SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

FACULTY OF CHEMICAL AND FOOD TECHNOLOGY

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Optimal Control of Industrial Storage Tanks

MASTER THESIS

2018 **Bc. Juraj Kukla**

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Study programme:	Process Control
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Thesis supervisor:	Ing. Martin Klaučo, PhD.

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Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation and Mathematics

Faculty of Chemical and Food Technology
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MASTER THESIS TOPIC

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Study programme:	Automation and Information Engineering in Chemistry and Food Industry
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Language of thesis: English

Specification of Assignment:

This thesis deals with the mathematical modeling and optimal control of storage tanks found in industry. Objectives of the thesis:

1. derive a nonlinear mathematical model for conical, horizontal cylindrical, and spherical storage tanks,
2. linearisation and analysis of step responses,
3. design of simple control strategy for each tank
4. design of advanced control strategy, including non-linear model predictive control.

Length of thesis: 60

Selected bibliography:

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Abstract

This master thesis deals with the design of optimal control strategies that are used for liquid level control in industrial storage tanks. In the process industry, we can come into the contact with tanks of various geometry where the cross-section is dependent on the level of the liquid. The most widespread tank types are of conical, spherical and horizontal-cylindrical shapes. This thesis consists of four parts. The goal of the first part is to derive a nonlinear dynamic mathematical model of each tank in continuous and discrete time. After that, we will compare the step responses of linear and nonlinear mathematical models. The second part of this thesis deals with the design of simple control strategies represented by PID and LQ controllers. In the third part, we will focus on the design of an advanced control strategies based on the predictive controller (MPC). The last part describes the case studies, where all control strategies are compared and evaluated.

Abstrakt

Táto diplomová práca sa venuje návrhu optimálnych riadiacich algoritmov slúžiacich na riadenie výšky hladiny v priemyselných zásobníkoch kvapaliny. V priemysle sa často stretávame so zásobníkmi s rôznou geometriou, v ktorých výška hladiny sa dynamicky mení v závislosti od prierezu daného zásobníka. Najrozšírenejšie sú zásobníky kužeľového, guľového a horizontálne valcového tvaru, ktorým sa budeme podrobnejšie venovať. Táto práca pozostáva zo štyroch častí. V prvej časti odvodíme dynamické matematické modely jednotlivých zásobníkov kvapaliny v spojitom aj diskrétnom čase a porovnáme odozvy lineárneho a nelineárneho modelu na skokové zmeny. Druhá časť sa zaoberá návrhom jednoduchých riadiacich algoritmov reprezentovaných PID a LQ regulátormi. V tretej časti sa budeme venovať návrhu pokročilého riadenia pomocou prediktívneho regulátora (MPC). V poslednej časti porovnáme jednotlivé prístupy riadenia v prípadových štúdiách a zhodnotíme výsledky.

Contents

CHAPTER 1

Introduction

Chemical processes present many challenging control problems due to nonlinear dynamic behavior, uncertain and time varying parameters, constraints on manipulated variable, interaction between manipulated and controlled variables, unmeasured and frequent disturbances, dead time on input and measurements, etc [11].

Liquid level control in storage tanks is a basic problem in process industry. The goal of this thesis is to design and evaluate several control strategies used for liquid level control in industrial storage tanks of various geometry. Among the most frequent storage tanks in the process industry are tanks of conical, spherical and horizontal cylindrical shapes that cover the core of this work.

In the first part of this thesis, we will derive a dynamic mathematical model of each tank in continuous and discrete time. Each storage tank represents a first order SISO system with single input q_{in} and single output q_{out} that is the volumetric flow. Then, we need to derive a linear mathematical model of each tank via calculating the Jacobian in order to design the controllers. After that, we compare the step responses of nonlinear and linear mathematical model in continuous and in discrete time.

The second part of this thesis consist of the theoretical base where various control strategies are presented. First, we will focus on simple control strategies represented by PID and LQ controllers. We derive the relations how to design such controllers and what are their advantages and disadvantages. Then, we focus on an advanced control strategy which is presented by a model predictive control (MPC). This part consists of two MPC formulations, the first one for linear MPC with time-varying Kalman Filter and the second one for nonlinear MPC.

The last part of this work deals with the simulation case studies. We will go through the control with various controllers, particularly PID, LQ and MPC, and evaluate each control via performance indexes like integral square error (ISE) criterion or calculation the sums of objective functions. At the end of this part we will show the simulation for one of the tanks when using nonlinear MPC we are able to control on maximal liquid level in the tank with respect all constraints.

Chapter 2

Mathematical Modeling

This chapter deals with the development of mathematical models of conical, spherical and horizontal cylindrical tanks in continuous and discrete time. In order to derive a mathematical model, we need to know the physical and chemical principles of the studied process that are expressed by mass and enthalpy balances[5]. In this thesis, we consider a continuous processes with mass accumulation represented by water tanks of various geometry. Particular dynamical models are based on equations in [8].

The dynamical mathematical model of a tank with one inlet stream denoted as $q_{\text{in}}(t)$ and one outlet stream given by $q_{\text{out}}(t)$ is given by a mass balance equation of the following form

$$q_{\text{in}}(t) = q_{\text{out}}(t) + \frac{\mathrm{d}V(t)}{\mathrm{d}t}. \tag{2.1}$$

The volume of the liquid inside the tank $V(t)$ is given by relation

$$V(t) = f\big(F\big(h(t)\big)\big), \tag{2.2}$$

where $F(h(t))$ represents the base of the tank that depends on the liquid level $h(t)$ in the tank. Also, it is known from Bernoulli's principle that outlet stream is the function of liquid level in the tank and can be expressed as follows

$$q_{\text{out}}(t) = k_{\text{v}}\sqrt{h(t)}, \tag{2.3}$$

where k_{v} represents the valve coefficient. Once the assumptions (2.2) and (2.3) have been accepted, we can rewrite the model in (2.1) to

$$q_{\text{in}}(t) = k_{\text{v}}\sqrt{h(t)} + \frac{\mathrm{d}V(t)}{\mathrm{d}h}\frac{\mathrm{d}h(t)}{\mathrm{d}t}. \tag{2.4}$$

After the substitution $\dfrac{\mathrm{d}V(t)}{\mathrm{d}h}$ to $F(h)$ we obtain the nonlinear mathematical model in form

$$q_{\text{in}}(t) = k_{\text{v}}\sqrt{h(t)} + F(h)\frac{\mathrm{d}h(t)}{\mathrm{d}t}. \tag{2.5}$$

In general, the nonlinear state space model is given by the following relations

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f\left(x(t), u(t), t\right), \tag{2.6a}$$

$$y(t) = g\left(x(t), u(t), t\right), \tag{2.6b}$$

where states are denoted as $x \in \mathbb{R}^{n_\mathrm{x}}$, measured signals are denoted by $y \in \mathbb{R}^{n_\mathrm{y}}$ and by $u \in \mathbb{R}^{n_\mathrm{u}}$ is represented the vector of manipulated variables. In this thesis, we consider the state $x(t)$ is equal to the measured variable $y(t)$ and is represented by the liquid level $h(t)$ in tank. The manipulated variable is the input stream $q_{\text{in}}(t)$.

Rewriting the model in (2.5) to general state space model form defined in (2.6) we get a general nonlinear mathematical model of tank

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \frac{q_{\text{in}}(t) - k_\mathrm{v}\sqrt{h(t)}}{F(h)}, \tag{2.7a}$$

$$y(t) = h(t), \tag{2.7b}$$

with initial condition $h(0) = h^\mathrm{s}$, where the variable h^s stands for the steady state value of the process variable.

In steady state the level of the liquid in the tank is constant and time derivation of the height of level is equal to zero. Then mathematical model of tank defined in (2.5) is in steady state expressed as follows

$$q_{\text{in}}^\mathrm{s} = k_\mathrm{v}\sqrt{h^\mathrm{s}}. \tag{2.8}$$

Let the input stream q_{in}^s in steady state be known. Then from (2.8) we can evaluate the liquid level in steady state as

$$h^\mathrm{s} = \left(\frac{q_{\text{in}}^\mathrm{s}}{k_\mathrm{v}}\right)^2. \tag{2.9}$$

Linear state space model which is sufficiently accurate for simple control purposes is expressed as

$$\dot{x}(t) = \widetilde{A}x(t) + \widetilde{B}u(t), \tag{2.10a}$$

$$y(t) = Cx(t) + Du(t), \tag{2.10b}$$

where $\widetilde{A} \in \mathbb{R}^{n_\mathrm{x} \times n_\mathrm{x}}$, $\widetilde{B} \in \mathbb{R}^{n_\mathrm{x} \times n_\mathrm{u}}$, $C \in \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{x}}$ and $D \in \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{u}}$ with zero initial conditions. All matrix elements of linear state space model are constant and therefore the model is said to be time invariant. Matrices in (2.10) can be obtained by calculating the Jacobian based on the following relation

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_\mathrm{n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_\mathrm{m}}{\partial x_1} & \cdots & \frac{\partial f_\mathrm{m}}{\partial x_\mathrm{n}} \end{bmatrix}_{x^\mathrm{s}, u^\mathrm{s}}, \tag{2.11}$$

where the subscript $x^\mathrm{s}, u^\mathrm{s}$ indicates that all entries in the matrix are calculated at the stationary (linearization) points. However, in this thesis, we assume SISO systems. Then matrices $\tilde{A}$, $\tilde{B}$, C and D are given as

$$\widetilde{A} = \left.\frac{\mathrm{d}f(\cdot)}{\mathrm{d}h^\mathrm{s}}\right|_{h^\mathrm{s}}, \tag{2.12a}$$

$$\widetilde{B} = \left.\frac{\mathrm{d}f(\cdot)}{\mathrm{d}q_\mathrm{in}^\mathrm{s}}\right|_{q_\mathrm{in}^\mathrm{s}}, \tag{2.12b}$$

$$C = \left.\frac{\mathrm{d}g(\cdot)}{\mathrm{d}h^\mathrm{s}}\right|_{h^\mathrm{s}}, \tag{2.12c}$$

$$D = \left.\frac{\mathrm{d}g(\cdot)}{\mathrm{d}q_\mathrm{in}^\mathrm{s}}\right|_{q_\mathrm{in}^\mathrm{s}}, \tag{2.12d}$$

where the functions $f(\cdot)$ and $g(\cdot)$ represent the nonlinear mathematical model derived in (2.6).

It is often practical to reformulate the mathematical model in continuous time to discrete time in order to design an advanced discrete controllers like MPC. As we defined the nonlinear state space model in continuous time in (2.13) we can also define the nonlinear state space model in discrete time as follows

$$x(t + T_\mathrm{s}) = x(t) + T_\mathrm{s} f(x(t), u(t), t), \tag{2.13a}$$

$$y(t) = g\left(x(t), u(t), t\right), \tag{2.13b}$$

where the functions $f(\cdot)$ and $g(\cdot)$ are the same as in (2.7) and T_s stands for the sampling time.

Whereas the linear mathematical model in continuous time (2.10) is defined as a set of first order differential equation and equation of output, the discrete time formulation can be expressed as a set of difference equation and output equation in a following form

$$x(t + T_\mathrm{s}) = Ax(t) + Bu(t), \tag{2.14a}$$

$$y(t) = Cx(t) + Du(t), \tag{2.14b}$$

where the matrices A and B are calculated by utilizing the following property

$$\mathrm{e}^{\begin{bmatrix}\widetilde{A} & \widetilde{B}\\ 0 & 0\end{bmatrix}T_{\mathrm{s}}} = \begin{bmatrix}M_{11} & M_{12}\\ 0 & I\end{bmatrix}, \tag{2.15}$$

and then

$$A = M_{11}, \tag{2.16a}$$
$$B = M_{12}. \tag{2.16b}$$

The matrices C and D in discrete time are the same as in (2.10b). The discrete time system matrices (A, B) can be calculated also via a zero-order hold discretization in MATLAB.

To derive individual models for the conical, spherical and horizontal-cylindrical tanks, we adopt the procedure from the Section 2.

2.1 Conical Storage Tank

We consider an inverted frustum of a right cone as a conical tank process that is characterized by technological parameters in the Table 2.1.

Table 2.1: Technological parameters of the conical storage tank.

Variable	Unit	Value
R_1	m	1.000
R_2	m	0.200
$h_{\max}$	m	2.000
k_{v}	$\mathrm{m}^{2.5}\mathrm{s}^{-1}$	0.075

The geometrical representation of the conical tank is shown in the Fig. 2.1. The mathematical model of such a process is derived by expressing the volume of the frustum as a function of level of the liquid. Conical storage tank is characterized by variables R_1 and R_2 which are radii of the bottom and upper base, and by the maximal height of the storage tank $h_{\max}$. The volume of the liquid inside the frustum is given by

$$V_{\mathrm{f}}(h(t)) = \frac{\pi h(t)}{3}\left(r_{\mathrm{f}}^2(h(t)) + R_2 r_{\mathrm{f}}(h(t)) + R_2^2\right), \tag{2.17}$$

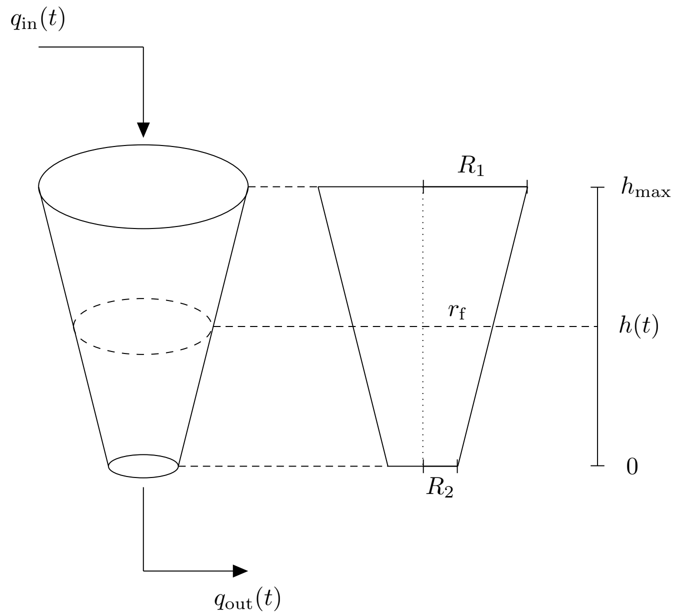

Figure 2.1: Illustration of the conically-shaped tank.

where the variable $r_{\text{f}}(h(t))$ represents the radius of a disc representing the surface of the liquid at level $h(t)$. The radius $r_{\text{f}}(h(t))$ is explicit function of the liquid level, expressed as

$$r_{\text{f}}(h(t)) = R_2 + \frac{R_1 - R_2}{h_{\max}} h(t). \tag{2.18}$$

By substituting the expression in (2.18) to (2.17) we obtain

$$V_{\text{f}}(h(t)) = \frac{\pi h(t)}{3} \left(3R_2^2 + 3R_2 \frac{R_1 - R_2}{h_{\max}} h(t) + \left(\frac{R_1 - R_2}{h_{\max}} \right)^2 h^2(t) \right). \tag{2.19}$$

Now, we can combine the expression for the volume in (2.19) and the general mass balance model in (2.5), which results in

$$q_{\text{in}}(t) = k_{\text{v}} \sqrt{h(t)} + \pi \left(R_2 + h(t) \frac{R_1 - R_2}{h_{\max}} \right)^2 \frac{\mathrm{d}h(t)}{\mathrm{d}t}. \tag{2.20}$$

Hence, the nonlinear dynamical mathematical model of conical storage tank with output equation based on (2.6) can be expressed as

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \frac{q_{\mathrm{in}}(t) - k_{\mathrm{v}}\sqrt{h(t)}}{\pi\left(R_2 + h(t)\dfrac{R_1 - R_2}{h_{\max}}\right)^2}, \tag{2.21a}$$

$$y(t) = h(t), \tag{2.21b}$$

with initial condition $h(0) = h^{\mathrm{s}}$. The liquid level in the tank in steady state is known. Then based on the equation (2.8), we can calculate the inlet flow in steady state. Linearized mathematical model of the conical storage tank that is defined in general as (2.10), we obtain by calculating the Jacobian (2.12) as follows

$$\widetilde{A} = -\frac{h_{\max}^2\left(h_{\max} R_2\, k_{\mathrm{v}} - 3\, R_1\, h^{\mathrm{s}}\, k_{\mathrm{v}} + 3\, R_2\, h^{\mathrm{s}}\, k_{\mathrm{v}} + 4\, R_1 \sqrt{h^{\mathrm{s}}}\, q_{\mathrm{in}}^{\mathrm{s}} - 4\, R_2 \sqrt{h^{\mathrm{s}}}\, q_{\mathrm{in}}^{\mathrm{s}}\right)}{2\sqrt{h^{\mathrm{s}}}\,\pi\left(R_1\, h^{\mathrm{s}} - R_2\, h^{\mathrm{s}} + h_{\max}\, R_2\right)^3}, \tag{2.22a}$$

$$\widetilde{B} = \frac{1}{\pi\left(\dfrac{h^{\mathrm{s}}\left(R_1 - R_2\right)}{h_{\max}} + R_2\right)^2}, \tag{2.22b}$$

$$C = 1, \tag{2.22c}$$

$$D = 0. \tag{2.22d}$$

The nonlinear mathematical model of conical tank in discrete time is defined based on (2.13), specifically

$$h(t + T_{\mathrm{s}}) = h(t) + T_{\mathrm{s}} \cdot \frac{q_{\mathrm{in}}(t) - k_{\mathrm{v}}\sqrt{h(t)}}{\pi\left(R_2 + h(t)\dfrac{R_1 - R_2}{h_{\max}}\right)^2} \tag{2.23}$$

Linear mathematical model in discrete time is expressed by (2.14) where matrices A and B are calculated from (2.15) and (2.16).

