mikleš, 2007) Mass balance of reboiler (index = 9):

$$(\dot{n}_L(t) + \dot{n}_F(t))x_8(t) = \dot{n}_G(t)y_9(t) + \dot{n}_W(t)x_9(t) + \frac{d(Z_9x_9(t))}{dt} \tag{3.2}$$

Mass balance of stripping part (index = 8,7,6):

$$(\dot{n}_L(t) + \dot{n}_F(t))x_7(t) + \dot{n}_G(t)y_9(t) = (\dot{n}_L(t) + \dot{n}_F(t))x_8(t) + \dot{n}_G(t)y_8(t) + \frac{d(Z_8x_8(t))}{dt} \tag{3.3}$$

$$(\dot{n}_L(t) + \dot{n}_F(t))x_6(t) + \dot{n}_G(t)y_8(t) = (\dot{n}_L(t) + \dot{n}_F(t))x_7(t) + \dot{n}_G(t)y_7(t) + \frac{d(Z_7x_7(t))}{dt} \tag{3.4}$$

$$(\dot{n}_L(t) + \dot{n}_F(t))x_5(t) + \dot{n}_G(t)y_7(t) = (\dot{n}_L(t) + \dot{n}_F(t))x_6(t) + \dot{n}_G(t)y_6(t) + \frac{d(Z_6x_6(t))}{dt} \tag{3.5}$$

Mass balance of feed tray (index = 5):

$$\dot{n}_F(t))x_F(t) + \dot{n}_L(t)x_4(t) + \dot{n}_G(t)y_6(t) = (\dot{n}_L(t) + \dot{n}_F(t))x_5(t) + \dot{n}_G(t)y_5(t) + \frac{d(Z_5x_5(t))}{dt} \tag{3.6}$$

Mass balance of enriching part (index = 4,3,2,1):

$$\dot{n}_L(t)x_3 + \dot{n}_G(t)y_5(t) = \dot{n}_L(t)x_4(t) + \dot{n}_G(t)y_4(t) + \frac{d(Z_4x_4(t))}{dt} \tag{3.7}$$

$$\dot{n}_L(t)x_2 + \dot{n}_G(t)y_4(t) = \dot{n}_L(t)x_3(t) + \dot{n}_G(t)y_3(t) + \frac{d(Z_3x_3(t))}{dt} \tag{3.8}$$

$$\dot{n}_L(t)x_1 + \dot{n}_G(t)y_3(t) = \dot{n}_L(t)x_2(t) + \dot{n}_G(t)y_2(t) + \frac{d(Z_2x_2(t))}{dt} \tag{3.9}$$

$$\dot{n}_L(t)x_10 + \dot{n}_G(t)y_2(t) = \dot{n}_L(t)x_1(t) + \dot{n}_G(t)y_1(t) + \frac{d(Z_1x_1(t))}{dt} \tag{3.10}$$

Mass balance of condenser (index = 10)

$$\dot{n}_G(t)y_1(t) = \dot{n}_L(t)x_{10}(t) + \dot{n}_D(t)x_{10}(t) + \frac{d(Z_{10}x_{10}(t))}{dt} \tag{3.11}$$

Vector x represents concentration in liquid phase on trays and vector y represents concentration in vapour phase. Relation between these vectors is shown in x-y diagram 1.1. Initial conditions of (3.2)-(3.11):

$$x_i(0) = x_{i0} \qquad i = \{1, 2, \dots 10\} \tag{3.12}$$

$$\dot{n}_k(0) = \dot{n}_{k0} \qquad k = \{G, L, F, W, D\} \tag{3.13}$$

Our differential equations of process look as follows:

$$\dot{x}_9(t) = -\frac{\dot{n}_W}{Z_9}x_9(t) + \frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_9}x_8(t) - \frac{\dot{n}_G(t)}{Z_9}y_9(t) \tag{3.14}$$

$$\dot{x}_8(t) = -\frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_8}x_8(t) + \frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_8}x_7(t) + \frac{\dot{n}_G(t)}{Z_8}y_9(t) - \frac{\dot{n}_G(t)}{Z_8}y_8(t) \tag{3.15}$$

$$\dot{x}_7(t) = -\frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_7}x_7(t) + \frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_7}x_6(t) + \frac{\dot{n}_G(t)}{Z_7}y_8(t) - \frac{\dot{n}_G(t)}{Z_7}y_7(t) \tag{3.16}$$

$$\dot{x}_6(t) = -\frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_6}x_6(t) + \frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_6}x_5(t) + \frac{\dot{n}_G(t)}{Z_6}y_7(t) - \frac{\dot{n}_G(t)}{Z_6}y_6(t) \tag{3.17}$$

$$\dot{x}_5(t) = -\frac{(\dot{n}_L(t) + \dot{n}_F(t))}{Z_5}x_5(t) + \frac{(\dot{n}_L(t))}{Z_5}x_4(t) + \frac{\dot{n}_F}{Z_5}x_F(t)\frac{\dot{n}_G(t)}{Z_5}y_6(t) - \frac{\dot{n}_G(t)}{Z_5}y_5(t) \tag{3.18}$$

$$\dot{x}_4(t) = -\frac{\dot{n}_L(t)}{Z_4}x_4(t) + \frac{\dot{n}_L(t)}{Z_4}x_3(t) + \frac{\dot{n}_G(t)}{Z_4}y_5(t) - \frac{\dot{n}_G(t)}{Z_4}y_4(t) \tag{3.19}$$

$$\dot{x}_3(t) = -\frac{\dot{n}_L(t)}{Z_3}x_3(t) + \frac{\dot{n}_L(t)}{Z_3}x_2(t) + \frac{\dot{n}_G(t)}{Z_3}y_4(t) - \frac{\dot{n}_G(t)}{Z_3}y_3(t) \tag{3.20}$$

$$\dot{x}_2(t) = -\frac{\dot{n}_L(t)}{Z_2}x_2(t) + \frac{\dot{n}_L(t)}{Z_2}x_1(t) + \frac{\dot{n}_G(t)}{Z_2}y_3(t) - \frac{\dot{n}_G(t)}{Z_2}y_2(t) \tag{3.21}$$

$$\dot{x}_1(t) = -\frac{\dot{n}_L(t)}{Z_1}x_1(t) + \frac{\dot{n}_L(t)}{Z_1}x_{10}(t) + \frac{\dot{n}_G(t)}{Z_1}y_2(t) - \frac{\dot{n}_G(t)}{Z_1}y_1(t) \tag{3.22}$$

$$\dot{x}_{10}(t) = -\frac{\dot{n}_L(t) + \dot{n}_D(t)}{Z_{10}}x_{10}(t) + \frac{\dot{n}_G(t)}{Z_{10}}y_1(t) \tag{3.23}$$

Control action is described by these two equations:

$$\dot{n}_D(t) = R\dot{n}_G(t) \tag{3.24}$$

Base form of state-space model looks like as in (3.25) and in (3.26). To create matrices A, B, C, D is needed Taylor's linearisation of non-linear equations (3.14)-(3.23). Relation between $y(t)$ and $y^*(t)$ is in (3.1). Relation between $x(t)$ and $y^*(t)$ is in 1.1 and relation between temperature of boiling mixture and composition is in 1.2.

$$\dot{z}(t) = Az(t) + Bu(t) \tag{3.25}$$

$$p(t) = Cz(t) + Du(t) \tag{3.26}$$

where $z(t)$ are state variables of the model , $p(t)$ are output variables from the model and $u(t)$ are input variables to the model.

This subsection of identification was about introducing state-space model matrices using mass balances of more volatile component. This is only one way, how to get analytical model of distillation column. The another way is identification of step responses to identify process transfer function and then from transfer function we can get state-space matrices A, B, C, D of our process. In this project was used identification using recursive least squares which is described in separate subsection of identification.

3.2 Recursive Least Squares

The Recursive least squares (RLS) adaptive filter is an algorithm which recursively finds the filter coefficients that minimizes a weighted linear least square cost function relating to the input signals. This is in contrast to other algorithms such as the least mean squares (LMS) that aim to reduce the mean square error. In the derivation of the RLS, the input signals are considered deterministic, while for the LMS and similar algorithm they are considered stochastic. Compared to most of its competitors, the RLS exhibits extremely fast convergence. However, this benefit comes at the cost of high computational complexity Hayes (1996)

Suppose, that our signal, can be defined as follows:

$$y = \theta_1 x_1 + \theta_2 x_2 + \ldots + \theta_s x_s + v = x^T\theta + v = \tilde{y} + v \tag{3.27}$$

The identification aim is to determine the parameter vector θ based on information of measured process output $y(n)$, the data vector $x(n)$ and $v(n)$ represents additive noise.

$$\begin{aligned} y_1 &= \theta_1 x_{11} + \theta_2 x_{12} + \ldots + \theta_S x_{1S} + v_1 \\ \vdots \quad & \quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \\ y_n &= \theta_1 x_{n1} + \theta_2 x_{n2} + \ldots + \theta_S x_{nS} + v_n \end{aligned} \tag{3.28}$$

$$\underbrace{\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}}_{Y} = \underbrace{\begin{pmatrix} x_1^T \\ \vdots \\ x_n^T \end{pmatrix}}_{X} \theta + \underbrace{\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}}_{V} \tag{3.29}$$

We will look for such an estimate $\hat{\theta}$ that minimises sum of squares of errors between measured and modelled outputs We have to minimize cost function with respect to θ

$$J^*(\theta, X) = \min V^T V = \min_{\theta} (Y - X\theta)^T (Y - X\theta) \tag{3.30}$$

$$\frac{\partial J^*}{\partial \theta} = 0 \tag{3.31}$$

If matrix X is invertible and gradient of this function with respect to θ is equal to zero:

$$\frac{\partial J^*}{\partial \theta} = 2X^T X \hat{\theta}^* - 2X^T Y = 0 \Rightarrow \hat{\theta}^* = (X^T X)^{-1} X^T Y \tag{3.32}$$

$$\hat{\theta}^* = \underbrace{(X^T X)^{-1}}_{P} X^T Y \tag{3.33}$$

where $\hat{\theta}^*$ is vector of estimated parameters. The matrix P is called the covariance matrix if the stochastic part has unit variance.

