Comparison of Two Approaches to Agile Manoeuvres via MPC

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Abstract

Model predictive control (MPC) is a widely used optimization-based control strategy. In this work, MPC is used to perform agile manoeuvres in pendulum-on-a-cart system. The objective is to quickly move the cart from the initial position to the target position while suppressing oscillations of the pendulum, which is mounted on the cart using a free joint. Moreover, the pendulum's endpoint is required to stay within prescribed constraints as to avoid collisions with obstacles. Two MPC strategies are explored. The first one is based on a linearized model, coupled with standard QP-based optimization. The second approach employs the full nonlinear dynamical model and uses a random-shooting algorithm to select control moves. Both approaches are compared by means of real-time experiments.

Process description

The nonlinear model of the laboratory pendulum considered in this contribution is described by

$$
\begin{aligned}
\dot{x_1} &= x_2,\\
\dot{x_2} &= \frac{3}{4L} u\cos(x_1) + \frac{3g}{4L}\sin(x_1) - bx_2,\\
\dot{x_3} &= x_4,\\
\dot{x_4} &= u,
\end{aligned}
$$

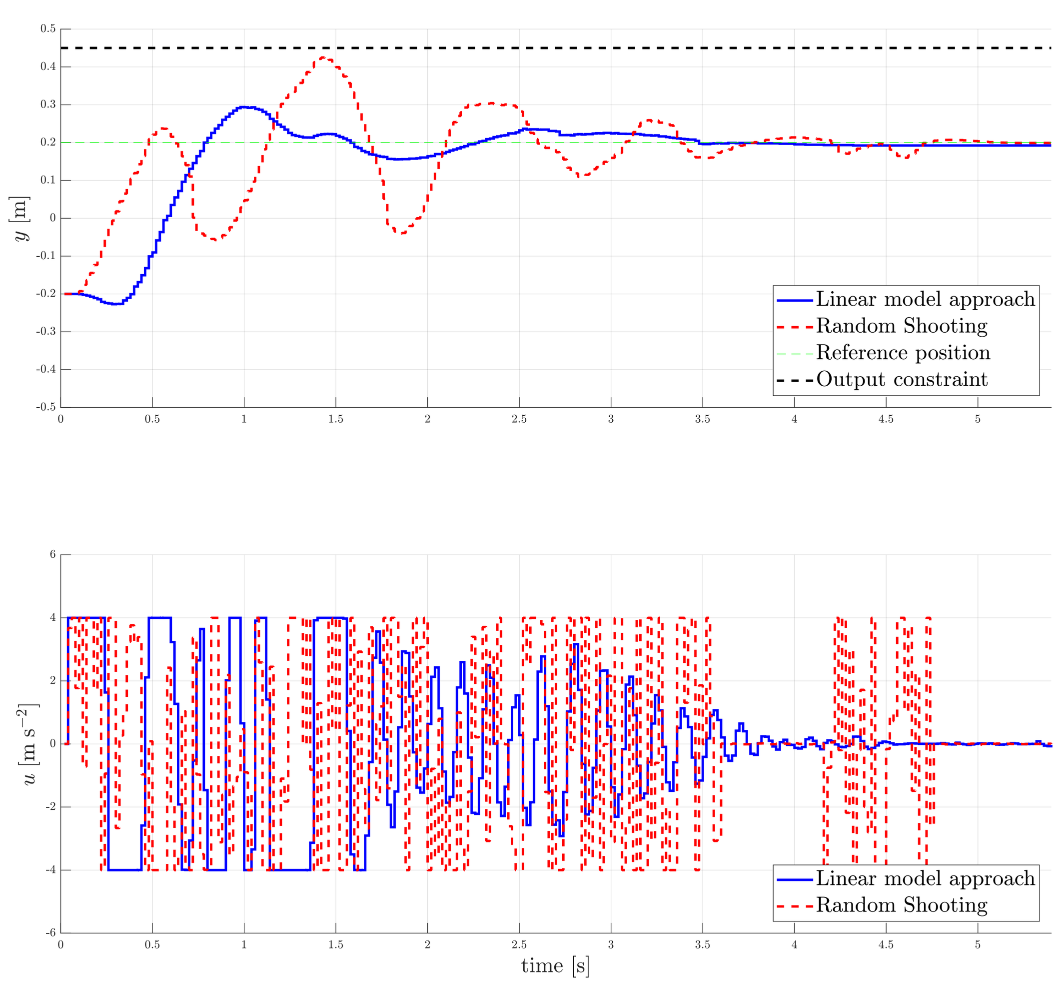
along with the output equation

$$y = L\sin(x_1) + x_3$$

where x_1 is angle of the pendulum, x_2 is angular velocity, x_3 is position of a cart, x_4 is speed of the cart and u is the acceleration of the cart (the manipulated variable). The output from the system y is the x-position of the pendulum's endpoint. There are no constraints for states x_1, x_2 and the rest is defined as follows

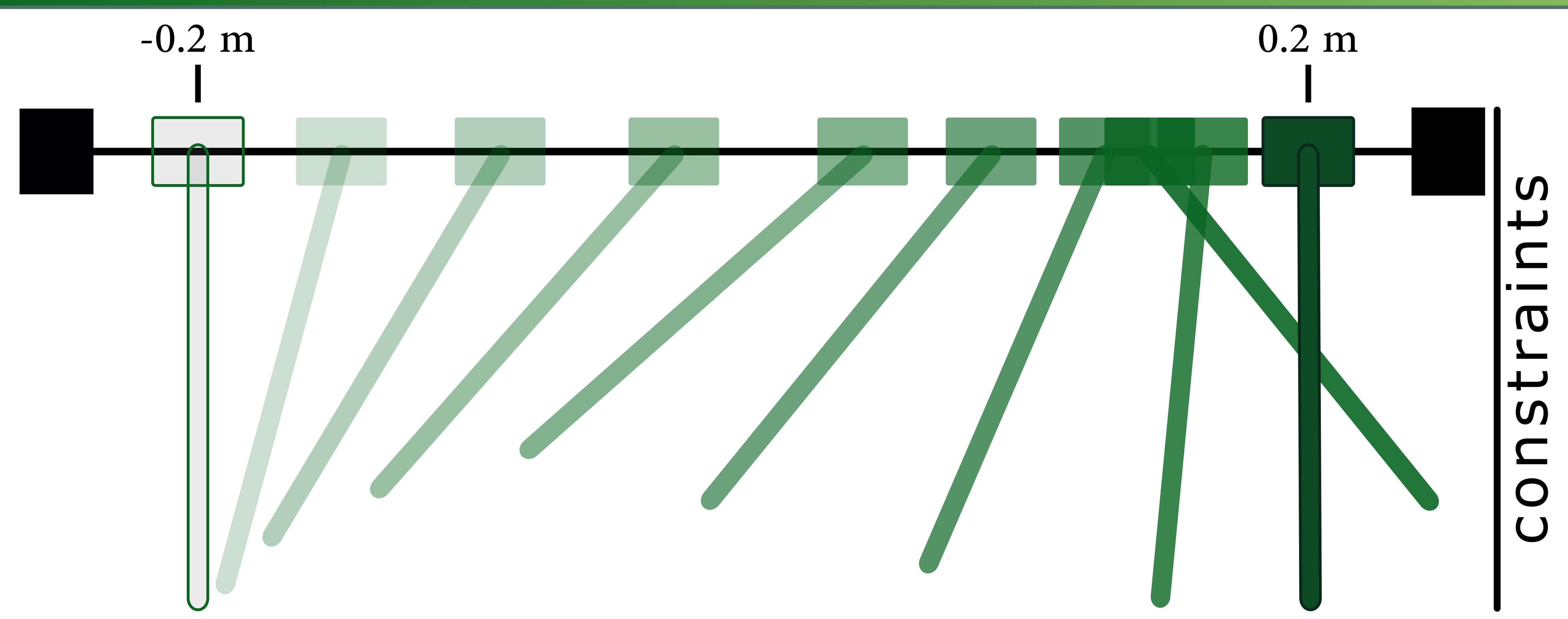
$$
\begin{aligned}
-0.25 \leq x_3 &\leq 0.25,\\
-2.0 \leq x_4 &\leq 2.0,\\
-\infty \leq y &\leq 0.45,\\
-4 \leq u &\leq 4.
\end{aligned}
$$