2.2 Spherical Storage Tank

The process considering in this section is represented by spherical storage tank with technological parameters shown in Table 2.2 and the geometrical representation of this storage tank is depicted in the Fig. 2.2.

Table 2.2: Technological parameters of the spherical storage tank.

Variable	Unit	Value
R	m	2.00
k_{v}	$\mathrm{m}^{2.5}\mathrm{s}^{-1}$	0.75

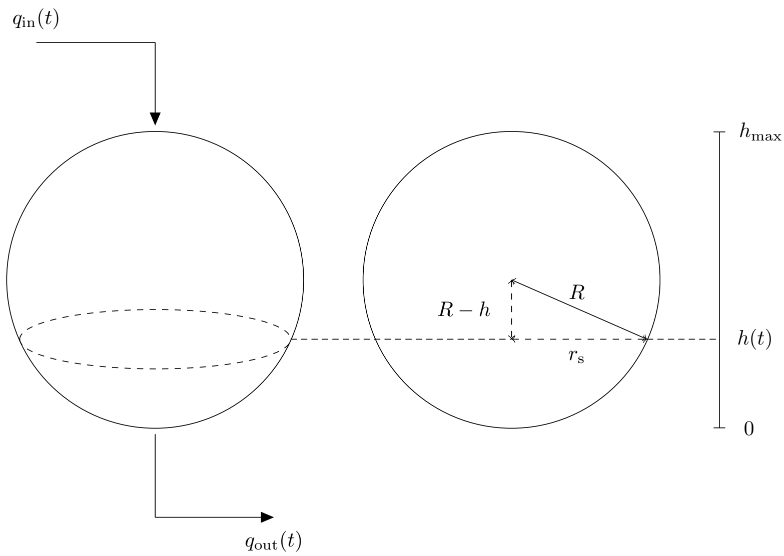

Figure 2.2: Illustration of the spherically-shaped tank.

The mathematical model of this process is derived by expressing the volume of the spherical segment of one base as a function of liquid level. This process is characterized by variable R, that represents the radius of the sphere. The volume of the liquid inside the spherical tank is given by

$$V_{\mathrm{s}}(h(t)) = \frac{\pi h(t)}{6}\left(3r_{\mathrm{s}}^2(h(t)) + h^2(t)\right), \tag{2.24}$$

where the variable $r_{\mathrm{s}}(h(t))$ represents the radius of a disc that is surface of the liquid at level $h(t)$. The radius as a function of the liquid level is expressed as

$$r_{\mathrm{s}}(h(t)) = \sqrt{R^2 - (R - h(t))^2} \tag{2.25}$$

By substituting the expression in (2.25) to (2.24) we obtain

$$V_{\mathrm{s}}(h(t)) = \frac{\pi h(t)}{3}\left(3Rh(t) - h^2(t)\right). \tag{2.26}$$

Then, we combine the expression for the volume in (2.26) and the general mass balance model defined in (2.5), which results

$$q_{\text{in}}(t) = k_{\text{v}}\sqrt{h(t)} + \pi\left(2Rh(t) - h^2(t)\right)\frac{\mathrm{d}h(t)}{\mathrm{d}t}. \tag{2.27}$$

Rewriting the model in (2.27) to general state space model form defined in (2.7), we get a nonlinear mathematical model of spherical storage tank

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \frac{q_{\text{in}}(t) - k_{\text{v}}\sqrt{h(t)}}{\pi\left(2Rh(t) - h^2(t)\right)}, \tag{2.28a}$$

$$y(t) = h(t), \tag{2.28b}$$

with initial condition $h(0) = h^{\text{s}}$. The liquid level in the tank is known in steady state. Then based on the equation (2.8), we can calculate the inlet flow in steady state. Linearized mathematical model of the conical storage tank, that is defined in general as (2.10), we obtain by calculating the Jacobian (2.12) as follows

$$\widetilde{A} = \frac{4q_{\text{in}}^{\text{s}}\left(h^{\text{s}} - R\right) + k_{\text{v}}\sqrt{h^{\text{s}}}\left(2R - 3h^{\text{s}}\right)}{2\pi {h^{\text{s}}}^2\left(h^{\text{s}} - 2R\right)^2}, \tag{2.29a}$$

$$\widetilde{B} = -\frac{1}{\pi h^{\text{s}} - 2\pi h^{\text{s}} R}, \tag{2.29b}$$

$$C = 1, \tag{2.29c}$$

$$D = 0. \tag{2.29d}$$

The nonlinear mathematical model of spherical tank in discrete time is defined based on (2.13), specifically

$$h(t + T_{\text{s}}) = h(t) + T_{\text{s}} \cdot \frac{q_{\text{in}}(t) - k_{\text{v}}\sqrt{h(t)}}{\pi\left(2Rh(t) - h^2(t)\right)}. \tag{2.30}$$

Linear mathematical model in discrete time is expressed by (2.14) where matrices A and B are calculated from (2.15) and (2.16).

2.3 Horizontal Cylindrical Storage Tank

In this section we will focus on the horizontal cylindrical storage tank that is given by technological parameters in Table 2.3.

The geometrical representation of the horizontal cylindrical storage tank is shown in the Fig. 2.3. The model of such a process is derived as processes before, by expressing the volume of the circular segment as a function of level of the liquid.

Table 2.3: Technological parameters of the horizontal cylindrical storage tank.

Variable	Unit	Value
R	m	2.00
ℓ	m	4.00
k_{v}	$\mathrm{m}^{2.5}\mathrm{s}^{-1}$	0.75

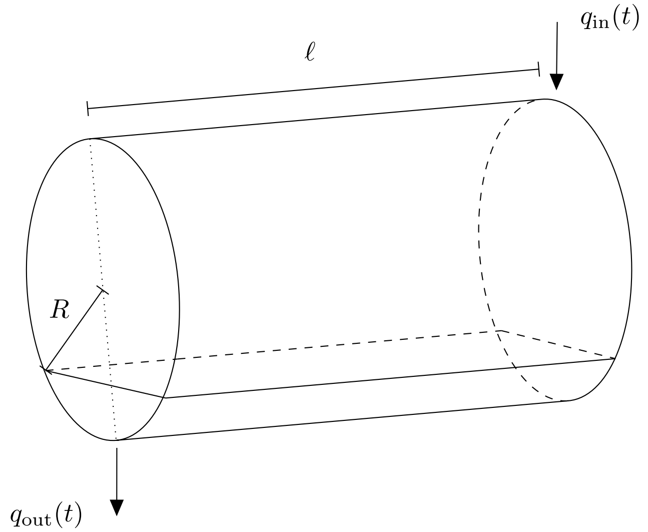

Figure 2.3: Illustration of the horizontal cylindrically-shaped tank.

The tank is characterized by variables R which represents the radius of the upper and bottom base of cylindrical tank and by ℓ that is the length of the cylindrical tank. The volume of the liquid inside the tank as a function of $h(t)$ is given by

$$V_{\mathrm{c}}(h(t)) = \left(R^2 \arccos\left(\frac{R-h(t)}{R}\right) - (R-h(t))\sqrt{2Rh(t)-h^2(t)}\right)\ell. \tag{2.31}$$

Combining the expression for the volume in (2.31) and the general mass balance defined in (2.5) we get

$$q_{\mathrm{in}}(t) = k_{\mathrm{v}}\sqrt{h(t)} + 2\ell\sqrt{h(t)(2R-h(t))}\,\frac{\mathrm{d}h(t)}{\mathrm{d}t} \tag{2.32}$$

Rewriting the model in (2.32) to general state space model form defined in (2.7), we get a nonlinear mathematical model of horizontal cylindrical storage tank

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \frac{q_{\mathrm{in}}(t) - k_{\mathrm{v}}\sqrt{h(t)}}{2\ell\sqrt{h(t)(2R - h(t))}}, \tag{2.33a}$$

$$y(t) = h(t), \tag{2.33b}$$

with initial condition $h(0) = h^{\mathrm{s}}$. The level of the liquid in the tank in the steady state is known. Then based on the equation (2.8), we can calculate the inlet flow in steady state. Linearized mathematical model of the conical storage tank that is defined in general as (2.10), we obtain by calculating the Jacobian (2.12) as follows

$$\widetilde{A} = \frac{2q_{\mathrm{in}}^{\mathrm{s}}\left(h^{\mathrm{s}} - R\right) - k_{\mathrm{v}}h^{\mathrm{s}\,3/2}}{4\ell\left(2Rh^{\mathrm{s}} - h^{\mathrm{s}\,2}\right)^{3/2}}, \tag{2.34a}$$

$$\widetilde{B} = \frac{1}{2\ell\sqrt{2Rh^{\mathrm{s}} - h^{\mathrm{s}\,2}}}, \tag{2.34b}$$

$$C = 1, \tag{2.34c}$$

$$D = 0. \tag{2.34d}$$

The nonlinear mathematical model of horizontal cylindrical tank in discrete time is defined based on (2.13) where the functions $f(\cdot)$ and $g(\cdot)$ are obtained in (2.33). Linear mathematical model in discrete time is expressed by (2.14) where matrices A and B are calculated from (2.15) and (2.16).

2.4 Analysis of the Step Responses

The step response of a system represents the time behavior of the outputs of a general system when its inputs change between values in a short time. In order to simulate the step responses of the nonlinear mathematical model in continuous time defined in (2.7) it is necessary to solve a differential equation for the particular value of the inlet stream. If we want to simulate the step responses of a linear mathematical model in a continuous time it is necessary to calculate the matrices $\widetilde{A}$, $\widetilde{B}$, C and D defined in (2.12). Regarding the step responses of the mathematical models in discrete time, the difference equation defined in (2.13) has to be evaluated in every sampling time T_{s} and compute the matrices of a linear state space model in discrete time defined in (2.14) by utilizing the property given in (2.15).

In this section, we will analyze and compare the step responses of the linear and nonlinear mathematical model in continuous and discrete time for the conical, spherical and horizontal cylindrical storage tank.

2.4.1 Step Responses of the Conical Storage Tank

All technological parameters of the conical storage tank are summarized in the Table 2.1. The liquid level in steady state is given as $h^{\mathrm{s}} = 0.3\,\mathrm{m}$. Based on the equation (2.8) we can calculate the inlet flow in steady state as $q_{\mathrm{in}}^{\mathrm{s}} = 0.041\,\mathrm{m}^3\mathrm{s}^{-1}$. Now, when all variables in steady state are known, we can calculate the matrices of of linear system (2.29) as follows

$$\widetilde{A} = -0.0677, \tag{2.35a}$$
$$\widetilde{B} = 0.9895, \tag{2.35b}$$
$$C = 1, \tag{2.35c}$$
$$D = 0. \tag{2.35d}$$

Matrices of the linear state space model in discrete time A and B can be calculated either by utilizing the property in (2.15) or by using the mathematical software MATLAB via `c2d` command. Matrices C and D are the same as for the continuous time. Then

$$A = 0.8733, \tag{2.36a}$$
$$B = 1.8507, \tag{2.36b}$$

where the sampling time is set to $T_{\mathrm{s}} = 2\,\mathrm{s}$.

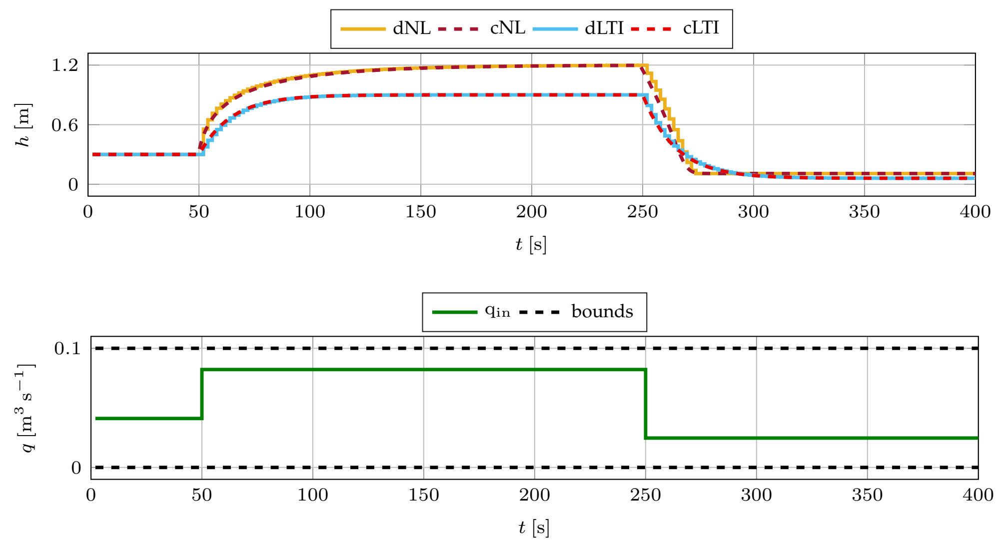

Figure 2.4: Step responses of the conical storage tank.

As we can see, in the Fig. 2.4 the big difference between the linear and nonlinear mathematical model is caused by a strong nonlinearity which is obvious from (2.21).

The more we get further from the operating point, the bigger differences between the nonlinear and linear model will be.

2.4.2 Step Responses of the Spherical Storage Tank

The same procedure will be adopt to the spherical storage tank. Technological parameters are displayed in the Table 2.2. The liquid level in steady state is given as $h^{\mathrm{s}} = 2\,\mathrm{m}$ and the inlet flow in steady state is $q_{\mathrm{in}}^{\mathrm{s}} = 1.0607\,\mathrm{m}^3\mathrm{s}^{-1}$. Then we can calculate the matrices of of linear system as

$$\widetilde{A} = -0.0211, \tag{2.37a}$$
$$\widetilde{B} = 0.0796, \tag{2.37b}$$
$$C = 1, \tag{2.37c}$$
$$D = 0. \tag{2.37d}$$

Matrices of the linear state space model in discrete time A and B are calculated as

$$A = 0.9587, \tag{2.38a}$$
$$B = 0.1558, \tag{2.38b}$$

and the sampling time is set to $T_{\mathrm{s}} = 5\,\mathrm{s}$.

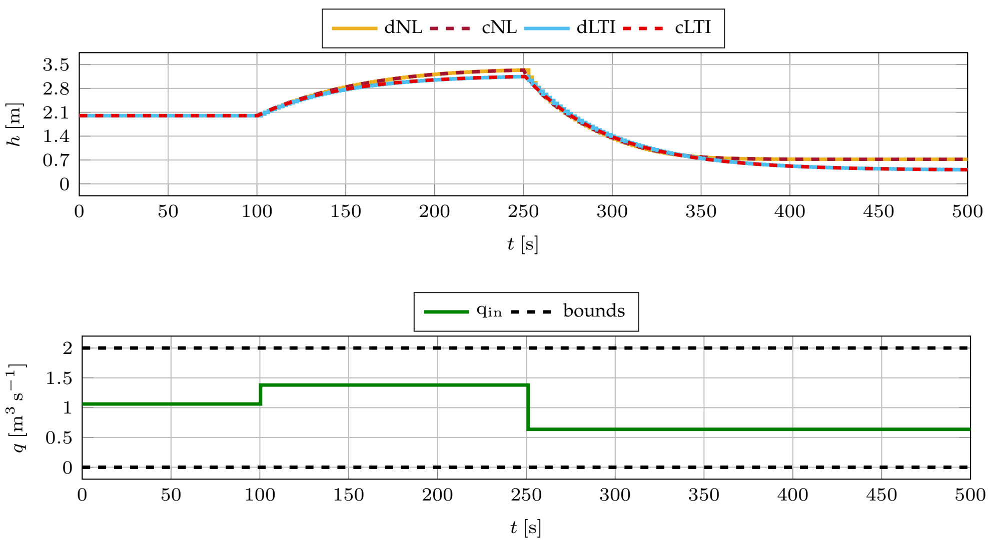

Figure 2.5: Step responses of the spherical storage tank.

2.4.3 Step Responses of the Horizontal Cylindrical Storage Tank

Technological parameters for horizontal cylindrical storage tanks are displayed in the Table 2.3. The liquid level in steady state is given as $h^{\mathrm{s}} = 2\,\mathrm{m}$ and the inlet flow in steady state is $q_{\mathrm{in}}^{\mathrm{s}} = 1.0607\,\mathrm{m}^3\mathrm{s}^{-1}$. The matrices of linear system are calculated as

$$\widetilde{A} = -0.0166, \tag{2.39a}$$
$$\widetilde{B} = 0.0625, \tag{2.39b}$$
$$C = 1, \tag{2.39c}$$
$$D = 0. \tag{2.39d}$$

Matrices of the linear state space model in discrete time A and B are calculated as

$$A = 0.9674, \tag{2.40a}$$
$$B = 0.1230, \tag{2.40b}$$

and the sampling time is set to $T_{\mathrm{s}} = 5\,\mathrm{s}$.