Transfer function in s-domain can be transform into z-domain using following equation:

$$G(z^{-1}) = (1 - z^{-1})\mathcal{Z}(\mathcal{L}^{-1}\frac{G(s)}{s}) = \frac{Y(z^{-1})}{X(z^{-1})} \tag{3.34}$$

The function $G(z^{-1})$ is referred to as discrete-time transfer function of the system with continuous-time transfer function $G(s)$.

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_b} z^{-n_b}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{n_a} z^{-n_a}} \tag{3.35}$$

A general discrete-time linear model can be written in time domain as

$$(1 + a_1 z^{-1} + \ldots + a_{n_a} z^{-n_a})Y(z^{-1}) = (b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_b} z^{-n_b})X(z^{-1}) \tag{3.36}$$

$$y(k) + a_1 y(k-1) + \ldots + a_{n_a} y(k-n_a) = b_1 u(k-1) + \ldots + b_{n_b} u(k-n_b) + e(k) \tag{3.37}$$

where y are outputs from system, u are inputs to the system and e is error variable.

$$Z^T(k) = \begin{pmatrix} -y(k-1), & \ldots, & -y(k-n_a), & u(k-1), & \ldots, & u(k-n_b) \end{pmatrix} \tag{3.38}$$

$$\hat{\theta} = \begin{pmatrix} a_1 & \ldots & a_{n_a} & b_1 & \ldots & b_{n_b} \end{pmatrix}^T \tag{3.39}$$

$$Y(k) = Z^T(k)\hat{\theta} + e(k) \tag{3.40}$$

$$\hat{\theta} = (Z^T(k)Z(k))^{-1} Z^T(k)Y(k) \tag{3.41}$$

$$Y(k) = \begin{pmatrix} y(1) \\ y(0) \\ \vdots \\ y(k-1) \\ y(k) \end{pmatrix} \qquad Z(k) = \begin{pmatrix} u^T(1) \\ u^T(0) \\ \vdots \\ u^T(k-1) \\ u^T(k) \end{pmatrix} \tag{3.42}$$

covariance matrix:

$$cov(\hat{\theta}) = \sigma^2 (Z^T(k)Z(k))^{-1} \tag{3.43}$$

$$P = (Z_k^T Z_k)^{-1} \tag{3.44}$$

matrix inversion lemma:

$$(A + BC^{-1}D)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C)^{-1} \tag{3.45}$$

If we consider $C = 1, B = z_k, D = z^T andA = P_{k-1}^{-1}$ then the matrix inversion lemma yields (3.47), where the term that has to be inverted is only a scalar.

$$P_k^{-1} = Z_k^T Z_k = \left(Z_{k-1}^T, z_k\right)\begin{pmatrix} Z_{k-1} \\ z_k^T \end{pmatrix} = \underbrace{Z_{k-1}^T Z_{k-1}}_{P_{k-1}^{-1}} + z_k z_k^T \tag{3.46}$$

$$P_k = (P_{k-1}^{-1} + z_k z_k^T)^{-1} = P_{k-1} - P_{k-1} z_k \underbrace{(z_k^T P_{k-1} z_k + 1)^{-1}}_{\gamma(k)} z_k^T P_{k-1} \tag{3.47}$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \underbrace{\gamma_k P_{k-1} z_k}_{L_k} \underbrace{(y_k - z_k^T \hat{\theta}_{k-1})}_{\varepsilon_k} \tag{3.48}$$

$$L_k = \gamma_k P_{k-1} z_k \tag{3.49}$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + L_k \varepsilon_k \tag{3.50}$$

initial values:

$$\hat{\theta}_0, P_0 \tag{3.51}$$

minimize:

$$\min_{\hat{\theta}} \quad J = (\hat{\theta} - \hat{\theta}_0)^T P_0^{-1} (\hat{\theta} - \hat{\theta}_0) = \sum_{i=1}^{k} \left(y - z_i^T \hat{\theta}\right)^2 \tag{3.52a}$$

$$s.t. \quad P_0 = cI \tag{3.52b}$$

3.3 Identification of the models using RLS

Step changes on reflux ratio were performed in order to obtain model relating control input (reflux ratio) and controlled variable (T10 temperature). Model of the disturbance was obtain using same identification method.

We made 2 step changes Fig. 3.1(a) of reflux rate on whole working interval. The first step of our identification was filtering the data. As we can see in Fig. 3.1(a) our signal was with a large noise. This noise can be caused by the thermocouples, because the temperature is measured on sharp ending of metal wire which is also part of the thermocouple. Around this ending is bubbling liquid and vapour at the same time, but their temperatures are different. Using the right parameters of a filter we filtered data as is shown in Fig. 3.1(b). We needed to normalize measured data. Normalization was performed by subtracting steady state and then by dividing this values by steady state. So our normalized data had values between -1 and 1. These filtered and normalized data could be used for identification.

Values of steady states are as follows:

$$T6^S = 25°\text{C} \tag{3.53}$$
$$T10^S = 85°\text{C} \tag{3.54}$$
$$u^S = 1 \tag{3.55}$$

where value of reflux rate $u^S = 1$ represents fully opened three-way valve and everything from the top of the column flows to the product tank as a distillate and doesn't flow back to the column.

With sampling time:

$$T_S = 5s \tag{3.56}$$

This value of sampling time is suitable to catch dynamics of distillation column.

Transfer function of the model in s-domain can be written as follows:

$$G(s) = \frac{B(s)}{A(s)} \tag{3.57}$$

or in z^{-1}-domain

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} \tag{3.58}$$

We choose degree of denumerator of our transfer function will be 2 and degree of numerator will be 1. Identified transfer function will be look like

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{3.59}$$

in s-domain, or in z^{-1}-domain:

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_0 z^{-2}}{1 + a_1 z^{-1} + a_0 z^{-2}} \tag{3.60}$$

Identification was running in continuous time and identified models were transform into z-domain using function **c2d** in MATLAB. If we put values of identified parameters into transfer function we get identified model of reflux rate, which look like in s-domain:

$$G(s) = \frac{1.156.10^{-6}s + 0.0003393}{s^2 + 0.006301s + 1.381.10^{-5}} \tag{3.61}$$

or in z^{-1}-domain:

$$G(z^{-1}) = \frac{0.004203z^{-1} + 0.004148z^{-2}}{1 - 1.969z^{-1} + 0.969z^{-2}} \tag{3.62}$$

with sampling time $T_S = 5$ seconds.

Using function **tf2ss** in MATLAB we get state-space matrices from transfer function of reflux rate, which looks as follows:

$$A_{ref} = \begin{bmatrix} 1.9686 & -0.9690 \\ 1 & 0 \end{bmatrix} \qquad B_{ref} = \begin{bmatrix} 0.1250 \\ 0 \end{bmatrix} \tag{3.63}$$

$$C_{ref} = \begin{bmatrix} 0.0336 & 0.0332 \end{bmatrix} \qquad D_{ref} = \begin{bmatrix} 0 \end{bmatrix} \tag{3.64}$$

We had to compare our model with real data. So we created schema which compare step response of our model with real temperature on the top of the column as is shown in Fig. 3.1(c)

Using function **compare** in MATLAB we obtain fit of our model as 81.25%. As is shown in Fig. 3.1(c) our model behaves like real system and the dynamics of our model and real system are very similar. This procedure was also applied for identification of disturbance T6. This variable wasn't filtered, because the noise of temperature was lower compare with noise on variable T10.

Identified transfer function of disturbance in s-domain:

$$G(s) = \frac{7.749.10^{-6}s + 5.948.10^{-6}}{s^2 + 0.04057s + 0.0002287} \tag{3.65}$$

or in z^{-1}-domain:

$$G(z^{-1}) = \frac{0.0001046z^{-1} - 2.995.10^{-5}z^{-2}}{1 - 1.811z^{-1} + 0.8164z^{-2}} \tag{3.66}$$

with sample time $T_S = 5$ seconds. Fit of our model of disturbance is 81.98% as we can see on Fig. 3.2(c).

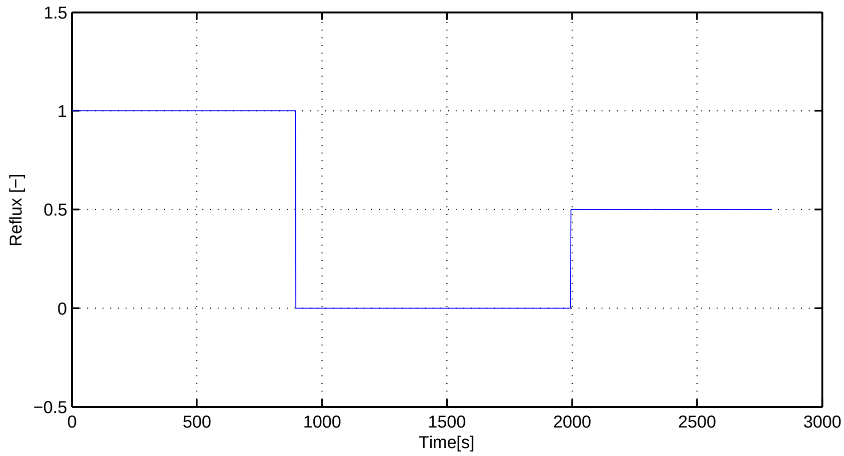
After transformation of transfer function into state-space matrices we obtain matrices of disturbance model:

$$A_{dist} = \begin{bmatrix} 1.8112 & -0.8164 \\ 1 & 0 \end{bmatrix} \qquad B_{dist} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{3.67}$$

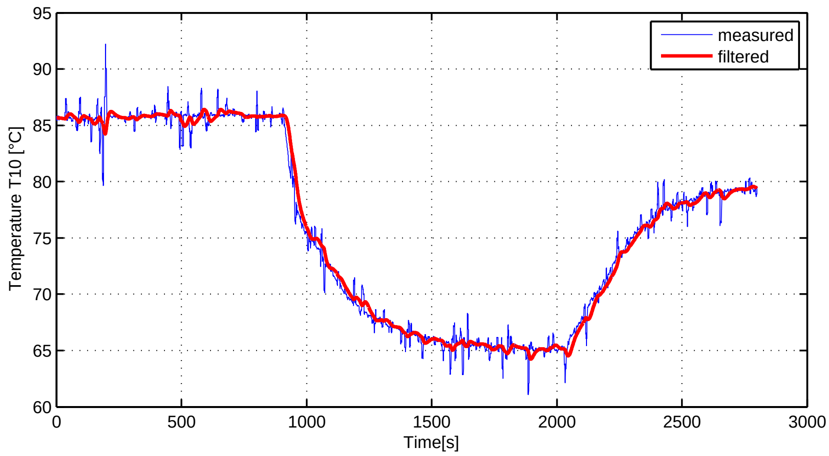
$$C_{dist} = \begin{bmatrix} 0.0083 & 0.0024 \end{bmatrix} \qquad D_{dist} = \begin{bmatrix} 0 \end{bmatrix} \tag{3.68}$$