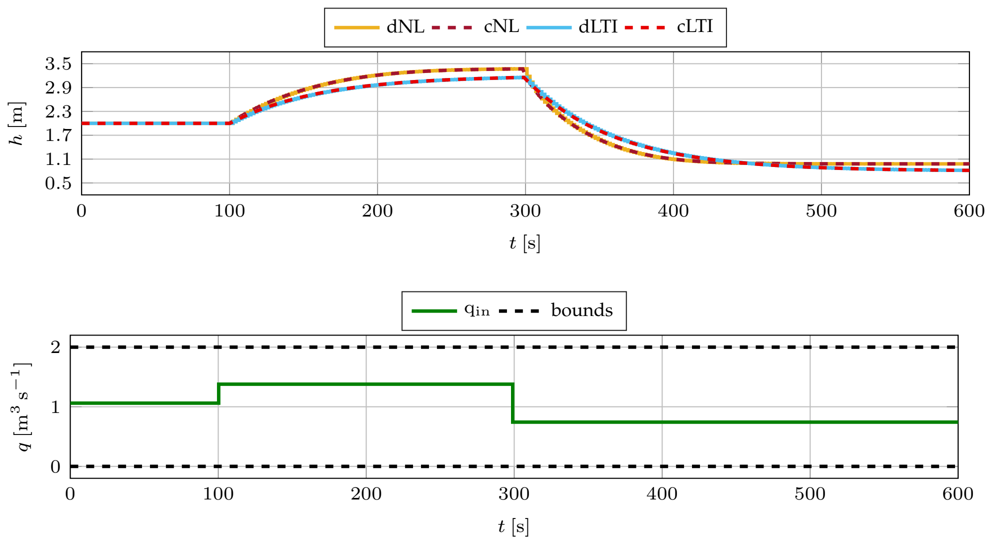

Figure 2.6: Step responses of the horizontal cylindrical storage tank.

If we look closer to the spherical and horizontal cylindrical tank, their step responses are very similar to each other since their mathematical models are similar too, however, the nonlinearity is not so dominant as for the conical storage tank.

CHAPTER 3

Simple Control Strategies

This chapter deals with the design of simple control strategies that are commonly used in process industry mainly for their simplicity, robustness, implementation and low costs[16]. We will focus on two control approaches, PID and LQ control. While the PID controllers are mostly used for SISO systems, the LQ controllers represent the optimal state controllers used for control of MIMO processes.

In general, the closed-loop system represents a set of technical instruments that are used for the correct working of a measured process in automatic mode. The simplified block scheme of such a loop is depicted in the Fig. 3.1. The closed-loop scheme consists

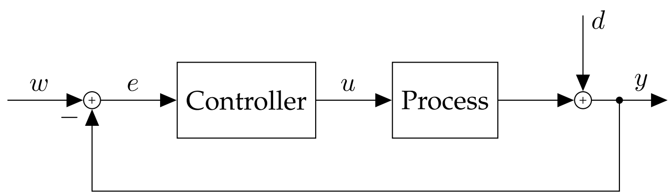

Figure 3.1: Closed-loop system block scheme.

of four basic components: controller, actuator, process and measurement device. To simplify the scheme, the actuator and the measurement device are included into the controlled process. The signals shown in here scheme are w as a reference, e as a control error, u as a manipulated (control) variable, d as a disturbance signal and y as an output (controlled) variable. The most important requirements on closed-loop system are stability, disturbance rejection, control performance and robustness to parameter changes [16].

3.1 PID Control

PID controller is by far the most widely used control algorithm in process industry. There are estimates that probably more than 90% of all controllers are of PID type. A lot of feedback loops are controlled via PID controllers. The main principle of the PID controller is to process its input signal that is a control error [7]. In general, the PID controller is defined by the following algorithm

$$u(t) = K\left(e(t) + \frac{1}{T_\mathrm{i}}\int_0^t e(\tau)\mathrm{d}\tau + T_\mathrm{d}\frac{\mathrm{d}e(t)}{\mathrm{d}t}\right), \tag{3.1}$$

where u represents the manipulated variable and e is the control error that is defined as a difference between the reference and actual output value. The manipulated variable is thus a sum of three terms. P - which is proportional to the error, I - that is proportional to the integral of the error gives and D - which is proportional to the derivative of the error. The parameters of the controller are proportional gain K, integral time T_i and derivative time T_d. The block scheme of process control via PID controller based on the general closed loop system in Fig. 3.1 is displayed in the Fig. 3.2.

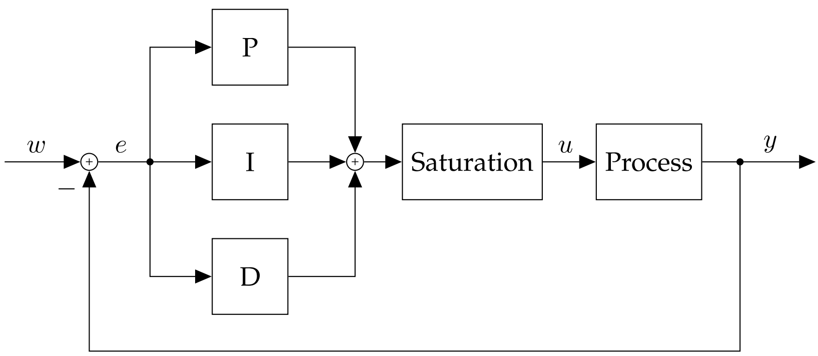

Figure 3.2: Block scheme of PID control.

Proportional Action

The control law for proportional (P) controller is defined as

$$u(t) = Ke(t). \tag{3.2}$$

From the practical point of view, such a controller is able to proportionally work only in a certain limited range of control errors since the manipulated variable can be only in a range between u_min and u_max. Proportional behavior of the controller can be characterized either by its gain K. The proportional controller acts like an on-off controller for large control errors. The proportional controller does not enforce an offset free control in steady state.

Integral Action

The main principle of the integral action is to make sure that the process output is coincident with the reference in steady state. A controller with integral action will always give zero steady-state error. For large values of the integration time constant, the response creeps slowly towards the reference. The smaller value of integral time constant T_i the bigger changes of manipulated variable u are generated.

Derivative Action

The objective of the derivative action is to improve the closed-loop stability and rise time. The derivative controller contains the derivative constant T_d which "predicts" the future value of the control error. The prediction is made by extrapolating the error by the tangent to the control error curve. However, the ideal derivative controller is not realizable. In order to use such a controller, we need to add the first order system in series as a filter.

Integrator Windup

In process industry, there are control systems with a wide range of operating conditions. This can lead to the situation that manipulated variable reaches the actuator limits. In the case that this happens, the feedback loop is broken and the system behaves as an open loop since the actuator will remain at its limit independently of the control process output [2]. If the controller with integrating action is used, the control error will continue to be integrated, hence, the integral term become very large. Then it is required that the error has opposite sign for a long period before things come back to normal. Typical symptoms of integrator windup are large overshoots that are caused by delayed activity of the controller. In this thesis, we will implement the integrator antiwindup based on the back-calculation method that is shown in the Fig. 3.3. The modification is made by adding another feedback loop. The input to this loop is the difference between the calculated and applied control action. In the case that calculated control action is in range u_min and u_max, the difference is equal to zero and the original controller remains. If not, it is necessary to change the integral action until the control action is again at the desired value. The speed of the integral rewind is given by the time constant T_b that is calculated based on the following empirical rule

$$T_{\mathrm{b}} = \sqrt{T_{\mathrm{i}} T_{\mathrm{d}}}, \tag{3.3}$$

where T_{i} and T_{d} are defined in (3.1).

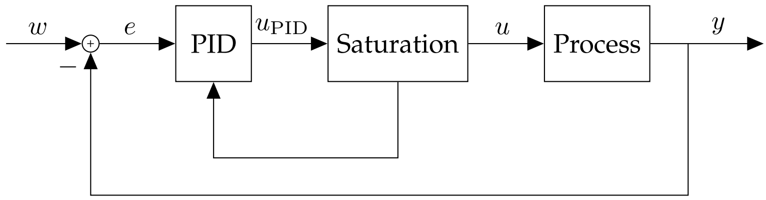

Figure 3.3: Block scheme of PID control with antiwindup.

Controller Synthesis

In order to design a PID controller, we can choose from various methods. These methods can be divided into two groups: analytical and experimental. Analytical methods assume that the transfer function of the controlled process is known, whereas the experimental methods require knowing the process dynamics in time. PID controller synthesis can be performed also in mathematical software MATLAB using the `pidTuner` command [19].

3.2 Optimal LQ Control

We consider a controllable continuous-time system with initial condition given as

$$\dot{x}(t) = \widetilde{A}x(t) + \widetilde{B}u(t), \quad x(0) = 0, \tag{3.4}$$

where matrices $\widetilde{A}$ and $\widetilde{B}$ represent the linear state space model defined in (2.10) and the infinite horizon cost function defined as

$$J = \int_0^\infty \left(x^\intercal(t) Q_{\mathrm{x}} x(t) + u^\intercal(t) Q_{\mathrm{u}} u(t)\right) \mathrm{d}t, \tag{3.5}$$

where the matrices Q_{x} and Q_{u} set relative weights of state deviation and input usage for which the following rules apply

$$Q_{\mathrm{x}} \succeq 0, \tag{3.6}$$

$$Q_{\mathrm{u}} \succ 0. \tag{3.7}$$

The main objective of the optimal control is to find such a feedback control law defined as

$$u(t) = -K_{\mathrm{c}} x(t), \tag{3.8}$$

that minimizes the cost function (3.5) subject to the constraint (3.4). The gain matrix K_c in (3.8) is given by the following relation

$$K_\mathrm{c} = -Q_\mathrm{u}^{-1} \widetilde{B}^\intercal P_\mathrm{c}, \tag{3.9}$$

where $P_\mathrm{c} \succ 0$ satisfies algebraic quadratic matrix Riccati equation (ARE)

$$\widetilde{A}^\intercal P_\mathrm{c} + P_\mathrm{c} \widetilde{A} - P_\mathrm{c} \widetilde{B} Q_\mathrm{u}^{-1} \widetilde{B}^\intercal P_\mathrm{c} + Q_\mathrm{x} = 0. \tag{3.10}$$

Matrices Q_x and Q_u can be designed in various ways, e.g.

$$Q_\mathrm{x} = \frac{1}{(x^\mathrm{s})^2}, \tag{3.11a}$$

$$Q_\mathrm{u} = \frac{1}{(u^\mathrm{s})^2}. \tag{3.11b}$$

The detailed solution of ARE to get K_c can be found in [6] or using the mathematical software MATLAB via `lqr` command. Since the K_c represents the constant matrix, such a controller implements the proportional action. From the Section 3.1 we know, that this action results in the steady-state control error. Our goal is to design such a LQ controller which will be able to track the reference without an offset in steady state. The control scheme with such a controller is depicted in the Fig. 3.4.

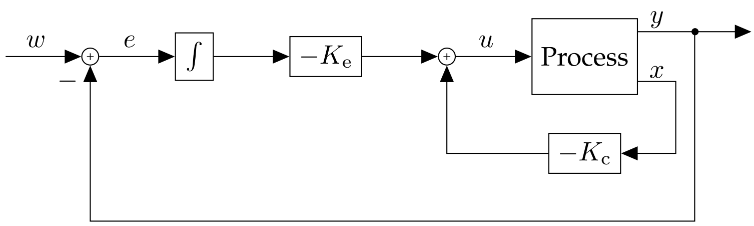

Figure 3.4: Block scheme of LQ control with integral action.

There are few possibilities how to get rid of a control error in steady-state. One of them is to define an additional dynamics for the control error [1]. It means that the difference between the reference and the output variable is equal to the new states defined as $\dot{x}_\mathrm{I}(t)$. Now we can extend the system dynamics in (3.4) by the control error dynamics that can be expressed in the matrix form as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_\mathrm{I}(t) \end{bmatrix} = \begin{bmatrix} \widetilde{A} & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_\mathrm{I}(t) \end{bmatrix} + \begin{bmatrix} \widetilde{B} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} w(t), \tag{3.12}$$

from which we obtain a new state-space representation as

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + Mw(t), \tag{3.13}$$

where the matrices $\bar{A}$ and $\bar{B}$ represent the extended matrices from (3.12). The cost function defined in (3.5) will change to the following form

$$J_{\mathrm{e}} = \int_0^{\infty} \big(\bar{x}^{\mathsf{T}}(t)\bar{Q}_{\mathrm{x}}\bar{x}(t) + u^{\mathsf{T}}(t)Q_{\mathrm{u}}u(t)\big)\,\mathrm{d}t. \tag{3.14}$$

By minimizing the cost function (3.14) subject to (3.13) and solving the ARE

$$\bar{A}^{\mathsf{T}}P_{\mathrm{e}} + P_{\mathrm{e}}\bar{A} - P_{\mathrm{e}}\widetilde{B}Q_{\mathrm{u}}^{-1}\bar{B}^{\mathsf{T}}P_{\mathrm{e}} + \bar{Q}_{\mathrm{x}} = 0, \tag{3.15}$$

we obtain a new optimal control law with integral action defined as

$$u(t) = -\bar{K}_{\mathrm{c}}\bar{x}(t) = -\begin{bmatrix}K_{\mathrm{c}} & K_{\mathrm{e}}\end{bmatrix}\begin{bmatrix}x(t)\\ x_{\mathrm{e}}(t)\end{bmatrix}, \tag{3.16}$$

where $\bar{K}_{\mathrm{c}}$ contains the information how to achieve an offset free control via K_{e}. The matrix $\bar{Q}_{\mathrm{x}}$ used in (3.15) is defined as

$$\bar{Q}_{\mathrm{x}} = \begin{bmatrix}Q_{\mathrm{x}} & 0\\ 0 & Q_{\mathrm{e}}\end{bmatrix}, \tag{3.17}$$

where the matrix Q_{e} that penalizes the control error is properly chosen, usually 10 times higher than Q_{x}.

CHAPTER 4

Model Predictive Control

As it was already mentioned in the Introduction, the main principle of the MPC is to find an optimal control action with respect to the state, input and output constraints. MPC control problem can be defined as an optimization problem which consists of a quadratic cost function subject to equality and inequality constraints [10]. The solution of such a problem is an optimal sequence of control actions over the prediction horizon. However, only the first sample is applied to the system. This control strategy is called receding horizon control [15]. In the Fig. 4.1 is shown a typical block diagram of a closed loop system where the MPC is used as a controller.

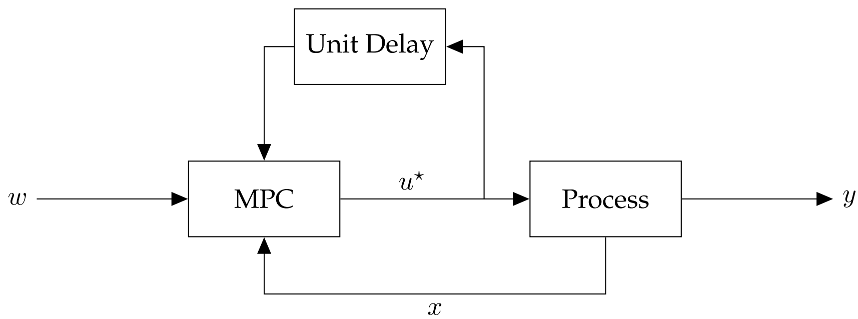

Figure 4.1: Closed-loop implementation with the MPC.

4.1 Quadratic programming

The MPC formulation needs to be transformed into the standard form of quadratic optimization problem in order to be solved using available solvers like GUROBI. The quadratic programming is an optimization problem that consists of quadratic cost function and all constraints are linear equalities or inequalities [3]. This class of optimization problems can be defined as

$$\min_{v} \quad v^\intercal P v + q^\intercal v + r, \tag{4.1a}$$

$$\text{s.t.} \quad F_{\text{eq}} v = g_{\text{eq}}, \tag{4.1b}$$

$$F v \preceq g \tag{4.1c}$$

where $v \in \mathbb{R}^n$ is a vector of optimization variables and P represents the Hessian which is a square matrix of dimensions $n \times n$. Next $q \in \mathbb{R}^n$, $F_{\text{eq}} \in \mathbb{R}^{m \times n}$, $F \in \mathbb{R}^{p \times n}$, $g_{\text{eq}} \in \mathbb{R}^m$ and $g \in \mathbb{R}^p$, where constants m and n represent the number of equality and inequality constraints. For control purposes there is a requirement that the optimization problem defined in (4.1) has a unique optimum since this problem results in control action and we can not obtain more than one solution. In order to achieve this requirement, the Hessian P must be a positive definite. When $P \succ 0$, then the quadratic problem is convex and we know that convex quadratic problems have one unique optima.