Matrices of model of reflux rate (3.63)-(3.64) and matrices of model of disturbance (3.67)-(3.68) we put together to create state-space model of column. Matrices of model with disturbance now looks like as follows:

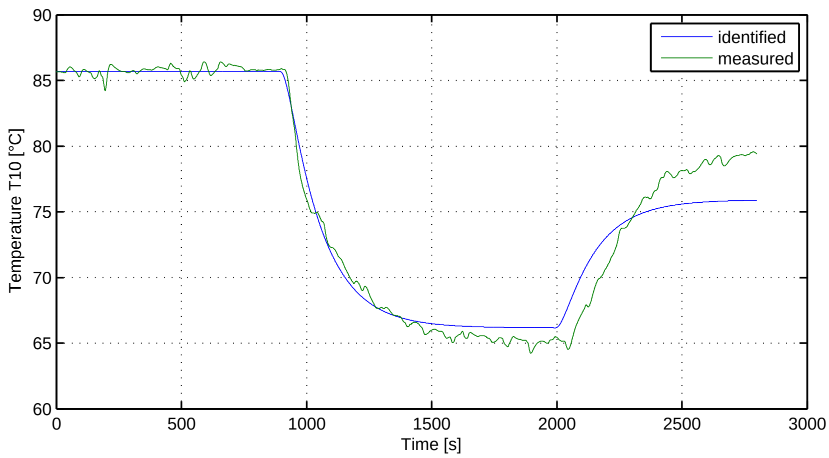
$$A = \begin{bmatrix} A_{ref} & 0 \\ 0 & A_{dist} \end{bmatrix} = \begin{bmatrix} 1.9686 & -0.9690 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.8112 & -0.8164 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix} \tag{3.69}$$



(a) Manipulated variable

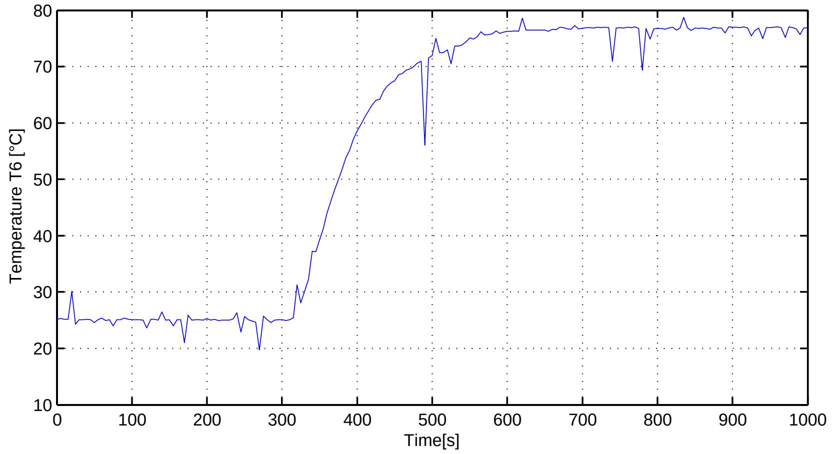


(b) Filtered step response

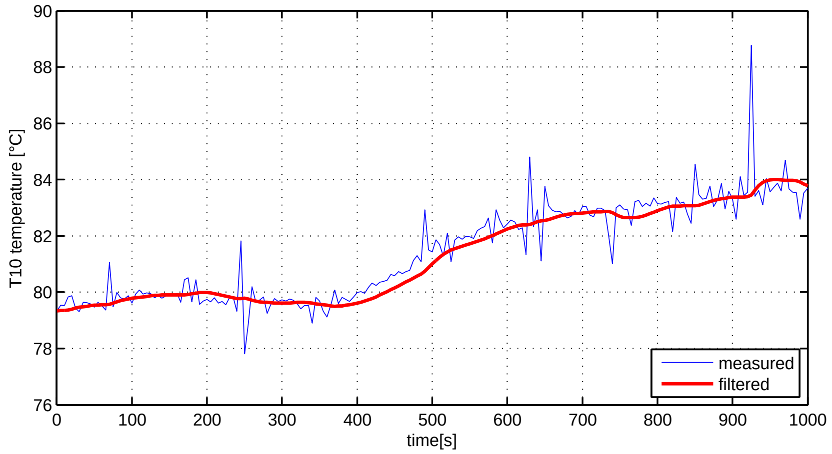


(c) Response of identified and measured data

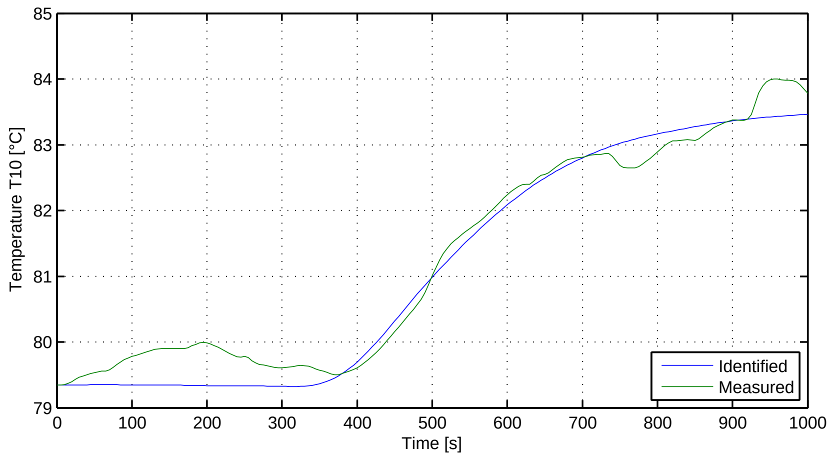
Figure 3.1: Identification of reflux rate



(a) Disturbance variable



(b) Filtered response on disturbance



(c) Response of identified and filtered data

Figure 3.2: Identification of disturbance

$$B_{ref} = \begin{bmatrix} 0.125 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.70}$$

$$B_{dist} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{3.71}$$

$$C = \begin{bmatrix} C_{ref} & C_{dist} \end{bmatrix} = \begin{bmatrix} 0.0336 & 0.0332 & 0.0083 & 0.0024 \end{bmatrix} \tag{3.72}$$

Identified second order of state space model for design control on laboratory distillation column.

Chapter 4

Model Predictive Control

Model predictive control (MPC) is an advanced method of process control that has been in use in the process industries in chemical plants and oil refineries since the 1980s. In recent years it has also been used in power system balancing models. Model predictive controllers rely on dynamic models of the process, most often linear empirical models obtained by system identification. Main advantage of MPC is the fact, that based on state space model and initial condition x_0 we predict future evolution of states, which is incorporated in optimization. This allows us to calculate optimal control inputs, which leads to optimal performance of the plant. MPC has the ability to anticipate future events and can take control actions accordingly. PID controllers do not have this predictive ability. MPC is a digital control.

The principle of MPC lies in solving quadratic objective function with linear constraints. Quadratic problem in standard form can be expressed as, (Boyd and Vandenberghe, 2009):

$$\min \quad \frac{1}{2} v^T H v + g^T v + r \tag{4.1a}$$

$$\text{s.t.} \quad Cv \preceq d \tag{4.1b}$$

$$Av = b \tag{4.1c}$$

where H is positive definite matrix $C \in \mathbb{R}^{m \times n}, A \in \mathbb{R}^{p \times n}$ and variables g, d, b are one column vectors, number of rows depends on problem which is solved.

Deriving cost function with the linear constraints is main part of the MPC implementation. Output regulation with constraints is considered and the optimizing variable will be the difference of control input. Formulation of MPC was taken from (Klauco, 2012).

General formulation of standard MPC problem with linear constraints can be expressed in following form:

$$\Phi = min \sum_{k=1}^{N} x^T Q x + \sum_{k=1}^{N-1} u^T R u \tag{4.2a}$$
$$s.t \qquad x_{k+1} = Ax + Bu_k \tag{4.2b}$$
$$x \in \mathbb{X} \qquad u \in \mathbb{U} \tag{4.2c}$$

where N is prediction horizon, $Q \in \mathbb{R}^{n_x \times n_x}, R \in \mathbb{R}^{n_u \times n_u}$ are weighting matrices.

By solving QP problem we calculate the optimal control inputs over prediction horizon. Once these inputs are applied to the plant, states of the plant are moved to different values, such application resembles open-loop implementation. In closed-loop application the optimization is performed at each sample, with new initial condition $(x_0 = x_k)$.(Maciejowski, 2002)

Output regulation problem together with disturbance modelling can be formulated as follows:

$$\Phi = \frac{1}{2} \sum_{k=0}^{N} \|r_k - y_k\|_Q^2 + \frac{1}{2} \sum_{k=0}^{N-1} \|\Delta u_k\|_R^2 \tag{4.3}$$

where r_k is the reference value for the output and y_k is the measurement of the output. This objective function will suppress the changes in the control signal Δu_k.