4.2 Dense Approach to Reformulation

In order to obtain the sequence of optimal control inputs, the control problem has to be defined firstly. As it was already mentioned in previous section, the MPC formulation represents the quadratic optimization problem subject to linear equality and inequality constraints. In this thesis, we will derive and use the following MPC formulation

$$\min_{u_0,\dots,u_{N-1}} \quad \sum_{k=0}^{N-1} (y_k - w_k)^\intercal Q_{\text{y}} (y_k - w_k) + \sum_{k=0}^{N-1} \Delta u_k^\intercal Q_{\text{u}} \Delta u_k, \tag{4.2a}$$

$$\text{s.t.} \quad x_{k+1} = A x_k + B u_k + E_{\text{x}} d_k, \tag{4.2b}$$

$$y_k = C x_k + D u_k + E_{\text{y}} d_k, \tag{4.2c}$$

$$d_{k+1} = d_k, \tag{4.2d}$$

$$x_0 = \hat{x}(t), \tag{4.2e}$$

$$d_0 = \hat{d}(t), \tag{4.2f}$$

$$\Delta u_k = u_k - u_{k-1}, \tag{4.2g}$$

$$u_{\min} \leq u_k \leq u_{\max}, \tag{4.2h}$$

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max}, \tag{4.2i}$$

$$x_{\min} \leq x_k \leq x_{\max}, \tag{4.2j}$$

$$y_{\min} \leq y_k \leq y_{\max}, \tag{4.2k}$$

where $k = 0, ..., N-1$. The quadratic cost function (4.2a) penalizes tracking error $y_k - w_k$ which we require to converge to zero, along with increments of the control action, denoted as Δu. This Δu represents the difference between the current value of control action and the previous one. In this MPC formulation we require to have an access to the previous control action u_{k-1} which represents the previous control action already applied to the system. Since we used the formulation containing Δu_k as a discrete-time integrator we are able to achieve an offset free control [20, 17, 18]. However, integrating behavior introduced by Δu MPC formulation does not remove offset in tracking when the MPC designed based on the linear system is used for controlling nonlinear systems. We achieve an offset free control only in case when the prediction model is identical with a real system. However, we rarely arrive at such a situation and also, the process can be affected by disturbances which may not be measurable. In order to avoid the model mismatch, we can extend the design model by a set of disturbance signals which affect not only state variables but also output variables. This procedure is called disturbance modeling and the control scheme with MPC controller and estimation of variables is shown in the Fig. 4.2.

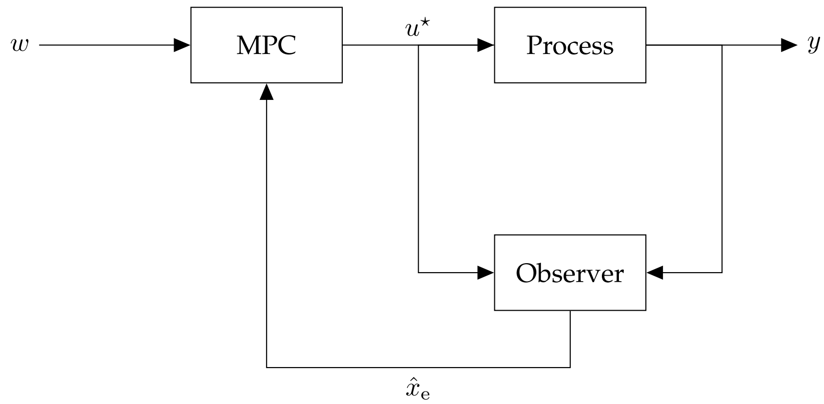

Figure 4.2: MPC control scheme with estimation of variables.

We consider an extended state space model in discrete time (4.2b) - (4.2d) where the unmeasured disturbances d enter through the matrices E_x and E_y of appropriate sizes. Also we assume that disturbances have a constant dynamics which is defined by (4.2d). The estimated extended state vector is given as

$$\hat{x}_\mathrm{e} = \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix}, \tag{4.3}$$

where $\hat{x}$ represents the estimated state vector of the process and $\hat{d}$ represents the estimated unmeasured disturbances. We assume that the number of disturbances is the same as the number of controller outputs. There exist several options how to obtain the estimated states and disturbances. One of them is to design a Luenberger observer or the second option is to use a Kalman Filter. In this thesis we will consider a time-varying Kalman Filter procedure which consists of two phases - prediction phase and update phase [9]. The prediction phase is made up of two equations

$$\hat{x}_{\mathrm{e},k|k-1} = A_\mathrm{e}\hat{x}_{\mathrm{e},k-1|k-1} + B_\mathrm{e}u_k, \tag{4.4a}$$
$$P_{k|k-1} = A_\mathrm{e}P_{k-1|k-1}A_\mathrm{e}^\intercal + Q_\mathrm{d}, \tag{4.4b}$$

where $\hat{x}_{\mathrm{e},k|k-1}$ represents the predicted state estimate on the previous time step and $P_{k,k-1}$ is a predicted value of the covariance matrix. The matrices A_e, B_e, C_e and D_e are defined as

$$A_\mathrm{e} = \begin{bmatrix} A & E_\mathrm{x} \\ 0 & I \end{bmatrix}, \quad B_\mathrm{e} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_\mathrm{e} = \begin{bmatrix} C & E_\mathrm{y} \end{bmatrix}, \quad D_\mathrm{e} = \begin{bmatrix} D \end{bmatrix}. \tag{4.5}$$

The update phase is represented by the following equations

$$\epsilon_k = (y_{\mathrm{m},k} - y_\mathrm{L}) - \left(C_\mathrm{e}\hat{x}_{\mathrm{e},k|k-1} + D_\mathrm{e}w_k\right), \tag{4.6a}$$
$$S_k = C_\mathrm{e}P_{k|k-1}C_\mathrm{e}^\intercal + R_\mathrm{e}, \tag{4.6b}$$
$$L_k = P_{k|k-1}C_\mathrm{e}^\intercal S_k^{-1}, \tag{4.6c}$$
$$\hat{x}_{\mathrm{e},k|k} = \hat{x}_{\mathrm{e},k|k-1} + L_k\epsilon_k, \tag{4.6d}$$
$$P_{k|k} = (I - L_kC_\mathrm{e})\,P_{k|k-1}, \tag{4.6e}$$

where the variable ϵ_k represents the estimation error, and it is calculated based on the process measurements $y_{\mathrm{m},k}$ and the linearization point y_L that is substracted from the measurement. The time-varying Kalman Filter gain L_k is calculated based on the (4.6b) and (4.6c). In order to determine the current estimate of the variables $\hat{x}_{\mathrm{e},k|k}$ the Kalman Filter gain L_k is used based on (4.6d). The covariance matrix P is updated at the end of the update phase.

To obtain an optimal control action from the formulation (4.2) we need to reformulate this optimization problem into the form that is given in (4.1). The reason for this step is that solvers require standard formulation of optimization problems. There exist two approaches how to obtain the quadratic programming form from the formulation (4.2) - dense [13] and sparse [10] formulation of quadratic problem. In this thesis we will focus on the dense approach to matrix reformulation.

We assume the full linear state space model in discrete time extended at disturbance signals in order to calculate the future evolution of the states based on the initial condition x_0, the first control action u_0 and the first disturbance input d_0

$$x_1 = Ax_0 + Bu_0 + E_\mathrm{x} d_0, \tag{4.7a}$$
$$y_0 = Cx_0 + Dx_0 + E_\mathrm{y} d_0. \tag{4.7b}$$

Since the value of states at the first sample of prediction x_1 is known from the (4.7a), we can substitute the value of x_1 for the next state and output predictions as

$$\begin{aligned} x_2 &= Ax_1 + Bu_1 + E_\mathrm{x} d_1 = \\ &= A\left(Ax_0 + Bu_0 + E_\mathrm{x} d_0\right) + Bu_1 + E_\mathrm{x} d_1 = \\ &= A^2 x_0 + ABu_0 + AE_\mathrm{x} d_0 + Bu_1 + E_\mathrm{x} d_1, \end{aligned} \tag{4.8a}$$

$$\begin{aligned} y_1 &= Cx_1 + Du_1 + E_\mathrm{y} d_1 = \\ &= C\left(Ax_0 + Bu_0 + E_\mathrm{x} d_0\right) + Du_1 + E_\mathrm{y} d_1 = \\ &= CAx_0 + CBu_0 + CE_\mathrm{x} d_0 + Du_1 + E_\mathrm{y} d_1, \end{aligned} \tag{4.8b}$$

$$\begin{aligned} x_3 &= Ax_2 + Bu_2 + E_\mathrm{x} d_2 = \\ &= A\left(A^2 x_0 + ABu_0 + AE_\mathrm{x} d_0 + Bu_1 + E_\mathrm{x} d_1\right) + Bu_2 + E_\mathrm{x} d_2 = \\ &= A^3 x_0 + A^2 Bu_0 + A^2 E_\mathrm{x} d_0 + ABu_1 + AE_\mathrm{x} d_1 + Bu_2 + E_\mathrm{x} d_2, \end{aligned} \tag{4.9a}$$

$$\begin{aligned} y_2 &= Cx_2 + Du_2 + E_\mathrm{y} d_2 = \\ &= C\left(A^2 x_0 + ABu_0 + AE_\mathrm{x} d_0 + Bu_1 + E_\mathrm{x} d_1\right) + Du_2 + E_\mathrm{y} d_2 = \\ &= CA^2 x_0 + CABu_0 + CAE_\mathrm{x} d_0 + CBu_1 + CE_\mathrm{x} d_1 + Du_2 + E_\mathrm{y} d_2. \end{aligned} \tag{4.9b}$$

Adopting previous expressions, we obtained an explicit formula for calculating value of states based on initial condition and future control actions as

$$x_{k+1} = A^k x_0 + \sum_{j=0}^{k-1} A^j \left(Bu_{k-1-j} + E_\mathrm{x} d_{k-1-j}\right), \tag{4.10}$$

and explicit formula representing y_k for an arbitrary k is given by

$$y_k = \begin{cases} Cx_0 + Du_0 + E_\mathrm{y} d_0 & \text{if} \quad k = 0, \\ CA^k x_0 + \sum\limits_{j=0}^{k-1} CA^j \left(Bu_{k-1-j} + E_\mathrm{x} d_{k-1-j}\right) + Du_k + E_\mathrm{y} d_k & \text{if} \quad k \geq 1. \end{cases} \tag{4.11}$$

Expression (4.10) allows us to formulate the state prediction equation in matrix form as

$$X = \Psi_\mathrm{x} x_0 + \Gamma_\mathrm{x} U + \Gamma_{\mathrm{d}_\mathrm{x}} \widetilde{D}, \tag{4.12}$$

where

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}, \quad U = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{N-1} \end{bmatrix} = v, \quad \widetilde{D} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_N \end{bmatrix}, \tag{4.13}$$

and

$$\Psi_\mathrm{x} = \begin{bmatrix} I \\ A \\ A^2 \\ A^3 \\ \vdots \\ A^N \end{bmatrix}, \quad \Gamma_\mathrm{x} = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ B & 0 & 0 & 0 & \dots & 0 \\ AB & B & 0 & 0 & \dots & 0 \\ A^2B & AB & B & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & A^2B & AB & B \end{bmatrix},$$

$$\Gamma_{\mathrm{d_x}} = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ E_\mathrm{x} & 0 & 0 & 0 & \dots & 0 \\ AE_\mathrm{x} & E_\mathrm{x} & 0 & 0 & \dots & 0 \\ A^2E_\mathrm{x} & AE_\mathrm{x} & E_\mathrm{x} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ A^{N-1}E_\mathrm{x} & A^{N-2}E_\mathrm{x} & \dots & A^2E_\mathrm{x} & AE_\mathrm{x} & E_\mathrm{x} \end{bmatrix}. \tag{4.14}$$

Using the (4.11), we obtain the expression for Y as a function of initial conditions x_0 that can be formulated as

$$Y = \Psi_\mathrm{y} x_0 + \Gamma_\mathrm{y} U + \Gamma_{\mathrm{d_y}} \widetilde{D}, \tag{4.15}$$

which represents the output prediction equation in matrix form. First we define

$$Y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{N-1} \end{bmatrix}, \tag{4.16}$$

and then

$$\Psi_{\mathrm{y}} = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N-1} \end{bmatrix} \quad \Gamma_{\mathrm{y}} = \begin{bmatrix} D & 0 & 0 & 0 & \dots & 0 \\ CB & D & 0 & 0 & \dots & 0 \\ CAB & CB & D & 0 & \dots & 0 \\ CA^2B & CAB & CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \dots & CAB & CB & D \end{bmatrix}, \tag{4.17}$$

$$\Gamma_{\mathrm{d_y}} = \begin{bmatrix} E_{\mathrm{y}} & 0 & 0 & 0 & \dots & 0 \\ CE_{\mathrm{x}} & E_{\mathrm{y}} & 0 & 0 & \dots & 0 \\ CAE_{\mathrm{x}} & CE_{\mathrm{x}} & E_{\mathrm{y}} & 0 & \dots & 0 \\ CA^2E_{\mathrm{x}} & CAE_{\mathrm{x}} & CE_{\mathrm{x}} & E_{\mathrm{y}} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ CA^{N-2}E_{\mathrm{x}} & CA^{N-3}E_{\mathrm{x}} & \dots & CAE_{\mathrm{x}} & CE_{\mathrm{x}} & E_{\mathrm{y}} \end{bmatrix}. \tag{4.18}$$

Based on the definition of Δu per (4.2g), the stacked vector ΔU is given by

$$\Delta U = \begin{bmatrix} \Delta u_0 \\ \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \\ \vdots \\ \Delta u_{N-1} \end{bmatrix} = \begin{bmatrix} u_0 - u_{-1} \\ u_1 - u_0 \\ u_2 - u_1 \\ u_3 - u_2 \\ \vdots \\ u_{N-1} - u_{N-2} \end{bmatrix}. \tag{4.19}$$

Subsequently, we obtain

$$\Delta U = \Lambda U + \lambda u_{-1}, \tag{4.20}$$

where

$$\Lambda = \begin{bmatrix} I_{n_u} & 0 & 0 & 0 & \dots & 0 \\ -I_{n_u} & I_{n_u} & 0 & 0 & \dots & 0 \\ 0 & -I_{n_u} & I_{n_u} & 0 & \dots & 0 \\ 0 & 0 & -I_{n_u} & I_{n_u} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & -I_{n_u} & I_{n_u} \end{bmatrix}, \quad \lambda = \begin{bmatrix} -I_{n_u} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{4.21}$$

In order to obtain final matrices of the quadratic problem (4.1) the origin objective function can be reformulate as follows

$$\sum_{k=0}^{N-1} (y_k - w_k)^{\intercal} Q_{\mathrm{y}} (y_k - w_k) + \sum_{k=0}^{N-1} \Delta u_k^{\intercal} Q_{\mathrm{u}} \Delta u_k = (Y - W)^{\intercal} \Phi_{\mathrm{y}} (Y - W) + \Delta U^{\intercal} \Phi_{\mathrm{u}} \Delta U, \tag{4.22}$$

in which the weighting matrices are given as

$$\Phi_\mathrm{y} = \begin{bmatrix} Q_\mathrm{y} & 0 & 0 & 0 \\ 0 & Q_\mathrm{y} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_\mathrm{y} \end{bmatrix}, \quad \Phi_\mathrm{u} = \begin{bmatrix} Q_\mathrm{u} & 0 & 0 & 0 \\ 0 & Q_\mathrm{u} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_\mathrm{u} \end{bmatrix} \tag{4.23}$$

where $\Phi_\mathrm{y} \in \mathbb{R}^{(Nn_y)\times(Nn_y)}$ and $\Phi_\mathrm{u} \in \mathbb{R}^{(Nn_u)\times(Nn_u)}$. The objective function also contains the reference vector which is defined as

$$W = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_{N-1} \end{bmatrix}. \tag{4.24}$$

Once cost function is introduced in matrix form and all matrices and vectors are defined, vectors Y and ΔU are substituted into the cost function (4.22) with formulas (4.15) and (4.20) as follows

$$\begin{aligned} &\left(\Psi_\mathrm{y}x_0 + \Gamma_\mathrm{y}U + \Gamma_{\mathrm{d_y}}\widetilde{D} - W\right)^\intercal \Phi_\mathrm{y} \left(\Psi_\mathrm{y}x_0 + \Gamma_\mathrm{y}U + \Gamma_{\mathrm{d_y}}\widetilde{D} - W\right) = \\ = \;& x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Psi_\mathrm{y}x_0 + x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}U + x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} - x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}W + \\ & U^\intercal\Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}\Psi_\mathrm{y}x_0 + U^\intercal\Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}U + U^\intercal\Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} - U^\intercal\Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}W + \\ & \widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}\mathrm{y}x_0 + \widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}U + \widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} - \widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}W - \\ & W^\intercal\Phi_y\Psi_\mathrm{y}x_0 - W^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}U - W^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} + W^\intercal\Phi_yW = \\ = \;& U^\intercal\Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}U + \left(2x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y} + 2\widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y} - 2W^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y}\right)U + \\ & x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Psi_\mathrm{y}x_0 + 2x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} - 2x_0^\intercal\Psi_\mathrm{y}^\intercal\Phi_\mathrm{y}W - 2\widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}\Phi_\mathrm{y}W + \\ & \widetilde{D}^\intercal\Gamma_{\mathrm{d_y}}^\intercal\Phi_\mathrm{y}\Gamma_{\mathrm{d_y}}\widetilde{D} + W^\intercal\Phi_\mathrm{y}W, \end{aligned} \tag{4.25a}$$

$$\begin{aligned} &(\Delta U + \lambda u_{-1})^\intercal\,\Phi_\mathrm{u}\,(\Delta U + \lambda u_{-1}) = \\ = \;& U^\intercal\Lambda^\intercal\Phi_\mathrm{u}\Lambda U + U^\intercal\Lambda^\intercal\Phi_\mathrm{u}\lambda u_{-1} + u_{-1}^\intercal\lambda^\intercal\Phi_\mathrm{u}\Lambda U + u_{-1}^\intercal\lambda\Phi_\mathrm{u}\lambda u_{-1} = \\ = \;& U^\intercal\Lambda^\intercal\Phi_\mathrm{u}\Lambda U + 2u_{-1}^\intercal\lambda^\intercal\Phi_\mathrm{u}\Lambda U + u_{-1}^\intercal\lambda\Phi_\mathrm{u}\lambda u_{-1}. \end{aligned} \tag{4.25b}$$

Expansion of respective parts of the cost function can be put together and then we obtain a standard formulation of quadratic programming problem defined in (4.1) with

$$P = \Gamma_\mathrm{y}^\intercal\Phi_\mathrm{y}\Gamma_\mathrm{y} + \Lambda^\intercal\Phi_\mathrm{u}\Lambda, \tag{4.26a}$$

$$q = \left(2x_0^\intercal \Psi_\mathrm{y}^\intercal \Phi_\mathrm{y} \Gamma_\mathrm{y} + 2\widetilde{D}^\intercal \Gamma_{\mathrm{d_y}}^\intercal \Phi_\mathrm{y} \Gamma_\mathrm{y} - 2W^\intercal \Phi_\mathrm{y} \Gamma_\mathrm{y} + 2u_{-1}^\intercal \lambda^\intercal \Phi_\mathrm{u} \Lambda \right)^\intercal , \tag{4.26b}$$

$$\begin{aligned} r = {} & x_0^\intercal \Psi_\mathrm{y}^\intercal \Phi_\mathrm{y} \Psi_\mathrm{y} x_0 + 2x_0^\intercal \Psi_\mathrm{y}^\intercal \Phi_\mathrm{y} \Gamma_{\mathrm{d_y}} \widetilde{D} - 2x_0^\intercal \Psi_\mathrm{y}^\intercal \Phi_\mathrm{y} W - 2\widetilde{D}^\intercal \Gamma_{\mathrm{d_y}} \Phi_\mathrm{y} W + \\ & \widetilde{D}^\intercal \Gamma_{\mathrm{d_y}}^\intercal \Phi_\mathrm{y} \Gamma_{\mathrm{d_y}} \widetilde{D} + W^\intercal \Phi_\mathrm{y} W + u_{-1}^\intercal \lambda \Phi_\mathrm{u} \lambda u_{-1}. \end{aligned} \tag{4.26c}$$

By performing the substitution in (4.22), we are actually removing equality constraints from optimization problem. The last step is to derive and reformulate only the remaining inequality constraints into matrix formulation in order to obtain a standard form of quadratic programming problem. Firstly, these constraints have to be rewritten into the following form

$$x_k \leq x_\mathrm{max}, \tag{4.27a}$$
$$-x_k \leq -x_\mathrm{min}, \tag{4.27b}$$
$$u_k \leq u_\mathrm{max}, \tag{4.27c}$$
$$-u_k \leq -u_\mathrm{min}, \tag{4.27d}$$
$$y_k \leq y_\mathrm{max}, \tag{4.27e}$$
$$-y_k \leq -y_\mathrm{min}, \tag{4.27f}$$
$$\Delta u_k \leq \Delta u_\mathrm{max}, \tag{4.27g}$$
$$-\Delta u_k \leq -\Delta u_\mathrm{min}, \tag{4.27h}$$

and then into the matrix form as follows

$$X \leq X_\mathrm{max}, \tag{4.28a}$$
$$-X \leq -X_\mathrm{min}, \tag{4.28b}$$
$$U \leq U_\mathrm{max}, \tag{4.28c}$$
$$-U \leq -U_\mathrm{min}, \tag{4.28d}$$
$$Y \leq Y_\mathrm{max}, \tag{4.28e}$$
$$-Y \leq -Y_\mathrm{min}, \tag{4.28f}$$
$$\Delta U \leq \Delta U_\mathrm{max}, \tag{4.28g}$$
$$-\Delta U \leq -\Delta U_\mathrm{min}. \tag{4.28h}$$

Next step in this derivation is to substitute vectors X from (4.12), Y from (4.15) and ΔU from (4.20), by straight forward mathematical modifications, we obtain the following expression

$$\begin{aligned}
\Gamma_{\mathrm{x}} U &\leq \quad X_{\max} - \Psi_{\mathrm{x}} x_0 - \Gamma_{\mathrm{d_x}} \widetilde{D}, && \text{(4.29a)}\\
-\Gamma_{\mathrm{x}} U &\leq -X_{\min} + \Psi_{\mathrm{x}} x_0 + \Gamma_{\mathrm{d_x}} \widetilde{D}, && \text{(4.29b)}\\
U &\leq \quad U_{\max}, && \text{(4.29c)}\\
-U &\leq -U_{\min}, && \text{(4.29d)}\\
\Gamma_{\mathrm{y}} U &\leq \quad Y_{\max} - \Psi_{\mathrm{y}} x_0 - \Gamma_{\mathrm{d_y}} \widetilde{D}, && \text{(4.29e)}\\
-\Gamma_{\mathrm{y}} U &\leq -Y_{\min} + \Psi_{\mathrm{y}} x_0 + \Gamma_{\mathrm{d_y}} \widetilde{D}, && \text{(4.29f)}\\
\Lambda U &\leq \quad \Delta U_{\max} - \lambda u_{-1}, && \text{(4.29g)}\\
-\Lambda U &\leq -\Delta U_{\min} + \lambda u_{-1}. && \text{(4.29h)}
\end{aligned}$$

Set of inequality constraints given in (4.29) are in standard form, which was introduced in (4.1). In order to simplify the expression, we can write the left hand side matrix F and right hand side vector g as follows

$$F = \begin{bmatrix} \Gamma_{\mathrm{x}} \\ -\Gamma_{\mathrm{x}} \\ I_{n_u} \\ -I_{n_u} \\ \Gamma_{\mathrm{y}} \\ -\Gamma_{\mathrm{y}} \\ \Lambda \\ -\Lambda \end{bmatrix}, \quad g = \begin{bmatrix} X_{\max} - \Psi_{\mathrm{x}} x_0 - \Gamma_{\mathrm{d_x}} \widetilde{D} \\ -X_{\min} + \Psi_{\mathrm{x}} x_0 + \Gamma_{\mathrm{d_x}} \widetilde{D} \\ U_{\max} \\ -U_{\min} \\ Y_{\max} - \Psi_{\mathrm{y}} x_0 - \Gamma_{\mathrm{d_y}} \widetilde{D} \\ -Y_{\min} + \Psi_{\mathrm{y}} x_0 + \Gamma_{\mathrm{d_y}} \widetilde{D} \\ \Delta U_{\max} - \lambda u_{-1} \\ -\Delta U_{\min} + \lambda u_{-1} \end{bmatrix}, \tag{4.30}$$

which is the form required by quadratic programming problems solvers.

4.3 Formulation of Nonlinear MPC problem

In the previous section, we formulated an optimization problem which consisted of the quadratic objective function subject to the linear equality and inequality constraints. Solving this optimization problem, we obtained a sequence of the optimal control actions, however, only the first of them is applied to the process. Also, in order to achieve an offset free control we had to assume an extended model of the process. Here we are going to formulate another optimization problem which also consists of quadratic objective function but the constraints are linear and nonlinear. Particularly, the constraints representing the model behavior based on which the predictions are made is nonlinear [14]. The MPC formulation is given as

$$\min_{u_0,\ldots,u_{N-1}} \quad \sum_{k=0}^{N-1} (y_k - w_k)^\intercal \, Q_\mathrm{y} \, (y_k - w_k) + \sum_{k=0}^{N-1} \Delta u_k^\intercal Q_\mathrm{u} \Delta u_k, \tag{4.31a}$$

$$\text{s.t.} \quad x_{k+1} = f\left(x_k, u_k\right), \tag{4.31b}$$

$$y_k = x_k \tag{4.31c}$$

$$x_0 = x(t), \tag{4.31d}$$

$$\Delta u_k = u_k - u_{k-1}, \tag{4.31e}$$

$$u_\mathrm{min} \leq u_k \leq u_\mathrm{max}, \tag{4.31f}$$

$$\Delta u_\mathrm{min} \leq \Delta u_k \leq \Delta u_\mathrm{max}, \tag{4.31g}$$

$$y_\mathrm{min} \leq y_k \leq y_\mathrm{max}, \tag{4.31h}$$

where $k = 0, ..., N-1$. Such an optimization problem can not be reformulated to the standard quadratic programming form since the constraint (4.31b) has a nonlinear character. In order to solve this quadratic problem, we can choose from various optimization methods which are able to handle nonlinear constraints. For solving this optimization problem, we will use a built-in function in mathematical software MATLAB called `fmincon` which implements four different algorithms: interior point, sequential quadratic programming, active set and trust region reflective. We will choose the interior point algorithm, which introduces logarithmic barriers in the cost function to deal with nonlinear constraints [4]. Command `fmincon` finds a constrained optimum of a function of several variables that is used as

```
fmincon(@(U) FUN,U0,A,B,Aeq,Beq,LB,UB,NONLCON,OPTIONS),
```

where the parameter `FUN` represents the quadratic objective function (4.31a) into which we substitute the nonlinear constraint (4.31b) and linear constraints (4.31d) - (4.31e). The next parameter is `U0` which is the starting point for optimization. Parameters `A` and `B` represent the linear inequality constraints, in our case it is the constraint (4.31g). `Aeq` and `Beq` represent the linear equality constraints which were already were substituted into the objective function. `LB` and `UB` defines a set of lower and upper bounds on the optimized variable U, so that a solution is found in the range `LB` $\leq$ `U` $\leq$ `UB`. In the MPC formulation is such a constraint (4.31f). `NONLCON` represents the nonlinear constraint function that is (4.31h) and the parameter `OPTIONS` gives us a possibility to choose what method will be used for optimization.

CHAPTER 5

Simulation Case Studies

In this chapter, we will illustrate the design and analysis of the simple and advanced control strategies explained in Chapters 3 and 4. The first section focuses on simple control strategies represented by PID and LQ controllers, on their design and evaluation, whereas the second section deals with the design of advanced MPC controllers and their evaluation. In all simulations we use continuous time nonlinear model as a controller process. For PID and LQ control we use Simulink model and for advanced control we use `ODE45` solver.

5.1 Simple control strategies

From the previous chapters, we know that there is a lot of methods how to design a PID controller. These methods are divided into two groups - analytical and experimental. In order to design a PID controller we need to know either mathematical model of the controlled process or the dynamics of the process based on the experimental data. We will design a PID controller in the mathematical software MATLAB via `pidTuner` whose input represents the transfer function of the controlled process. Transfer function can be obtained by the transformation from the state space model (2.10), or via MATLAB using `ss2tf` command. `pidTuner` enables on-line controller tuning through the graphical user interface. Hence, the controller is designed primary based on the linear model, however, this controller can be applied to control the nonlinear model with a little tuning.

Based on the process specification and our requirements, we have designed PI controllers. Our target is to achieve an offset free control in steady state and without the overshoot. It is obvious that we will need at least PI controller which is able to achieve a control without offset in steady state. After that, we have to tune the controller in order to ensure the control without the overshoot. The reason why we do not accept the overshoot during the control is simple - if the setpoint for the controlled variable would be near the maximal height of the storage tank, the liquid inside the tank can overflow. This can be a risky situation if the storage tank contains an acid, toxic and expensive materials or if the storage tank is the pre-process for another technology, like distillation column etc.

It is known that the constants of the PI controller are rarely designed in an optimal way. In order to achieve an optimal control, we need to find such a controller that is designed by the solving of the optimization problem. LQ controller is one of them and the design of this controller is described in the section 3.2. We need to minimize the objective function (3.14) subject to the process dynamics in order to achieve an optimal control action which can be applied to the controlled process. Now, we are going to take a closer look on the design and control performance of PID and LQ control of the conical, spherical and horizontal cylindrical storage tank. We consider a continuous PI and LQ control, where the simulation model is implemented in the Simulink.

5.1.1 PID and LQ Control of the Conical Storage Tank

We designed a PI controller, based on (3.1) with the following parameters in order to control a nonlinear mathematical model in continuous time

$$K = 0.52, \tag{5.1a}$$

$$T_{\mathrm{i}} = 0.05\,\mathrm{s}, \tag{5.1b}$$

and the time constant T_{b} which represents the speed of the integral rewind is set as $T_{\mathrm{b}} = 0.5\,\mathrm{s}$.

An optimal LQ controller is designed as follows. We assume the extended system defined in (3.13), where

$$\bar{A} = \begin{bmatrix} -0.0677 & 0 \\ -1.0000 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0.9895 \\ 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.2a}$$

The weighting matrix $\bar{Q}_{\mathrm{x}}$ defined in (3.17) consists of matrices Q_{x} and Q_{e}, where the matrix Q_{x} penalizes the state variables and the matrix Q_{e} penalizes the control error, is chosen as

$$\bar{Q}_{\mathrm{x}} = \begin{bmatrix} 0.0900 & 0 \\ 0 & 0.0010 \end{bmatrix}, \tag{5.3}$$

and the weighting matrix Q_{u} which penalizes the control actions is designed as

$$Q_{\mathrm{u}} = 0.1. \tag{5.4}$$

By minimizing the cost function (3.14) subject to (3.13) and solving the ARE (3.15) we obtain an optimal control law with integral action defined as (3.16), where

$$K_\mathrm{c} = -0.9836, \tag{5.5a}$$
$$K_\mathrm{e} = 0.1000. \tag{5.5b}$$

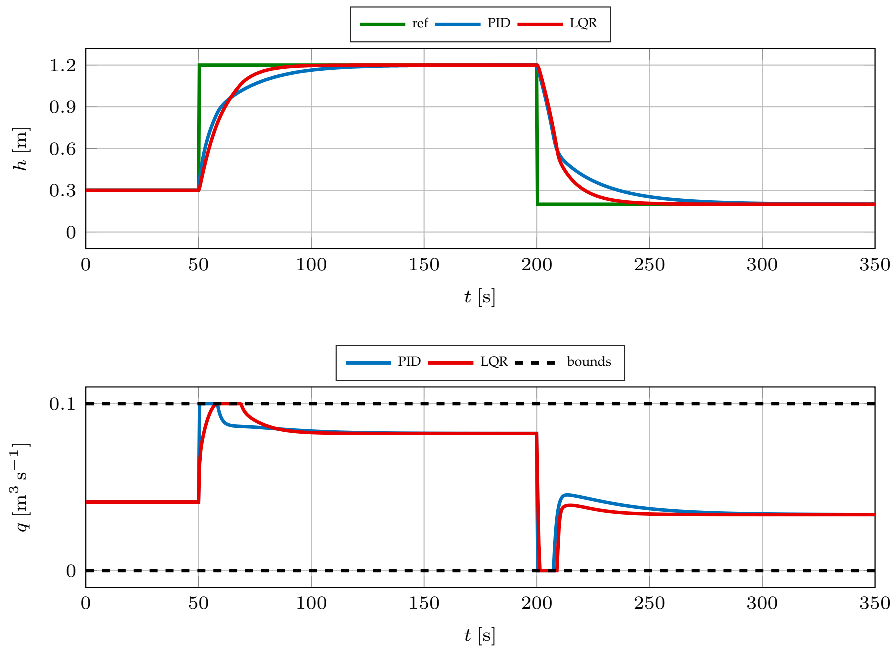

Figure 5.1: Comparison of the PID and LQ control of the conical storage tank.

In the figure 5.1 is depicted the comparison of the continuous PID and LQ control with integral action of the conical storage tank. For the evaluation of the designed control strategies we can calculate the performance index represented by ISE criterion which is defined as

$$\mathrm{ISE} = \int_0^{t_\mathrm{f}} e^2(t)\mathrm{d}t, \tag{5.6}$$

where the e stands for the control error. The value of the ISE criterion for PID control is 11.91 whereas for the LQ control it is 6.42. Based on these values, we can say that LQ control shows a better performance and it is more suitable for the control of the conical storage tank. The disadvantage of the LQ control is its implementation difficulty and also we need to calculate the algebraic Riccati equation which requires some time.

5.1.2 PID and LQ Control of the Spherical Storage Tank

The same procedure will be adopted for the spherical storage tank. Designed PI controller has the following parameters

$$K = 1.6, \tag{5.7a}$$
$$T_\mathrm{i} = 0.052\,\mathrm{s}, \tag{5.7b}$$

and the time constant representing the speed of the integral rewind is set as $T_\mathrm{b} = 0.5\,\mathrm{s}$. The matrices of the extended state space model for LQ control are calculated as

$$\bar{A} = \begin{bmatrix} -0.0211 & 0 \\ -1.0000 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0.0531 \\ 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.8a}$$

The weighting matrices $\bar{Q}_\mathrm{x}$ and Q_u are designed as follows

$$\bar{Q}_\mathrm{x} = \begin{bmatrix} 1000 & 0 \\ 0 & 100 \end{bmatrix}, \quad Q_\mathrm{u} = 2. \tag{5.9}$$

The constants of LQ controller K_c and K_e from (3.16) are calculated as

$$K_\mathrm{c} = -28.6780, \quad K_\mathrm{e} = 7.5000. \tag{5.10}$$

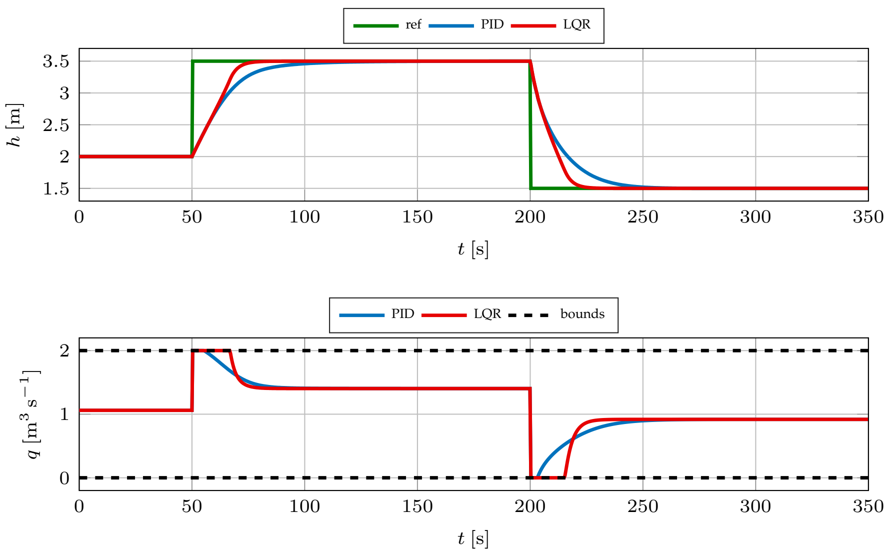

Figure 5.2: Comparison of the PID and LQ control of the spherical storage tank.