4.1 MPC

In order to solve optimization problem expressed in (4.3), this objective function has to be rewritten into form presented in (4.5). For the purpose of rewriting the MPC cost function into standard QP problem, we have to know relations between outputs y_k and inputs u_k. State space model can be described in following form:

$$x_{k+1} = Ax_k + Bu_k + E_x d_k \qquad k = 0...N \tag{4.4a}$$
$$y_k = Cx_k + Du_k + E_y d_k \qquad k = 0...N \tag{4.4b}$$

QP problem can be expressed as a weighted least square quadratic optimization problem with optimal solution U^*.

$$U^* = min \ \frac{1}{2}\|Y - R\|_Q^2 + \frac{1}{2}\|\Delta U\|_R^2 \tag{4.5}$$

This cost function can be then translated into standard QP problem by exploiting the evolution of the outputs over the prediction horizon base on state space model. First is considered model without disturbances d.

$$U^* = \frac{1}{2} U^T H U + g^T U + r \tag{4.6}$$

State space evolution for sample $k = 1$:

$$x_{k+1} = Ax_k + Bu_k \tag{4.7a}$$
$$y_k = Cx_k + Du_k \tag{4.7b}$$

State space evolution for sample $k = 2$:

$$\begin{aligned} x_{k+2} &= Ax_{k+1} + Bu_{k+1} && (4.8)\\ &= A(Ax_k + Bu_k) + Bu_{k+1} && (4.9)\\ &= A^2x_k + ABu_k + Bu_{k+1} && (4.10) \end{aligned}$$

$$\begin{aligned} y_{k+1} &= Cx_{k+1} + Du_{k+1} && (4.11)\\ &= C(Ax_k + Bu_k) + Du_{k+1} && (4.12)\\ &= CAx_k + CBu_k + Du_{k+1} && (4.13) \end{aligned}$$

State space evolution for sample $k = 3$:

$$\begin{aligned} x_{k+3} &= Ax_{k+2} + Bu_{k+2} && (4.14)\\ &= A(A^2x_k + ABu_k + Bu_{k+1})Bu_{k+2} && (4.15)\\ &= A^3x_k + A^2Bu_k + ABu_{k+1} + Bu_k && (4.16) \end{aligned}$$

$$\begin{aligned} y_{k+2} &= Cx_{k+2} + Du_{k+2} && (4.17)\\ &= C(A^2x_k + ABu_k + Bu_{k+1}) + Du_{k+2} && (4.18)\\ &= CA^2x_k + CABu_k + CBu_{k+1} + Du_{k+2} && (4.19) \end{aligned}$$

Based on these equations matrix form of prediction equation is expressed:

$$Y = \Psi x_k + \Gamma U \tag{4.20}$$

in which:

$$U = \begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ u_{k+3} \\ \vdots \\ u_{k+N-1} \end{bmatrix} \qquad Y = \begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ \vdots \\ y_{k+N-1} \end{bmatrix} \qquad \Psi = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N-1} \end{bmatrix} \tag{4.21}$$

$$\Gamma = \begin{bmatrix} D & 0 & 0 & 0 & \cdots & 0 \\ CB & D & 0 & 0 & \cdots & 0 \\ CAB & CB & D & 0 & \cdots & 0 \\ CA^2B & CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ CA^{N-2}B & CA^{N-3}B & CA^{N-4}B & \cdots & CAB & D \end{bmatrix} \tag{4.22}$$

where $\Psi \in \mathbb{R}^{Nn_u \times n_x}$

Control moves are defined as $\Delta u_k = u_k - u_{k-1}$. Using this definition, vector form of control moves over the control horizon is written:

$$\begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \Delta u_{k+2} \\ \vdots \\ \Delta u_{k+N-1} \end{bmatrix} = \begin{bmatrix} u_k - u_{k-1} \\ u_{k+1} - u_k \\ u_{k+2} - u_{k+1} \\ \vdots \\ u_{k+N-1} - u_{k+N-2} \end{bmatrix} \tag{4.23}$$

$$\begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \Delta u_{k+2} \\ \vdots \\ \Delta u_{k+N-1} \end{bmatrix} = \begin{bmatrix} I_u & 0 & 0 & 0 & \cdots & 0 \\ -I_u & I_u & 0 & 0 & \cdots & 0 \\ 0 & -I_u & I_u & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & -I_u & I_u \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ \vdots \\ u_{k+N-1} \end{bmatrix} - \begin{bmatrix} I_u \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_{k-1} \tag{4.24}$$

where $I_u \in \mathbb{I}^{n_u \times n_u}$.

$$\Delta U = \Lambda U - I_{1,u} u_{k-1} \tag{4.25}$$

By determining all matrices and vector, standard QP problem simplifying notation is formulated, (Muske and Rawlings, 1993)

$$U^* = min\left(\tfrac{1}{2}\|R - Y\|_Q^2 + \tfrac{1}{2}\|\Delta U\|_R^2\right) = min\ (\Phi_Y + \Phi_U) \tag{4.26}$$

$$Y - R = (\Gamma U + \Phi x_k) - R = \Gamma U - (R - \Phi x_k) = \Gamma U - c \tag{4.27}$$

$$\begin{aligned}
\Phi_Y &= \frac{1}{2}\|Y - R\|_Q^2 && (4.28)\\
&= \frac{1}{2}\|\Gamma U - c\|_Q^2 && (4.29)\\
&= \frac{1}{2}(\Gamma U - c)^T Q(\Gamma U - c) && (4.30)\\
&= \frac{1}{2}U^T\Gamma^T Q\Gamma U - (\Gamma^T Qc)^T U + \frac{1}{2}c^T Qc && (4.31)
\end{aligned}$$

$$\begin{aligned}
\Phi_U &= \frac{1}{2}\|\Delta U\|_R^2 && (4.32)\\
&= \frac{1}{2}\|\Lambda U - I_{1,u}u_{k-1}\|_R^2 && (4.33)\\
&= \frac{1}{2}(\Lambda U - I_{1,u}u_{k-1})^T R(\Lambda U - I_{1,u}u_{k-1}) && (4.34)\\
&= \frac{1}{2}U^T\Lambda^T R\Lambda U - (\Lambda^T R I_{1,u}u_{k-1})^T U && (4.35)
\end{aligned}$$

$$H = \Gamma^T Q\Gamma + \Lambda^T R\Lambda \tag{4.36}$$

$$\begin{aligned}
g &= -\Gamma^T Qc - \Lambda^T R I_{1,u}u_{k-1} && (4.37)\\
&= -\Gamma^T Q(R - \Phi_k) - \Lambda^T R I_{1,u}u_{k-1} && (4.38)\\
&= \Gamma^T QR + \Lambda^T R I_{1,u}u_{k-1} && (4.39)
\end{aligned}$$

H is curvature matrix and g is first order coefficient vector. In order to achieve offset of control, disturbances must be taken into account. In order to this, relation between outputs y and disturbances d is found. This equation shows the matrix form of prediction equation, in which are included states, control inputs and disturbances.

$$\Gamma_D = \begin{bmatrix}
E_y & 0 & 0 & 0 & \cdots & 0\\
CE_x & E_y & 0 & 0 & \cdots & 0\\
CAE_x & CE_x & E_y & 0 & \cdots & 0\\
CA^2E_x & CAE_x & CE_x & E_y & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \ddots & 0\\
CA^{N-2}E_x & CA^{N-3}E_x & CA^{N-4}E_x & \cdots & CAE_x & E_y
\end{bmatrix} \tag{4.40}$$

$$Y = \Psi x_k + \Gamma U + \Gamma_D D \tag{4.41}$$

This equation is then inserted into equation (4.27), yielding equation (4.42). By continuing derivation like it was presented in equation (4.32) through (4.37). The curvature matrix H will remain unchanged, but vector g will be changed as follows:

$$Y - R = (\Phi x_k + \Gamma U + \Gamma_D D) - R = \Gamma U - (R - \Phi x_k - \Gamma_D D) = \Gamma U - c \tag{4.42}$$

$$g = \Gamma^T Q R + \Gamma^T Q \Phi x_k - \Lambda^T R I_{1,u} u_{k-1} + \Gamma^T Q \Gamma_D D \tag{4.43}$$

For the purpose of simplifying notation, equation (4.43) is rewritten into:

$$g = M_R R + M_x x_k + M_u u_{k-1} + M_D D \tag{4.44}$$

Final formulation presented in (4.45) can be solved by numerous algorithms e.g. active-set algorithms, (Nocedal and Wright, 1999). The tools solving QP problems used in this project were namely **quadprog()** in MATLAB.

$$U^* = min\ \frac{1}{2} U^T H U + g^T U \tag{4.45}$$

4.2 Hard Constraints

The main advantage of MPC is to handle constraints. These constraints have form of linear inequality equations (4.46). Since we are dealing with stable system, constraints on system states may not be considered, so we are considering only hard constraints on control inputs u, control moves Δu and outputs y.

$$u_{min} \leq u_k \quad \leq u_{max} \qquad k = 0, \dots, N-1 \tag{4.46}$$

$$\Delta u_{min} \leq \Delta u_k \leq \Delta u_{max} \qquad k = 0, \dots, N-1 \tag{4.47}$$

$$y_{min} \leq y_k \quad \leq y_{max} \qquad k = 0, \dots, N \tag{4.48}$$

Constraints presented in (4.46) have to be rewritten into matrix form. The bounds on control signal are just stacked like in (4.49). Using definition of Λ matrix from (4.25), matrix form of inequality constraints for control moves are expressed in (4.50) yielding (4.51).

$$U_{min} = \begin{bmatrix} u_{min} \\ u_{min} \\ \vdots \\ u_{min} \end{bmatrix} \qquad U_{max} = \begin{bmatrix} u_{max} \\ u_{max} \\ \vdots \\ u_{max} \end{bmatrix} \tag{4.49}$$

$$\begin{bmatrix} \Delta u_{min} + u_{k-1} \\ \Delta u_{min} \\ \vdots \\ \Delta u_{min} \end{bmatrix} \leq \Lambda U \leq \begin{bmatrix} \Delta u_{max} + u_{k-1} \\ \Delta u_{max} \\ \vdots \\ \Delta u_{max} \end{bmatrix} \tag{4.50}$$

$$\Delta U_{min} + I_{1,u}u_{k-1} \quad \leq \quad \Lambda U \quad \leq \quad \Delta u_{max} + I_{1,u}u_{k-1} \tag{4.51}$$

Relation between matrix form of output and control input is used (4.41). Bounds on outputs Y_{min} and Y_{max} are created similarly as bound on inputs (4.49)

$$Y_{min} \leq \Psi x_k + \Gamma U + \Gamma_D D \leq Y_{max} \tag{4.52}$$

$$Y_{min} - (\Psi x_k + \Gamma_D D) \leq \Gamma U \leq Y_{max} - (\Psi x_k + \Gamma_D D) \tag{4.53}$$

Constraints defined in(4.51) and (4.53) can be put together resulting in (4.54).

$$\begin{bmatrix} \Delta U_{min} + I_{1,u}u_{k-1} \\ Y_{min} - (\Psi x_k + \Gamma_D D) \end{bmatrix} \leq \begin{bmatrix} \Lambda \\ \Gamma \end{bmatrix} U \leq \begin{bmatrix} \Delta U_{max} + I_{1,u}u_{k-1} \\ Y_{max} - (\Psi x_k + \Gamma_D D) \end{bmatrix} \tag{4.54}$$

Most of solvers require formulation like presented in (4.1a), so (4.54) must be reformulated as shown in (4.55).

$$\begin{bmatrix} \Lambda \\ \Gamma \\ -\Lambda \\ -\Gamma \end{bmatrix} U \leq \begin{bmatrix} \Delta U_{max} + I_{1,u}u_{k-1} \\ Y_{max} - (\Psi x_k + \Gamma_D D) \\ -(\Delta U_{max} + I_{1,u}u_{k-1}) \\ -(Y_{max} - (\Psi x_k + \Gamma_D D)) \end{bmatrix} \tag{4.55}$$

4.3 State Observer

The Kalman filter, also known as linear quadratic estimation (LQE), is an algorithm that uses a series of measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone. More formally, the Kalman filter operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state. The filter is named for Rudolf E. Kálmán, one of the primary developers of its theory.