The value of the ISE criterion for the PID control is 39.95 whereas for the LQ control it is 21.87.

5.1.3 PID and LQ Control of the Horizontal Cylindrical Storage Tank

The PID controller used for the control of the horizontal cylindrical storage tank consists of the following parameters

$$K = 2, \quad T_{\mathrm{i}} = 0.035\,\mathrm{s}, \tag{5.11}$$

and the time constant representing the speed of the integral rewind is set as $T_{\mathrm{b}} = 0.5\,\mathrm{s}$. The matrices of the extended state space model for LQ control are calculated as

$$\bar{A} = \begin{bmatrix} -0.0166 & 0 \\ -1.0000 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0.0625 \\ 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.12a}$$

The weighting matrices $\bar{Q}_{\mathrm{x}}$ and Q_{u} are designed as follows

$$\bar{Q}_{\mathrm{x}} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.0006 \end{bmatrix}, \quad Q_{\mathrm{u}} = 0.01. \tag{5.13}$$

The constants of LQ controller K_{c} and K_{e} from (3.16) are calculated as

$$K_{\mathrm{c}} = -5.0239, \quad K_{\mathrm{e}} = 0.2470. \tag{5.14}$$

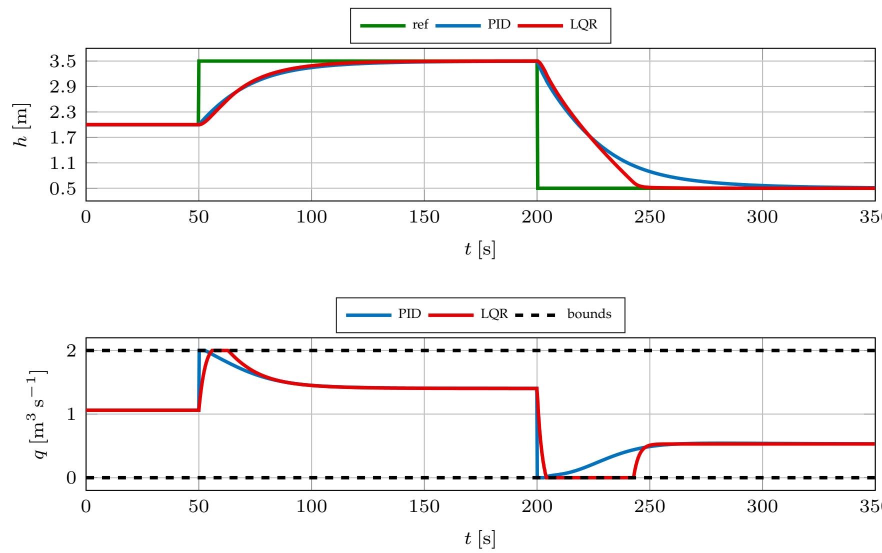

Figure 5.3: Comparison of the PID and LQ control of the horizontal cylindrical tank.

The value of the ISE criterion for the PID control is 142.6 whereas for the LQ control it is 127.3.

5.2 Advanced Control Strategies

This section deals with the comparison of the linear and nonlinear model predictive control of the conical, spherical and horizontal cylindrical storage tank. We consider a discrete implementation of LTI and NL MPC and the controlled process will be simulated via ODE45 solver. Linear MPC is designed using the YALMIP toolbox [12] and solved via GUROBI, for nonlinear MPC, we constructed the optimization problem which is being solved via fmincon.

5.2.1 MPC Control of the Conical Storage Tank

We consider an extended linear mathematical model in discrete time with disturbances (4.2b)-(4.2c), where the matrices A, B, C and D are calculated in (2.36) and unmeasured disturbances enter into the process via matrices E_{x} and E_{y} as follows

$$E_{\mathrm{x}} = 0, \tag{5.15a}$$
$$E_{\mathrm{y}} = 1. \tag{5.15b}$$

We usually set the matrix E_{x} as a zero matrix in case that the process does not contain unstable open loop dynamics. Since we often do not have an access to the future values of disturbances, we can substitute d_k into d_0. The matrices of the Kalman Filter are calculated based on (4.5) as follows

$$A_{\mathrm{e}} = \begin{bmatrix} 0.8733 & 0 \\ 0 & 1 \end{bmatrix}, \tag{5.16a}$$
$$B_{\mathrm{e}} = \begin{bmatrix} 1.8507 \\ 0 \end{bmatrix}, \tag{5.16b}$$
$$C_{\mathrm{e}} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \tag{5.16c}$$
$$D_{\mathrm{e}} = 0, \tag{5.16d}$$

and the initial value for the covariance matrix P is chosen as

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}. \tag{5.17}$$

The weighting matrices Q_{d} in the prediction phase (4.4b) and R_{e} in the update phase (4.6b) of the Kalman Filter are chosen as

$$Q_{\mathrm{d}} = \begin{bmatrix} 1 & 0 \\ 0 & 1000 \end{bmatrix}, \tag{5.18a}$$
$$R_{\mathrm{e}} = 0.001. \tag{5.18b}$$

The conical storage tank represents the SISO system where the liquid level is the state variable as well as the output variable. Hence, the constraints for these variables are the same. We can define the upper and lower bounds for all variables as follows

$$x_{\mathrm{min}} = 0\,\mathrm{m} - h^{\mathrm{s}}, \tag{5.19a}$$
$$x_{\mathrm{max}} = 2\,\mathrm{m} - h^{\mathrm{s}}, \tag{5.19b}$$
$$u_{\mathrm{min}} = 0\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^{\mathrm{s}}, \tag{5.19c}$$
$$u_{\mathrm{max}} = 0.15\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^{\mathrm{s}}, \tag{5.19d}$$
$$\Delta u_{\mathrm{min}} = -0.015 \text{ per sample}, \tag{5.19e}$$
$$\Delta u_{\mathrm{max}} = 0.015 \text{ per sample}. \tag{5.19f}$$

The weighting matrices Q_{y} and Q_{u} are the same for LTI and NL MPC which are chosen as

$$Q_{\mathrm{y}} = 5 \tag{5.20a}$$
$$Q_{\mathrm{u}} = 800. \tag{5.20b}$$

The prediction horizon is set to $N = 10$. Also, we consider model predictive control with trajectory preview setting. In the simulation, we consider step changes beginning at the steady states defined in the Section 2.4.1.

In the Fig. 5.4 is depicted the liquid level control of the conical storage tank via linear and nonlinear MPC. As we can see, all constraints are fulfilled in an optimal way. Also, the MPC allows us to define the constraints on Δu which represent the practical aspect in the process industry. The biggest difference between the LTI and NL MPC is that in case of NL MPC, we achieve a more precise setpoint tracking. This is caused since the NL MPC was designed based on the model which is identical to the process. Also we can see that LTI MPC shows a bigger overshoots than NL MPC with the same settings of the weighting matrices. The last subfigure shows the behavior of unmeasured disturbances. We can see, the cumulative unmeasurable disturbances settles when the reference is reached.

In the Fig. 5.5 we can see the evolution of the estimation error in the update phase of the Kalman Filter and the activity of the covariance matrix. The last two subfigures deals with the optimization quality criteria - the value of the objective function and the number of iterations necessary in each simulation step in order to converge to the optima. We can compare the value of objective functions for LTI and NL MPC as

$$J_{\mathrm{MPC}} = \sum_{i=0}^{t_{\mathrm{f}}} \left(\sum_{k=0}^{N-1} (y_k - w_k)^{\intercal} \, Q_{\mathrm{y}} \, (y_k - w_k) + \sum_{k=0}^{N-1} \Delta u_k^{\intercal} Q_{\mathrm{u}} \Delta u_k \right)_i , \tag{5.21}$$

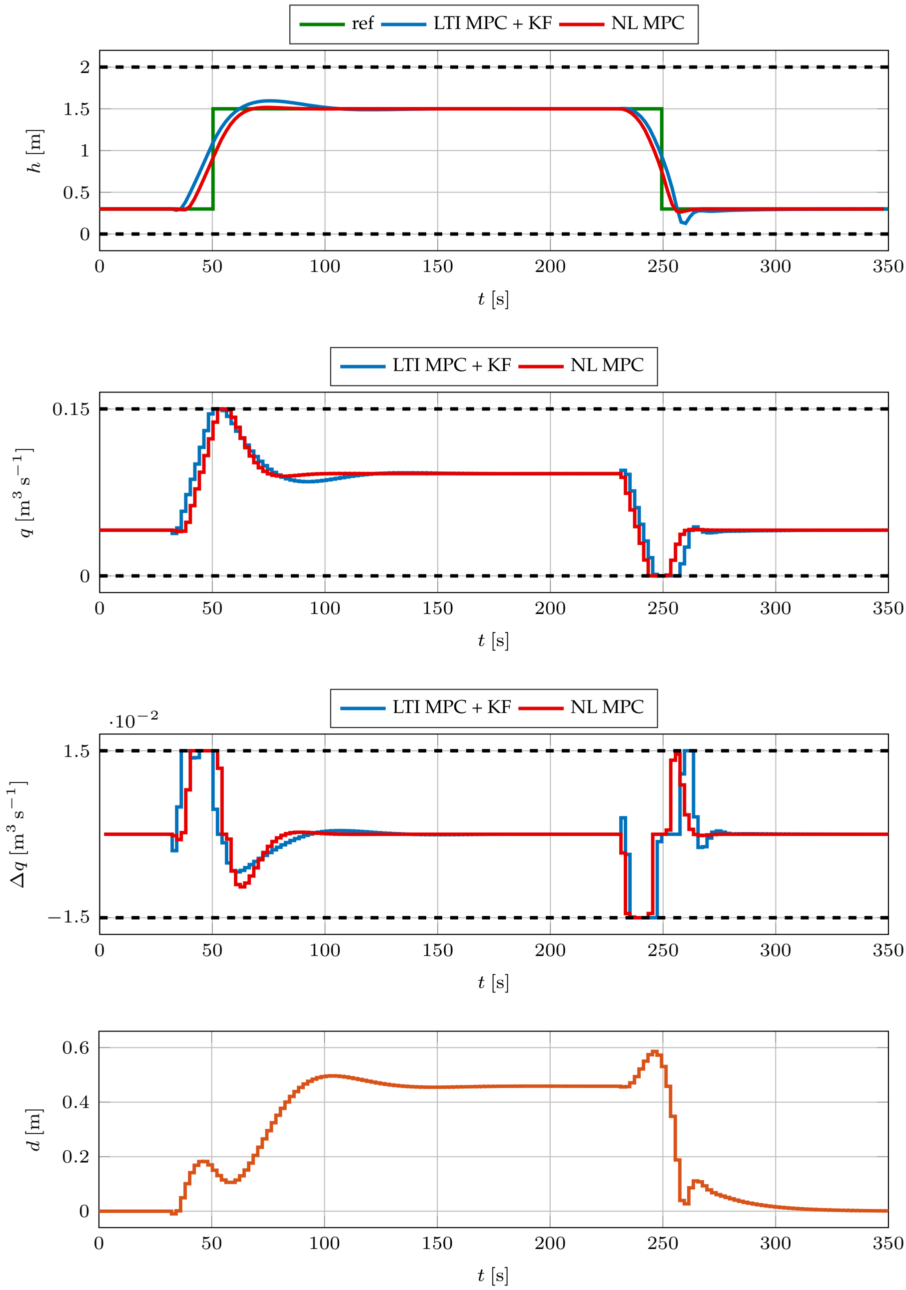

Figure 5.4: Comparison of the LTI MPC with Kalman Filter and NL MPC control of the conical storage tank.

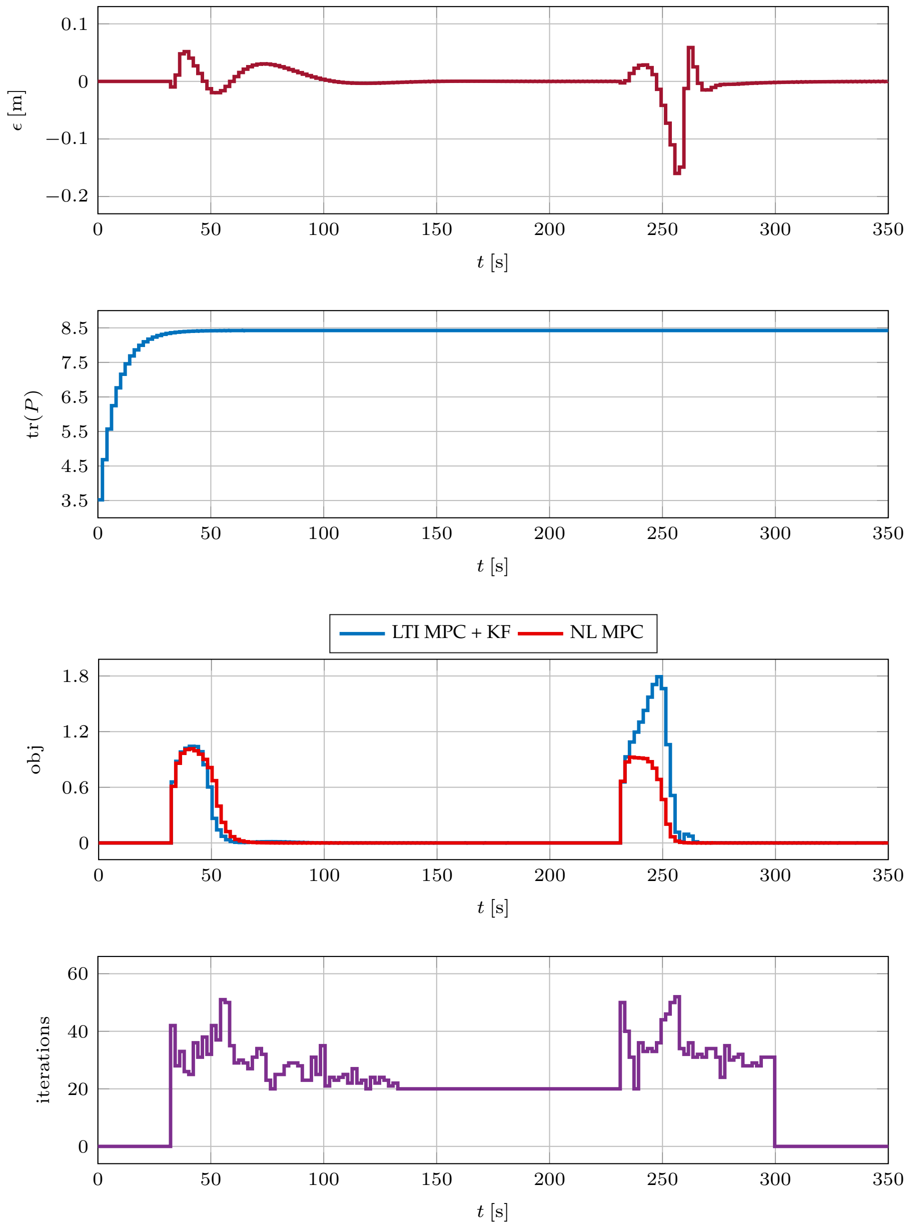

Figure 5.5: Estimation of disturbances, estimation error and covariance matrix of Kalman Filter for the conical storage tank.

where i represents the simulation step.

The final sum of the objective function for LTI MPC is $J_{\mathrm{LTI}} = 59.1258$ whereas the final sum of the objective function for NL MPC is $J_{\mathrm{NL}} = 45.1682$. It means, that nonlinear MPC is better then linear MPC by 23%. The figure with iterations has only informative character for nonlinear MPC, however, we can see that the number of iteration is not a big and we are able to solve this optimization problem in one sample time, which is $T_{\mathrm{s}} = 2\,\mathrm{s}$. It means that such a controller can be used in the process industry.