The algorithm works in a two-step process. In the prediction step, the Kalman filter produces estimates of the current state variables, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed, these estimates are updated using a weighted average, with more weight being given to estimates with higher certainty. Because of the algorithm's recursive nature, it can run in real time using only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required.

$$\begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_k = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k-1} + L\left(y_{m,k} - \hat{y}_{k-1}\right) \tag{4.56}$$

$$\begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k+1} = \begin{bmatrix} A & E_x \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_k + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \tag{4.57}$$

$$\hat{y}_k = \begin{bmatrix} C & E_y \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_k + D u_k \tag{4.58}$$

$\hat{x}$ is estimate of state variables and $\hat{d}$ is estimate of unmeasured disturbance variables. Matrices A, B, C, D represents discrete time state space model of laboratory distillation column. Matrices E, F can be tuned in order to reject all disturbances that might occur during operation.

Adding of integrator to (4.57) causes that our new matrices will have one row more. This integrator can reject noise in signal from column. Matrices E and F was chosen as follows:

$$E_x = \begin{bmatrix} 0 \end{bmatrix} \qquad E_y = \begin{bmatrix} 1 \end{bmatrix} \tag{4.59}$$

Matrices Q and R, which are used for calculation of Kalman gain L, we choose as follows:

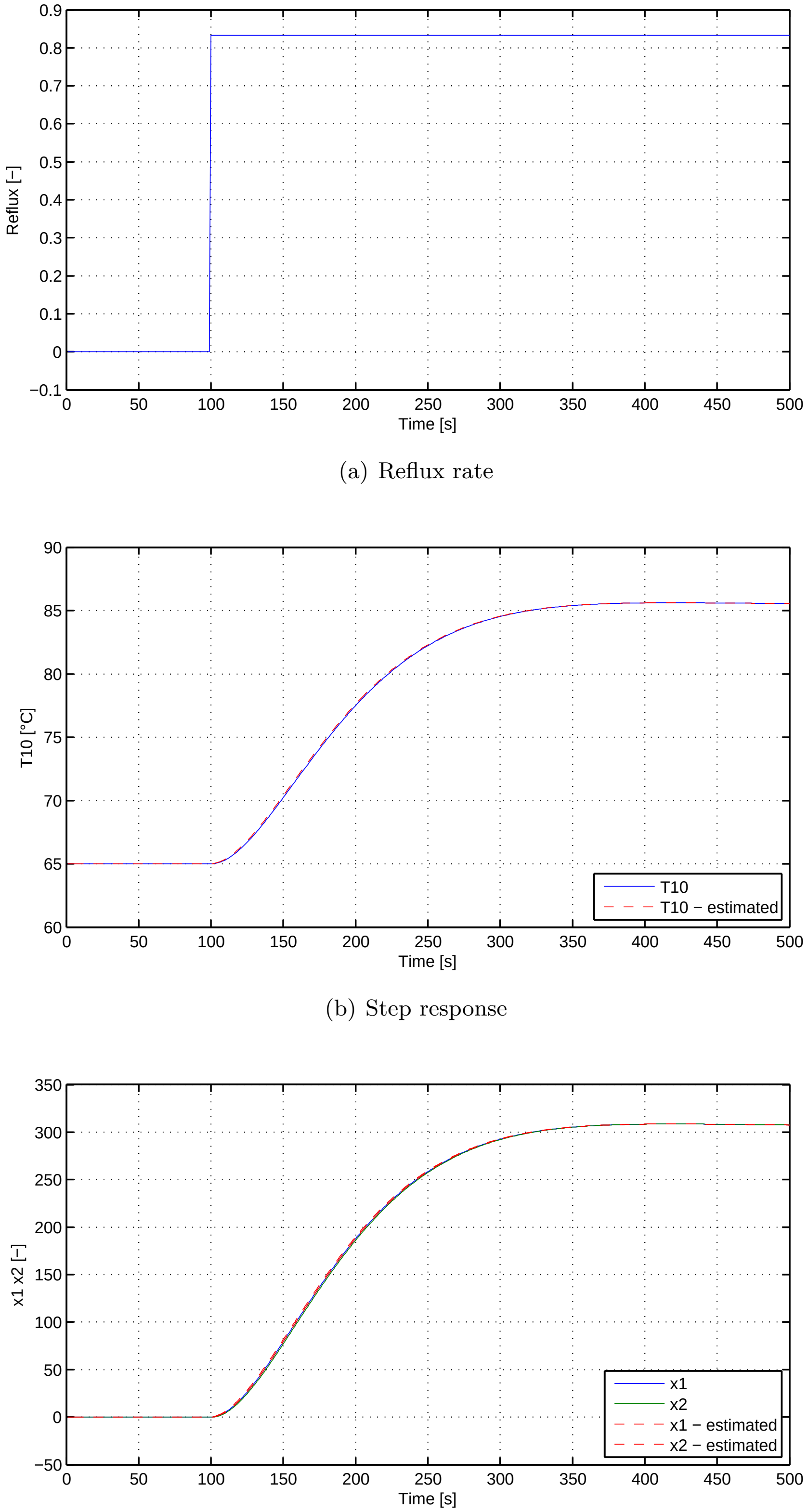
$$Q_{Kalman} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.60}$$

$$R_{Kalman} = \begin{bmatrix} 1.10^{-5} \end{bmatrix} \tag{4.61}$$

Using MATLAB function **dlqe** we enumerated values of vector L shown in (4.62).

$$L = \begin{bmatrix} 1.0098 \\ 0.0424 \\ -0.0548 \\ -0.0942 \\ 0.9653 \end{bmatrix} \tag{4.62}$$

Simulations Fig. (4.1(a), 4.1(b), 4.1(c)) tests our kalman filter and estimated variables are correct. State variables shown in Fig. 4.1(c) are very close together with its values, it is caused by dynamics of the system and state-space matrices have its values very similar.



(a) Reflux rate

(b) Step response

(c) Process variables

Figure 4.1: Simulation of kalman filter

Chapter 5

Control of Distillation Column

5.1 Performance of PI Controller

A proportional-integral-derivative controller (PID controller) is a control loop feedback mechanism (controller) widely used in industrial control systems. A PID controller calculates an "error" value as the difference between a measured process variable and a desired setpoint. The controller attempts to minimize the error in outputs by adjusting the process control inputs.

PID control is by far the dominating control structure in industrial practice. The textbook PID controller has following basic structure (Åström and Hägglund, 1995):

$$\frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s \tag{5.1}$$

Using Euler's method for numerical integration $s = \frac{z-1}{T_S z}$ can be created discrete transfer function of the PID controller (Herjólfsson and Hauksdóttir, 2004):

$$\frac{U(z)}{E(z)} = K_P + \frac{K_I T_S z}{z-1} + \frac{K_D(z-1)}{T_S z} \tag{5.2}$$

We are using to control distillation column only PI controller in discrete time, so the (5.2) has simpler form:

$$\frac{U(z)}{E(z)} = K_P + \frac{K_I T_S z}{z-1} \tag{5.3}$$

Using PID parameter tuning toolbox "pidtool" in Matlab we enumerated parameters of PI controller $K_P = 0.6$ and $K_I = \frac{0.6}{400}$. So parallel form of used discrete PI controller with sampling time $T_S = 5s$ is as follows:

$$\frac{U(z)}{E(z)} = 0.6 + \frac{3z}{400(z-1)} \tag{5.4}$$

Filtered data of control are shown in Fig. 5.1. In the Fig. 5.2(b) and 5.3(b) is shown control effort of PI controller. Controller is very aggressive and this control looks like bang-bang control. Such performance of controller causes oscillations around the setpoint of controlled temperature

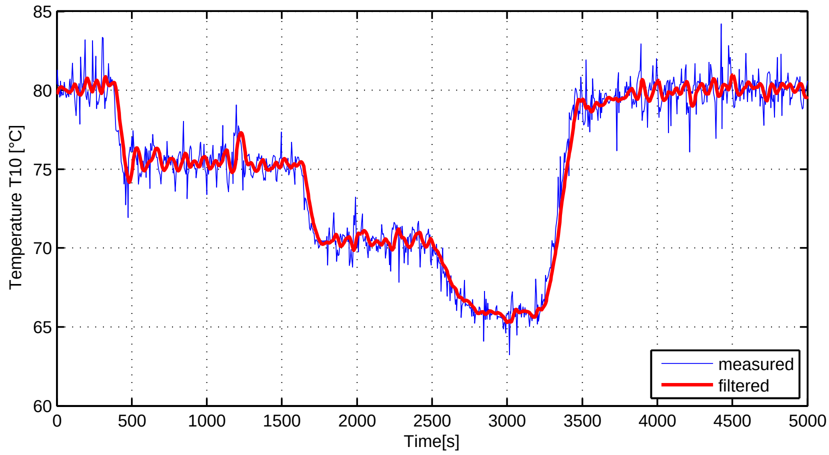
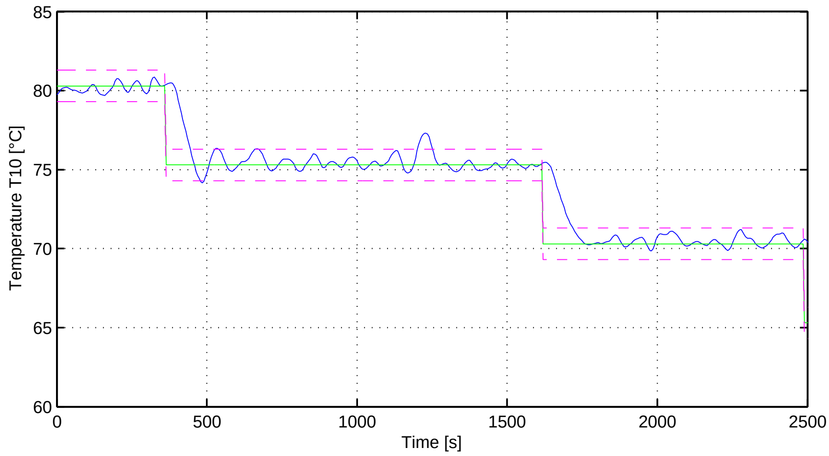
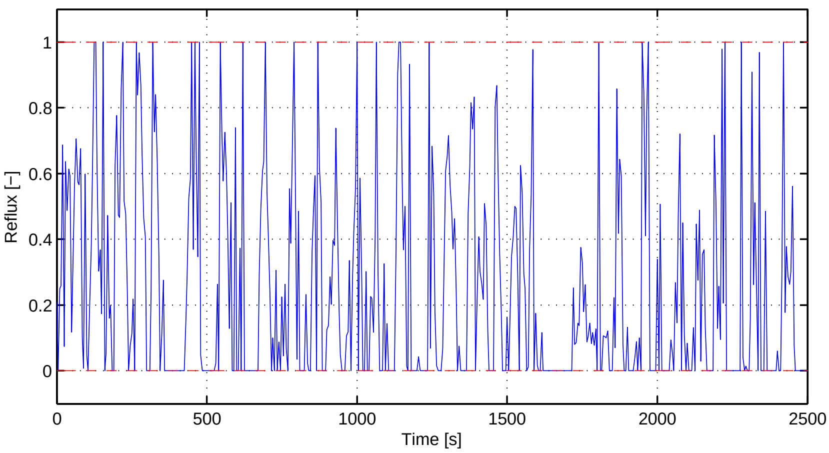


Figure 5.1: Filtered data

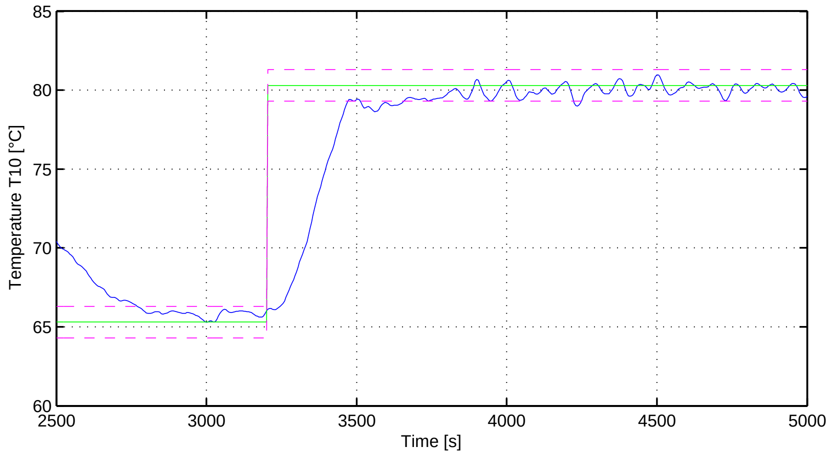


(a) First half of control

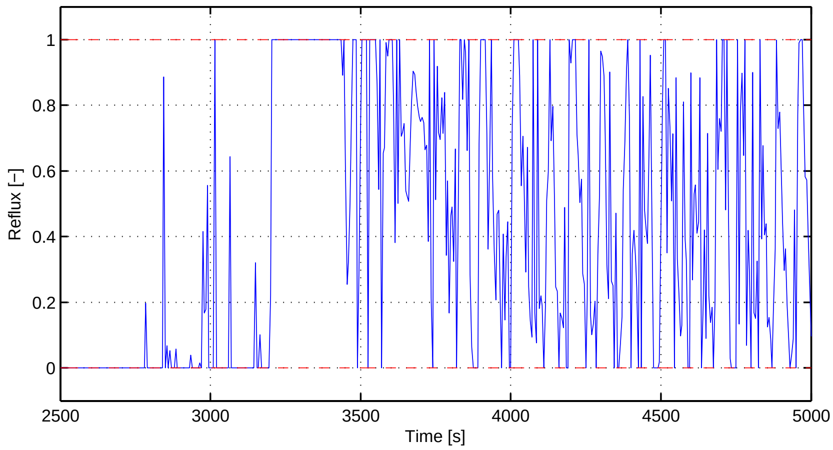


(b) First half of control effort

Figure 5.2: First half of performance of PI controller



(a) Second half of control



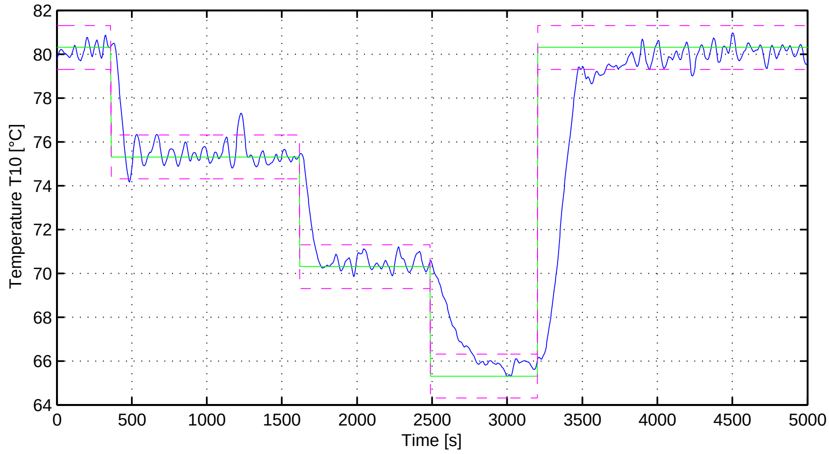
(b) Second half of control effort

Figure 5.3: Second half of performance of PI controller

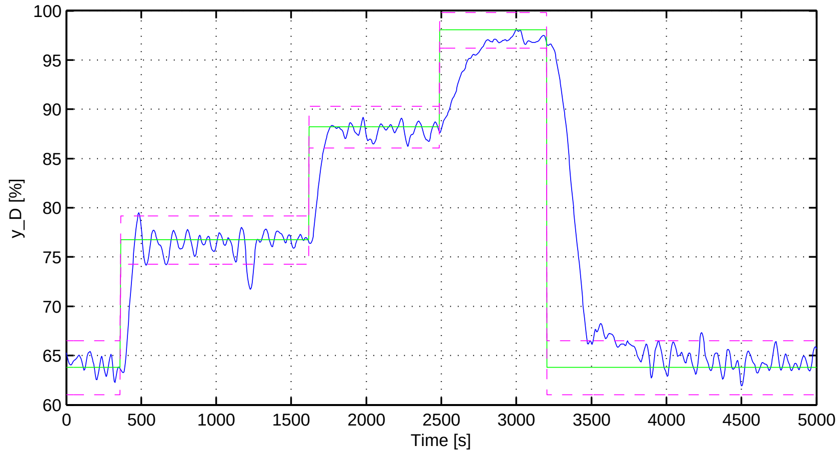
on the top of the column T10. Delta area represents satisfactory control and its value is $\delta = \pm 1$ degree of Celsius.

Whole control sequence is shown in Fig. 5.4(a) and in 5.4(b). Concentration of methanol in distillate was obtained from t-xy diagram, which is shown in 1.2.

PI controller was suitable for control of distillation column, but sometimes values of controlled variable leaved delta-area and this scenario couldn't be accepted on all of devices of this type in industry.



(a) Temperature control



(b) Concentration control

Figure 5.4: Performance of PI controller

5.2 Performance of MPC

To design functional predictive controller the weighting matrices have to be properly chosen. Our weighting matrices from (4.5) are Q which represents penalty on reference tracking and R which represents penalty on control effort. Since we are controlling SISO system, weighting factors are scalar values.

$$Q = 0.5 \qquad\qquad R = 10000 \tag{5.5}$$

Value of R is quite a large number compare to Q. This is caused by numerical structure of objective function and matrices of the model. Because state variables represented by model of reflux rate are acquiring greater values and if they are squared, the difference between value of regulation and value of control input are different by 4 orders of magnitude. Laboratory distillation column is very sensitive on large changes of reflux rate and high value of R penalizes high controller activity so the whole device is more stable with this value of R.

Matrices of model (3.69)-(3.72) and matrices of disturbance (4.59) was substitute into (4.4a)

Our system has some constraints, that have to be satisfied. Constraint Our system has constraints, that have to be satisfied. Constraint on reflux rate should be only between zero and one and mathematical equation looks as follows:

$$0 \leq u(t) \leq 1 \tag{5.6}$$

From t-xy diagram 1.2 we can see, that exist some limitation of concentration of methanol in vapour phase and on the top of the distillation column is only vapour phase which condensates in condenser. This fact creates another constraints on outputs from our system. This constraint can be mathematically written as follows:

$$64.5°C \leq y_{T10} \leq 100°C \tag{5.7}$$

with respect to temperature or with respect to composition:

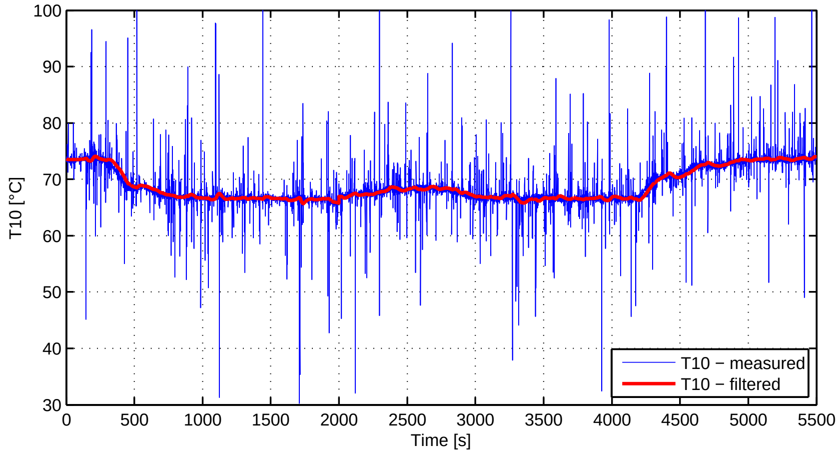
$$0\% \leq y_c \leq 100\% \tag{5.8}$$

In Fig. 5.5(b) is shown whole control sequence of control with MPC.