5.2.2 MPC Control of the Spherical Storage Tank

We consider an extended linear mathematical model in discrete time with disturbances (4.2b)-(4.2c), where the matrices A, B, C and D are calculated in (2.38) and unmeasured disturbances enter into the process via matrices E_{x} and E_{y} as follows

$$E_{\mathrm{x}} = 0, \tag{5.22a}$$

$$E_{\mathrm{y}} = 1. \tag{5.22b}$$

We usually set the matrix E_{x} as a zero matrix in case that the process does not contain unstable open loop dynamics. Since we often do not have an access to the future values of disturbances, we can substitute d_k into d_0. The matrices of the Kalman Filter are calculated based on (4.5) as follows

$$A_{\mathrm{e}} = \begin{bmatrix} 0.9587 & 0 \\ 0 & 1 \end{bmatrix}, \tag{5.23a}$$

$$B_{\mathrm{e}} = \begin{bmatrix} 1.558 \\ 0 \end{bmatrix}, \tag{5.23b}$$

$$C_{\mathrm{e}} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \tag{5.23c}$$

$$D_{\mathrm{e}} = 0, \tag{5.23d}$$

and the initial value for the covariance matrix P is chosen as

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}. \tag{5.24}$$

The weighting matrices Q_{d} in the prediction phase (4.4b) and R_{e} in the update phase (4.6b) of the Kalman Filter are chosen as

$$Q_{\mathrm{d}} = \begin{bmatrix} 30 & 0 \\ 0 & 750 \end{bmatrix}, \tag{5.25a}$$

$$R_{\mathrm{e}} = 0.01. \tag{5.25b}$$

The conical storage tank represents the SISO system where the liquid level is the state variable as well as the output variable. Hence, the constraints for these variables are the same. We can define the upper and lower bounds for all variables as follows

$$x_{\min} = 0\,\mathrm{m} - h^{\mathrm{s}}, \tag{5.26a}$$
$$x_{\max} = 4\,\mathrm{m} - h^{\mathrm{s}}, \tag{5.26b}$$
$$u_{\min} = 0\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^{\mathrm{s}}, \tag{5.26c}$$
$$u_{\max} = 2\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^{\mathrm{s}}, \tag{5.26d}$$
$$\Delta u_{\min} = -0.2 \text{ per sample}, \tag{5.26e}$$
$$\Delta u_{\max} = 0.2 \text{ per sample}. \tag{5.26f}$$

The wighting matrices Q_{y} and Q_{u} are the same for LTI and NL MPC which are chosen as

$$Q_{\mathrm{y}} = 0.9, \tag{5.27a}$$
$$Q_{\mathrm{u}} = 30. \tag{5.27b}$$

The prediction horizon is set to $N = 10$. Also, we consider model predictive control with trajectory preview setting. In the simulation, we consider step changes beginning at the steady states defined in the Section 2.4.2.

In the Fig. 5.6 is depicted the liquid level control of the spherical storage tank via linear and nonlinear MPC. As we can see, all constraints are fulfilled in an optimal way. Also, as for the conical tank, the MPC allows us to define the constraints on Δu which represent the practical aspect in the process industry. The biggest difference between the LTI and NL MPC is that in case of NL MPC, we achieve a more precise setpoint tracking. This is caused since the NL MPC was designed based on the model which is identical to the process. Also we can see that LTI MPC shows a bigger overshoots than NL MPC with the same settings of the weighting matrices. The last subfigure shows the behavior of unmeasured disturbances. We can see, the cumulative unmeasurable disturbances settles when the reference is reached.

In the Fig. 5.7 we can see the evolution of the estimation error in the update phase of the Kalman Filter and the activity of the covariance matrix. The last two subfigures deals with the optimization quality criteria - the value of the objective function and the number of iterations necessary in each simulation step in order to converge to the optima. We can compare the value of objective functions for LTI and NL MPC where the final sum of the objective function for LTI MPC is $J_{\mathrm{LTI}} = 194.6417$ whereas the final sum of the objective function for NL MPC is $J_{\mathrm{NL}} = 171.6715$. It means, that nonlinear MPC is better then linear MPC by 12%.

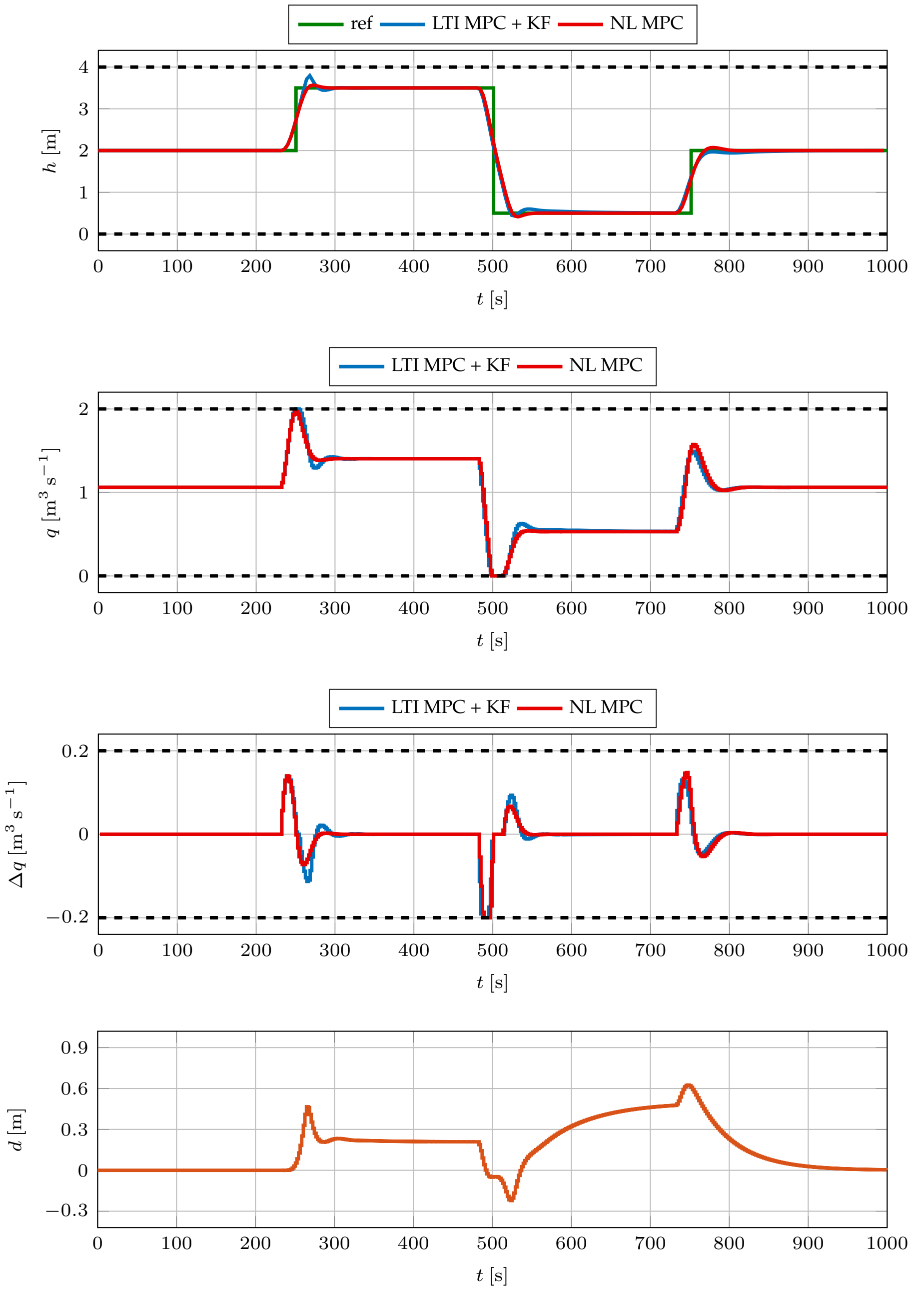

Figure 5.6: Comparison of the LTI MPC with Kalman Filter and NL MPC control of the spherical storage tank.

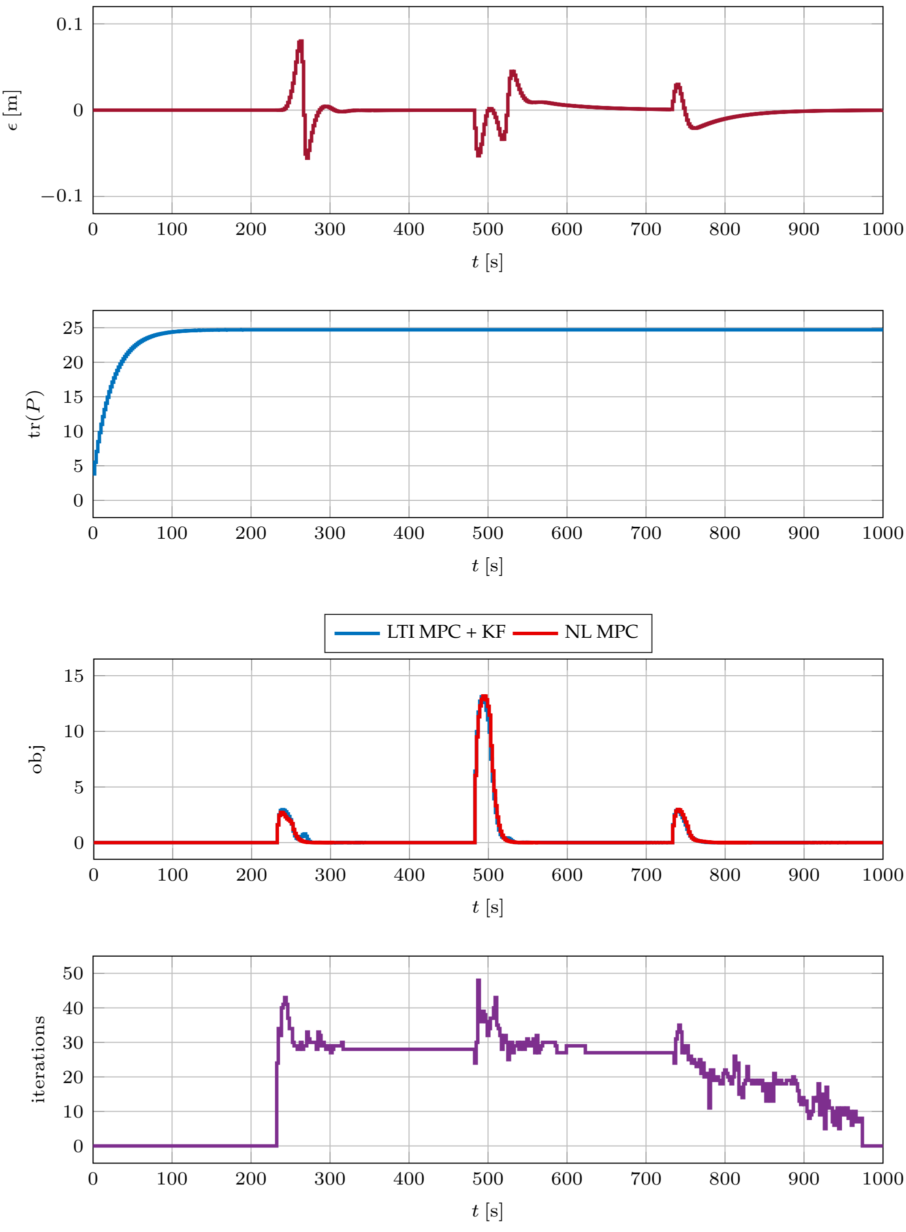

Figure 5.7: Estimation of disturbances, estimation error and covariance matrix of Kalman Filter for the spherical storage tank.

5.2.3 MPC Control of the Horizontal Cylindrical Storage Tank

We consider an extended linear mathematical model in discrete time with disturbances (4.2b)-(4.2c), where the matrices A, B, C and D are calculated in (2.36) and unmeasured disturbances enter into the process via matrices E_x and E_y as follows

$$E_\mathrm{x} = 0, \tag{5.28a}$$
$$E_\mathrm{y} = 1. \tag{5.28b}$$

We usually set the matrix E_x as a zero matrix in case that the process does not contain unstable open loop dynamics. Since we often do not have an access to the future values of disturbances, we can substitute d_k into d_0. The matrices of the Kalman Filter are calculated based on (4.5) as follows

$$A_\mathrm{e} = \begin{bmatrix} 0.9674 & 0 \\ 0 & 1 \end{bmatrix}, \tag{5.29a}$$
$$B_\mathrm{e} = \begin{bmatrix} 0.1230 \\ 0 \end{bmatrix}, \tag{5.29b}$$
$$C_\mathrm{e} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \tag{5.29c}$$
$$D_\mathrm{e} = 0, \tag{5.29d}$$

and the initial value for the covariance matrix P is chosen as

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}. \tag{5.30}$$

The weighting matrices Q_d in the prediction phase (4.4b) and R_e in the update phase (4.6b) of the Kalman Filter are chosen as

$$Q_\mathrm{d} = \begin{bmatrix} 0.1 & 0 \\ 0 & 150 \end{bmatrix}, \tag{5.31a}$$
$$R_\mathrm{e} = 0.001. \tag{5.31b}$$

The conical storage tank represents the SISO system where the liquid level is the state variable as well as the output variable. Hence, the constraints for these variables are the same. We can define the upper and lower bounds for all variables as follows

$$x_\mathrm{min} = 0\,\mathrm{m} - h^\mathrm{s}, \tag{5.32a}$$
$$x_\mathrm{max} = 2\,\mathrm{m} - h^\mathrm{s}, \tag{5.32b}$$
$$u_\mathrm{min} = 0\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^\mathrm{s}, \tag{5.32c}$$
$$u_\mathrm{max} = 0.1\,\mathrm{m}^3\,\mathrm{s}^{-1} - u^\mathrm{s}, \tag{5.32d}$$
$$\Delta u_\mathrm{min} = -0.01 \text{ per sample}, \tag{5.32e}$$
$$\Delta u_\mathrm{max} = 0.01 \text{ per sample}. \tag{5.32f}$$

The wighting matrices Q_{y} and Q_{u} are the same for LTI and NL MPC which are chosen as

$$Q_{\mathrm{y}} = 0.08, \tag{5.33a}$$
$$Q_{\mathrm{u}} = 0.57. \tag{5.33b}$$

The prediction horizon is set to $N = 10$. Also, we consider model predictive control with trajectory preview setting. In the simulation, we consider step changes beginning at the steady states defined in the Section 2.4.3.

In the Fig. 5.8 is depicted the liquid level control of the horizontal cylindrical storage tank via linear and nonlinear MPC. As we can see, all constraints are fulfilled in an optimal way. Also, the MPC allows us to define the constraints on Δu which represent the practical aspect in the process industry. The biggest difference between the LTI and NL MPC is that in case of NL MPC, we achieve a more precise setpoint tracking. This is caused since the NL MPC was designed based on the model which is identical to the process. Also we can see that LTI MPC shows a bigger overshoots than NL MPC with the same settings of the weighting matrices. The last subfigure shows the behavior of unmeasured disturbances. We can see, the cumulative unmeasurable disturbances settles when the reference is reached.

In the Fig. 5.9 we can see the evolution of the estimation error in the update phase of the Kalman Filter and the activity of the covariance matrix. The last two subfigures deals with the optimization quality criteria - the value of the objective function and the number of iterations necessary in each simulation step in order to converge to the optima. We can compare the value of objective functions for LTI and NL MPC where the final sum of the objective function for LTI MPC is $J_{\mathrm{LTI}} = 178.9290$ whereas the final sum of the objective function for NL MPC is $J_{\mathrm{NL}} = 148.3416$. It means, that nonlinear MPC is better then linear MPC by 16%.

5.2.4 Maximum Level Control via nonlinear MPC

In this section, we will present the nonlinear MPC control of liquid level in the conical storage tank where the reference is set to the value $h_{\max} = 4\,\mathrm{m}$ with respect to all constraints defined in (4.31). Such a control was not possible with linear MPC since this MPC was design based on the linear model which is not identical with the real (nonlinear) process.

Nonlinear MPC was designed based on the nonlinear mathematical model in discrete time, however, we simulated the continuous model via `ODE45`. We chose the

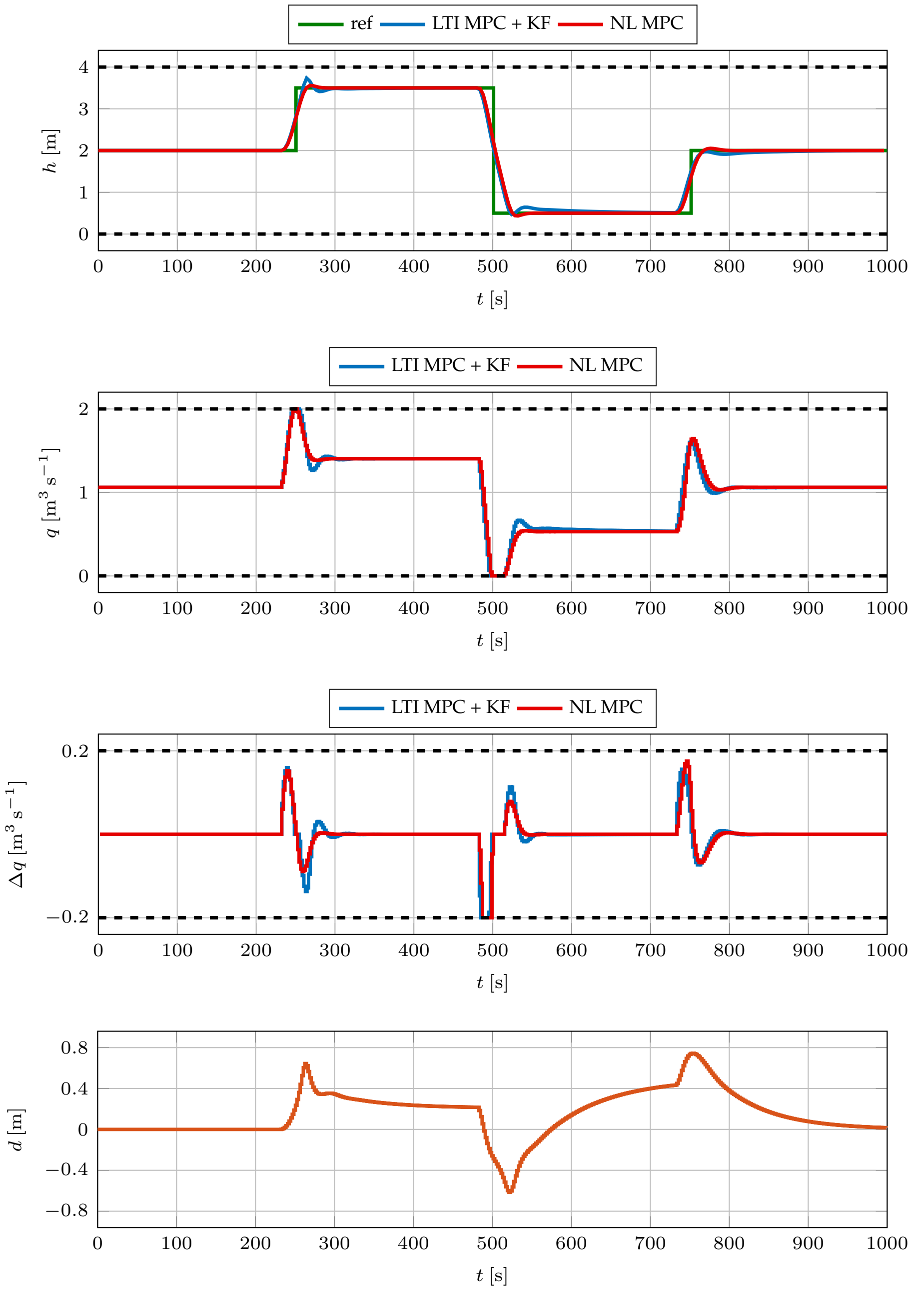

Figure 5.8: Comparison of the LTI MPC with Kalman Filter and NL MPC control of the horizontal cylindrical storage tank.