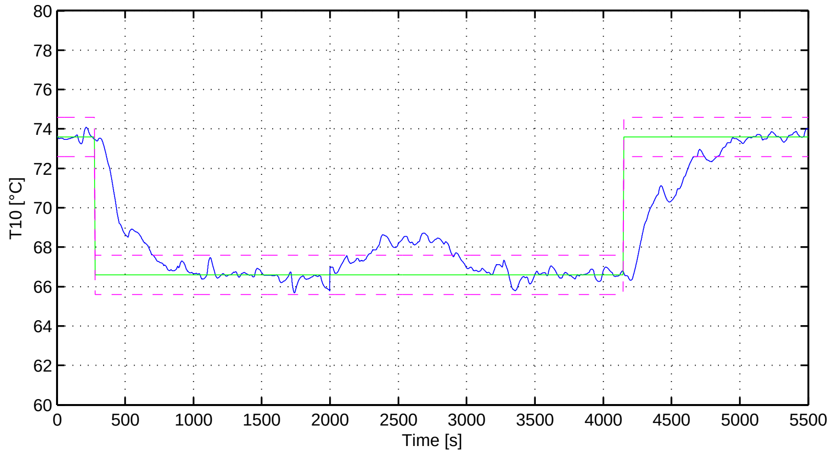
As we can see from Fig. 5.5(c) control effort of MPC controller is less aggressive as control effort of PI controller, which makes system stable and amplitudes of oscillations around reference value in Fig. 5.5(b) are smaller.

If we separate control sequence 5.5(b) into three different parts of control. First part shown in Fig. 5.6 represents negative step change of reference on temperature on the top of the column and we can see, that performance could be faster, but if we increase value of weighting matrix Q the system might goe unstable. Controlled variable T10 doesn't leaving delta-area, so this part of performance has satisfactory results.

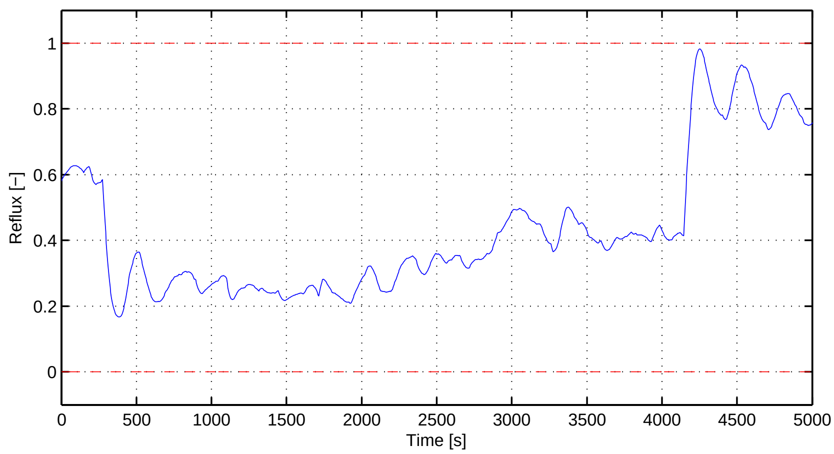
Second part of performance is positive set-point change of temperature T10 and is shown in Fig. 5.7. Result of this performance is satisfactory with respect to delta-area.



(a) Filtered data

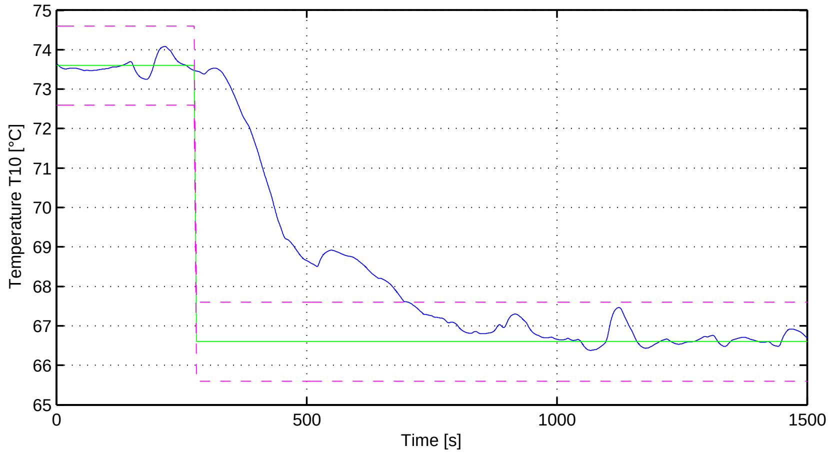


(b) Control sequence

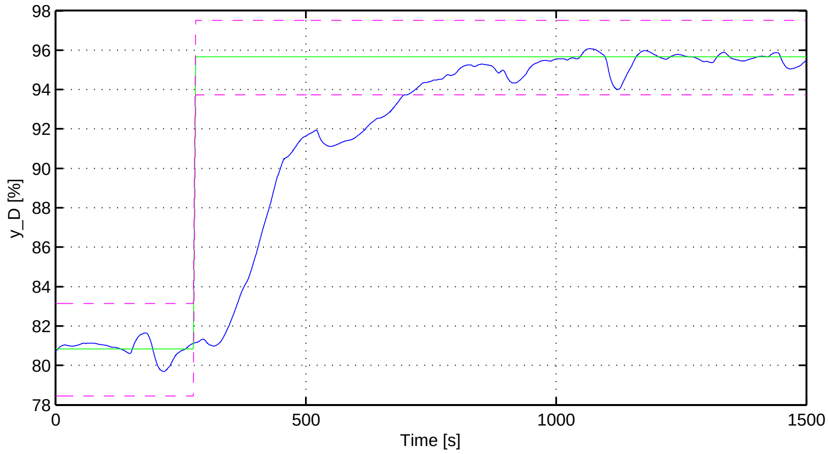


(c) Control effort sequence

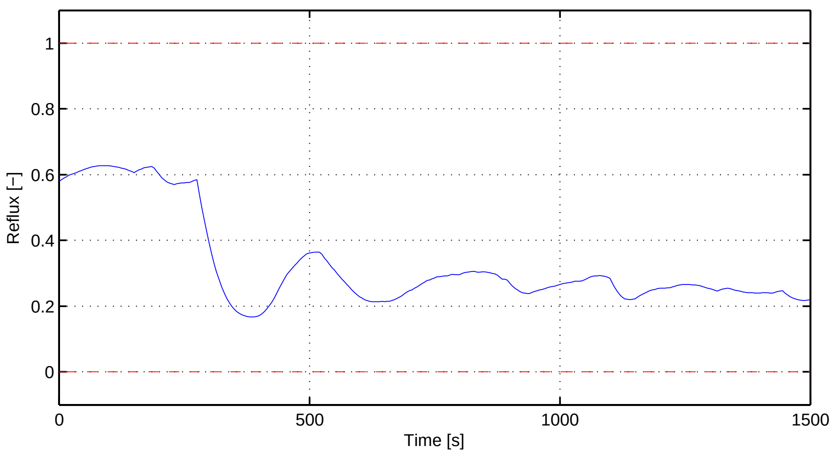
Figure 5.5: Performance of MPC



(a) First part of control. Temperature profile.

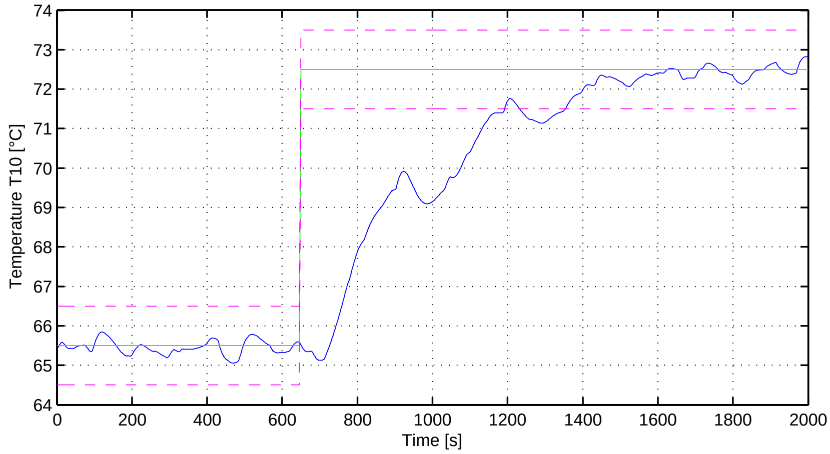


(b) First part of control. Concentration profile.

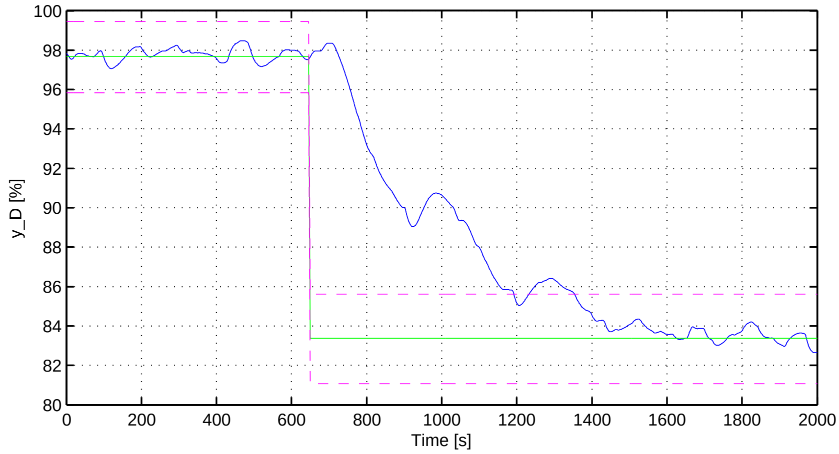


(c) First part of controller effort

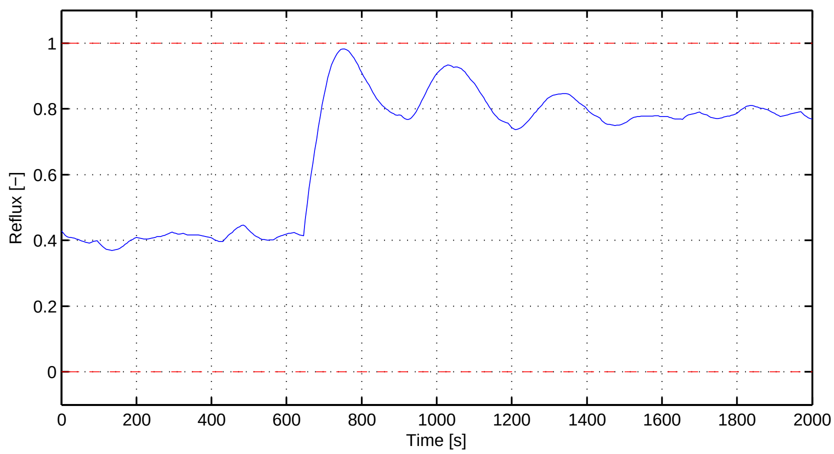
Figure 5.6: Negative set point change of temperature



(a) Second part of control. Temperature profile.