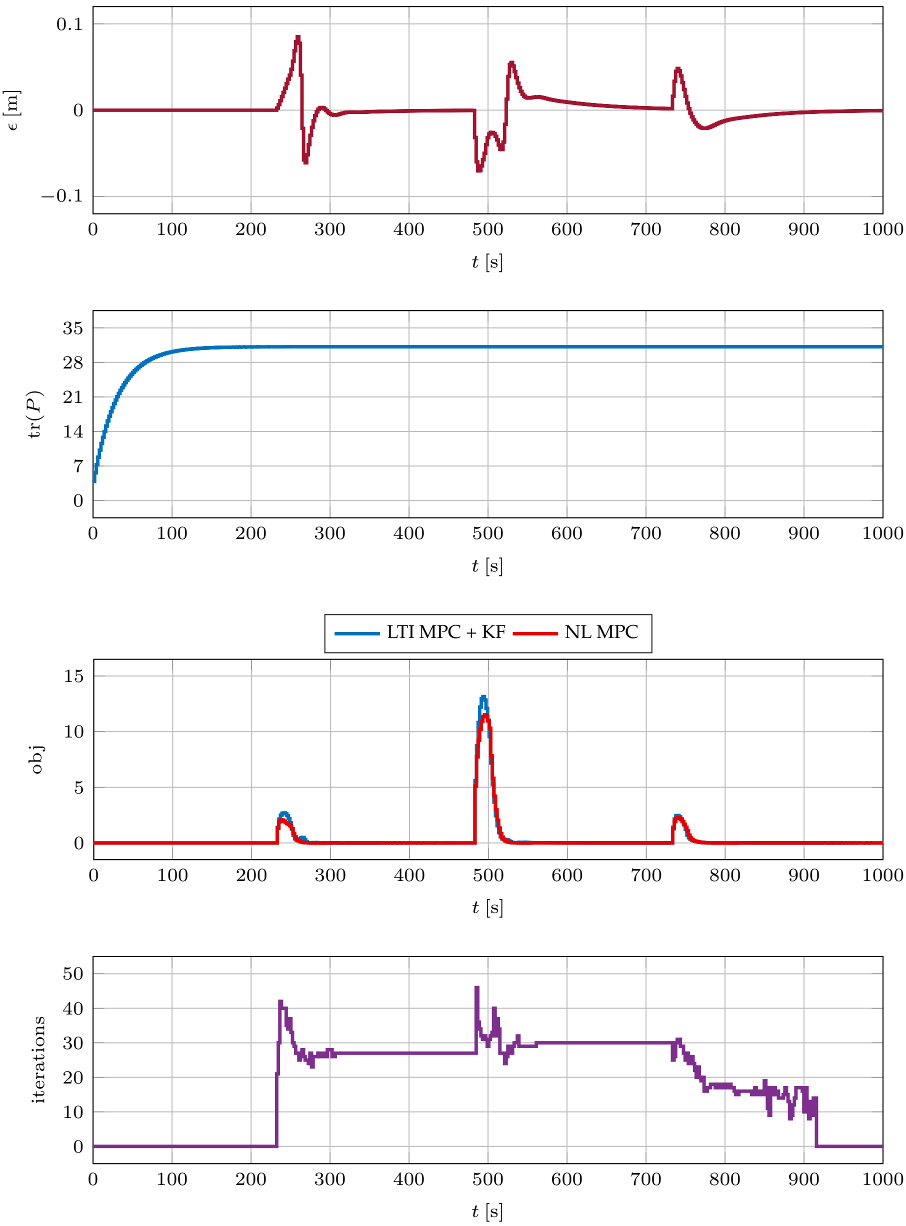

Figure 5.9: Estimation of disturbances, estimation error and covariance matrix of Kalman Filter for the horizontal cylindrical storage tank.

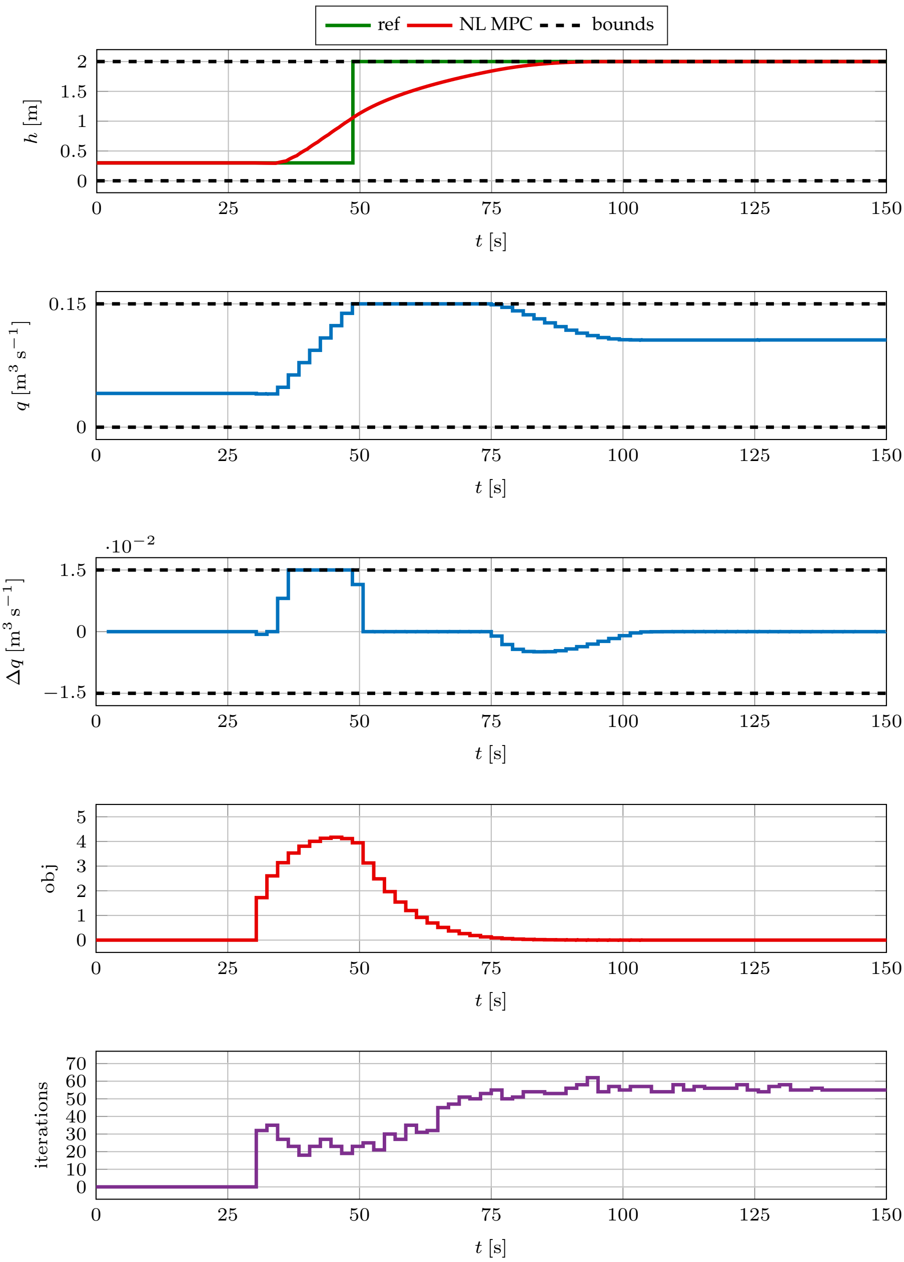

Figure 5.10: Maximum Level Control of the Conical Storage Tank using NL MPC.

weighting matrices Q_{y} and Q_{u} as follows

$$Q_{\mathrm{y}} = 0.945, \tag{5.34a}$$

$$Q_{\mathrm{u}} = 100. \tag{5.34b}$$

We considered the same constraints as for the comparison with linear MPC (5.19). The prediction horizon was set to $N = 10$. The liquid level control is depicted in the Fig. 5.10. We can see that we achieve an offset free control in steady state without overshoots and with respect all constraints. If we look at the optimization performance indexes, the sum of an objective function for the nonlinear MPC is $J_{NL} = 48.8402$ and the number of iterations is not so big. We are able to calculate the optimal control action in every sample time, that is $T_{\mathrm{s}} = 2s$.

Hence, we can see that using a NL MPC, we are able to utilize the whole working volume with respect to all constraints which is not possible with PID, LQ or other traditional control approaches [8] which do not allow us to consider various constraints.

CHAPTER 6

Conclusions

In this master thesis, we designed and implemented simple (PID and LQ) and advanced (LTI MPC with Kalman Filter and NL MPC) control strategies which were used for liquid level control in tanks with various geometry. We have shown, how to derive a linear and nonlinear mathematical model in continuous and discrete time for a conical, spherical and horizontal cylindrical storage tank. Then, we designed PID and LQ controller for each tank and we evaluated the performance criterion (ISE) in order to compare them. As we could see in the simulation figures, the LQ control was able to achieve the reference sooner without overshoots and also, the value of ISE was lower as for the PID control. If we could choose between these two approaches, the optimal LQ control is a better choice.

The problem of above mentioned strategies is that they are not able to handle a various constraints on manipulated, state and control variables or on technological parameters of the process, like the maximal height of the storage tank. That is why we designed a model predictive control which allowed us to consider these constraints. We designed two types of MPC - linear and nonlinear. However, in order to achieve an offset free control with linear MPC we had to assume an extended prediction model with disturbances which were observed via time-varying Kalman Filter. If we compare the LTI and NL MPC based on the simulations, we can say that NL MPC did not show such a big overshoots as the linear MPC. In both of cases we achieved the reference with respect to all constraints defined in MPC formulations. But, the biggest advantage of the nonlinear MPC is the fact, that we are able to control on the maximal height of the tank without overshoots and also with respect to all the constraints defined in the optimization problem. Such a control is not able with any other control strategy mentioned before. In process industry we often do not use the full working volume, since we are not able to ensure such a good control as NL MPC is. The disadvantage of such a control is that it is necessary to have a professional solver which is able to compute the optimal control action that us applied to the controlled process. However, as we could see in the simulation case studies, `fmincon` was able to solve the optimization problem in each sampling time wich was $2s - 5s$. It means, that such a controller is able to be used in the process industry.

APPENDIX A

Resumé

Predkladaná diplomová práca sa venuje návrhu a implementácii jednoduchých (PID a LQ) a pokročilých (MPC) riadiacich stratégií, ktoré sú následne použité na riadenie výšky hladiny v zásobníkoch s rôznou geometriou. V praxi zvyčajne nevyužívame celý objem zásobníkov na riadenie výšky hladiny, nakoľko nie je jednoduché navrhnúť regulátor takej kvality, aby bolo možné využívať celý pracovný objem. V tejto práci sa pozrieme bližšie na tento problém a pokúsime sa navrhnúť také regulátory, ktoré môžu byť reálne v praxi nasadené.

Ako už bolo spomenuté, technologický proces je reprezentovaný zásobníkom kvapaliny, pričom uvažujeme až tri rôzne tvary týchto zásobníkov, a to kužeľový, guľový a horizontálne valcový zásobník. Na to, aby sme mohli navrhnúť riadenie pre takýto technologický proces, musíme získať jeho matematický model. Tento sme získali na základe materiálovej bilancie zásobníka. Podrobnému odvodeniu matematickému modelu jednotlivých zásobníkov sa venujeme v kapitole 2. Súčasťou tejto kapitoly je aj samotné porovnanie ako sa správa výška hladiny pri zmenách akčného zásahu.

Po odvodení matematických modelov môžeme pristúpiť k návrhom riadenia. Ako prvé riadenie je navrhnuté PID, ktoré predstavuje najrozšírenejší typ riadenia v procesnom priemysle. Následne je navrhnuté optimálne LQ riadenie, ktoré je založené na minimalizácii kvadratického kritéria. Odvodeniu jednotlivých regulátorov sa venujeme v kapitole 3.

Nakoľko ani PID, ani LQ riadenie nie je schopné uvažovať ohraničenia na procesné ako aj technologické veličiny, pristúpime k návrhu pokročilejších metód riadenia, a to k prediktívnemu riadeniu. Podstata prediktívneho riadenia spočíva v minimalizácii kvadratickej účelovej funkcie vzhľadom na stavové, vstupné a výstupné ohraničenia. V tejto práci uvažujeme dva typy prediktívnych regulátorov - lineárne a nelineárne. Lineárne MPC je zostavené na základe diskrétneho lineárneho modelu v YALMIPe, pričom optimalizačný problém je riešený pomocou riešiteľa GUROBI.

Vzhľadom na to, že lineárny model nie je identický s reálnym procesom, pri riadení nedosiahneme žiadanú veličinu v ustálenom stave. Tento jav sa v zahraničnej literatúre nazýva ako *"model mismatch"*. Na to, aby sme dosiahli riadenie na žiadanú hodnotu, potrebujeme tento predikčný model rozšíriť o poruchy, ktoré si namodelujeme. Tieto poruchy ako aj samotné stavy procesu sú následne odhadované pomocou navrhnutého časovo premenlivého Kalmanovho filtra. Podrobné odvodenie formulácie MPC ako aj Kalmanovho filtra je popísané v kapitole 4. Súčasťou tejto kapitoly je aj formulácia nelineárneho prediktívneho riadenia, ktorý nie je možné formulovať v YALMIP toolboxe, nakoľko tento pracuje len s lineárnymi ohraničeniami. Preto sme na riešenie takéhoto optimalizačného problému zvolili riešiteľa v MATLABe, ktorý sa nazýva `fmincon`.

5. kapitola je venovaná samotnej implementácií jednotlivých regulátorov na riadenie zásobníkov kvapaliny, kde sú následne jednotlivé prístupy aj vyhodnotené na základe rôznych kvalitatívnych ukazovateľov v prípadových štúdiách. Porovnávali sme zvlášť jednoduché stratégie riadenia a zvlášť pokročilé metódy. V prípade PID a LQ riadenia môžeme vidieť, že LQ riadenie dosiahne žiadanú hodnotu rýchlejšie ako PID riadenie a bez preregulovania. Lepšiu kvalitu riadenia potvrdzuje aj kvalitatívny ukazovateľ ISE, ktorého hodnota je v prípade LQ riadenia menšia. Čím menšia je hodnota tohto ukazovateľa, tým kvalitnejšie riadenie sme navrhli. V oboch prípadoch sme regulátory navrhli na základe lineárneho modelu a následným ladením sme ich použili na riadenie nelineárneho modelu.

V prípade pokročilejších metód riadenia, kedy riešime optimalizačný problém na získanie optimálnych akčných zásahov, je na porovnanie lineárneho a nelineárneho MPC použitý ako kvalitatívny ukazovateľ hodnota účelovej funkcie počas trvania simulácie. Teda vyhodnocuje sa suma hodnôt účelových funkcií v každom kroku simulácie. Ako vyplýva z grafov jednotlivých priebehov riadenia, nelineárne MPC reprezentuje vždy lepšiu alternatívu na riadenie výšky hladiny ako lineárne MPC. Je to spôsobené tým, že nelineárne MPC je navrhnuté na základe nelineárneho modelu, a teda lepšie opisuje správanie sa procesu. Ďalšou nespornou výhodou nelineárneho prediktívneho riadenia je možnosť riadiť na maximálnu hodnotu výšky hladiny v zásobníku bez preregulovania a s rešpektovaním všetkých ohraničení, a teda využívať celý pracovný objem zásobníka na rozdiel od PID, LQ alebo iných tradičných riadiacich algoritmov, ktoré tohoto nie sú schopné. Ďalej sme ukázali, že pri nelineárnom MPC sme boli schopní vyriešiť optimalizačný problém v rámci jednej periódy vzorkovania, čo predstavuje 2 sekundy v prípade kužeľového zásobníka a 5 sekúnd v prípade guľového a horizontálne valcového zásobníka. Teda, takýto MPC regulátor je možné použiť v priemysle na riadenie výšky hladiny v zásobníkoch kvapaliny.

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