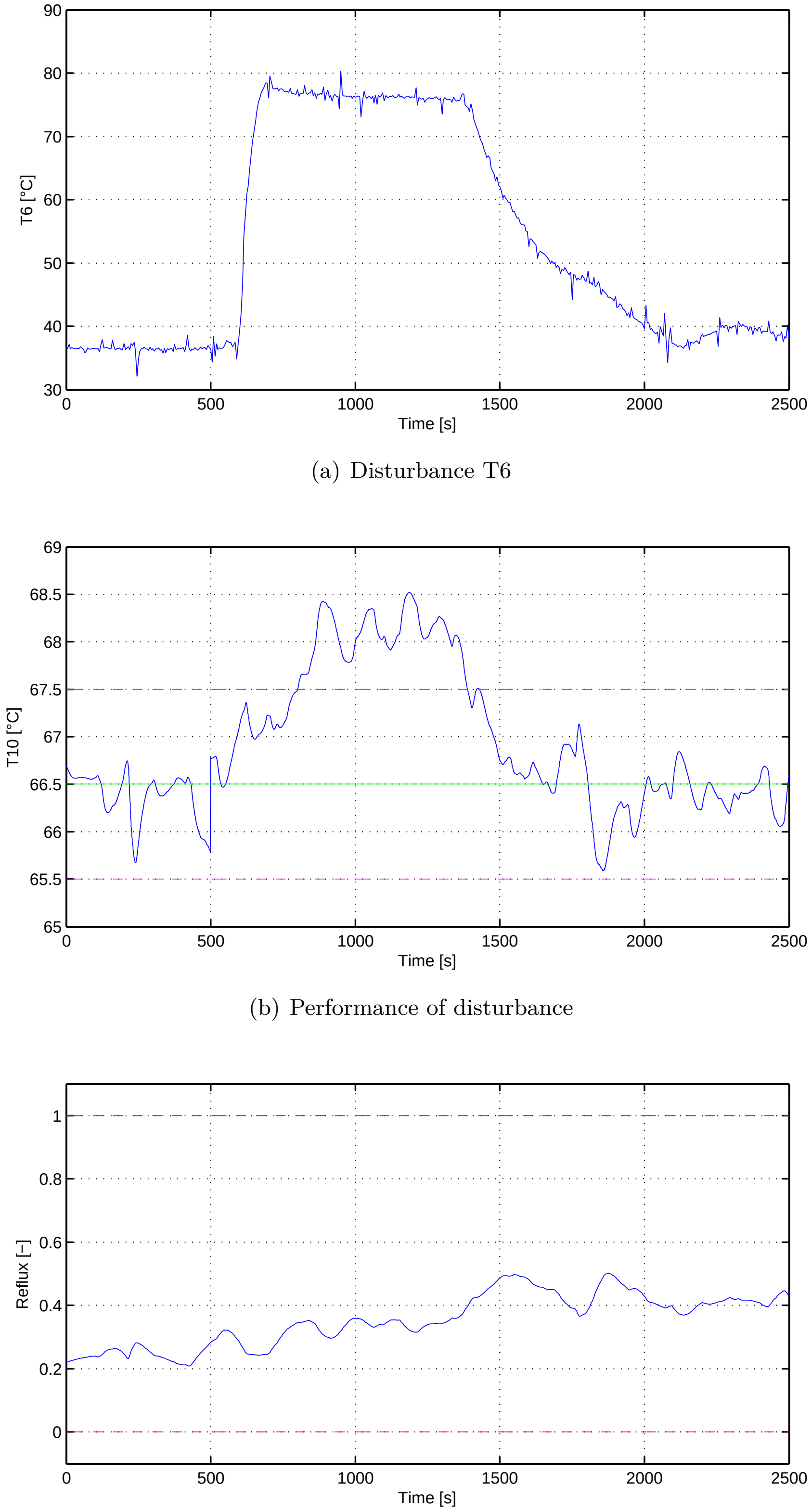


(b) Second part of control. Concentration profile.



(c) Second part of controller effort

Figure 5.7: Positive set point change of temperature



(a) Disturbance T6

(b) Performance of disturbance

(c) Controller effort

Figure 5.8: Performance of disturbance

Third part deals with eliminating measured disturbance. As we can see in Fig. 5.8(a) the value of temperature of feed increased two times and MPC controller slightly increased reflux rate (5.8(c)), this change start eliminating the disturbance influence on T10 measurement, and measurement of temperature goes back to delta-area.

Chapter 6

Conclusions

In this diploma project we made step changes on reflux rate on laboratory distillation column in order to identify transfer function of model of reflux rate using recursive least squares. Then we made change on disturbance variable, which represents temperature of feed, to identify transfer function between temperature on the top of the column and temperature of feed. We had to normalized data to unit step change and because measured data had noise, so we had to filtered measured data using Buttersworth filter.

To identification we used recursive least squares method. The first step was derivation of vector of parameters $\tilde{\theta}$ and then we created simulink schemes which including s-function block RLS to enumerating values of parameters of our identified model. This model was compared with measured data. Using identification toolbox we found that fit of our model is 81.25% and fit of our model of disturbance is 81.98%.

To design MPC controller, state observer was needed. Optimal state observer also known as kalman filter was created and than tested on simulations. Estimated variables were the same as process variables, that was the proof, that our kalman filter is correct.

To control of distillation column was used PI controller. This controller had satisfactory results, but its oscillations sometimes goes out of delta area and the system was on the bound of the stability. Designed MPC had to be first tested via simulations and then was implicated on our laboratory distillation column. Oscillations around the setpoint were smaller than using PI controller. Controller effort was slower, but it was caused by values of penalty matrices Q and R in (5.5). Decreasing value of matrix S could caused instability, but the controller will be faster.

Chapter 7

Resumé

Diplomová práca s názvom Modelling and Controlling Laboratory Distillation Column sa zaoberá identifikáciou modelov laboratórnej rektifikačnej kolóny, následnou verifikáciou identifikovaných modelov na reálnom zariadení. Po získaní prenosových funkcií medzi teplotou na hlave kolóny a refluxným pomerom a medzi teplotou na hlave kolóny a teplotou suroviny na nástrekovej etáži, ktorá reprezentuje merateľnú poruchu, sme vytvorili stavové matice modelu systému aj s poruchou. Po verifikácii získaného modelu sme pristúpili k návrhu optimálneho pozorovača, ktorým bol kalmanov filter. Simulačne sme overili správnosť kalmanovho filtra a odhadnuté stavové veličiny mohli byť použité na návrh prediktívneho riadenia rektifikačnej kolóny. Aby sme mohli porovnať výsledky prediktívneho regulátora, bol na kolónu navrhnutý PI regulátor. Jednotlivé priebehy riadenia týchto regulátov boli následne porovnané za účelom vyzdvihnutia určitých výhod prediktívneho riadenia, ktoré aj priemyseľ v poslednom čase rozoznáva.

Na identifikáciu bola použitá rukurzívna metóda najmenších štvorcov, ktorá identifikuje vektor parametrov prenosovej funkcie modelu. Zvolené identifikované prenosy boli druhého rádu, ktoré majú za úlohu zachytiť dynamiku systému. Ukázalo sa, že zvolený rád je postačujúci a po diskretizácii mohli byť tieto modely použité na vytvorenie stavových matíc, ktoré sú nevyhnutné pre návrh prediktívneho riadenia.

Kalmanov filter bol navrhnutý pomocou príkazu **dlqe** v softwareovom prostredí MATLAB. Simulačné overenie pozorovača ukázalo, že všetky odhadnuté veličiny sú totožné s veličinami identifikovaného modelu prenosu. Stavový pozorovač bol ďalšou nevyhnutnosťou na návrh prediktívneho riadenia, nakoľko MPC potrebuje poznať všetky hodnoty stavových veličín v každej perióde vzorkovania.

Na základe identifikovaného modelu refluxného pomeru, sme navrhli PI regulátor, ktorý bol následne použitý na riadenia laboratórnej rektifikačnej kolóny. Riadenie pomocou PI regulátora poskytlo uspokojivé výsledky, no riadená veličina oscilovala okolo žiadanej hodnoty s väčšou hodnotou amplitúdy ako pri použití prediktívneho riadenia. Niektoré oscilácie dokonca opustili prípustné delta okolie. Táto vlastnosť môže byť v priemysle nežiadúca a preto sa nedá jednoznačne prehlásiť, že PI regulátor vyhovoval požiadavkam na riadenie.

Výhodou nasadenia prediktívneho riadenia je zavedenie optimálnych akčných zásahov a dodržiavanie bezpečnostných ohraničení a požiadaviek, ktoré PI regulátor nie je schopný zabezpečiť.

Prediktívne regulátory majú dve nevýhody. Potrebujeme poznať presné matematické modely procesov a druhou nevýhodou môže byť, za určitých okolností, výpočtová náročnosť. Navrhnutý prediktívny regulátor bol následne použitý na riadenie laboratórnej rektifikačnej kolóny a priebeh riadenia ukázal, že riadená veličina oscilovala menej okolo žiadanej hodnoty a tieto oscilácie už neopúšťali prípustné delta okolie, čo je vlastnosť, ktorú PI regulátor nevedel zabezpečiť.

Riadením zariadenia pomocou prediktívneho regulátora sme dokázali, že táto forma zabezpečí dodržanie ohraničení a je vhodná na riadenie takýchto typov zariadení.

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