Results



The aim of the experiment was to change cart position from -0.2 m to 0.2 m without exceeding the constraints on y ($y \leq 0.45$ m). As can be seen the linearization-based method performs significantly better, exhibiting a smaller overshoot. The random shooting method shows more oscillations due to fact that it generates random control actions. However, both methods are able to move cart to desired position with similar settling time and without violating the constraints.

Agile Manoeuvre



Model Predictive Control

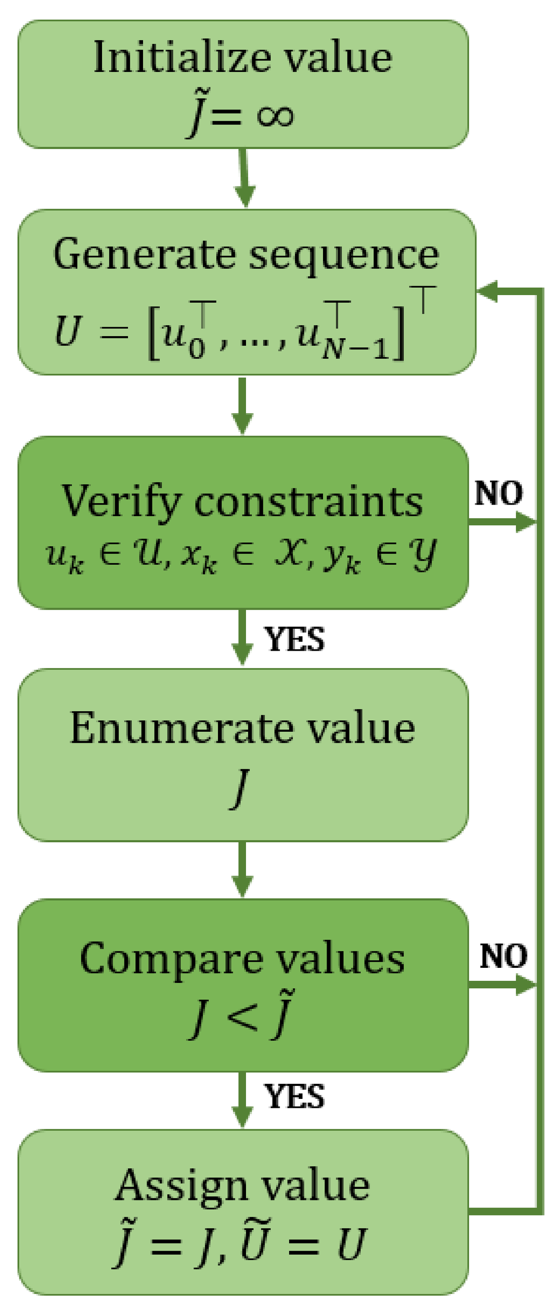
The first strategy is to use MPC. However, for this method the linearization is necessary. The linear model of the system is obtained by applying the truncated Taylor expansion.

$$
\begin{aligned}
\min_{u_0,\dots,u_{N-1}} \; & \sum_{k=0}^{N-1} (x_k^\top Q x_k + u_k^\top R u_k)\\
\text{s.t. } & x_{k+1} = Ax_k + Bu_k, && k = 0,\dots,N-1,\\
& y_k = Cx_k + Du_k, && k = 0,\dots,N-1,\\
& \underline{x} \leq x_k \leq \overline{x}, && k = 0,\dots,N-1,\\
& \underline{u} \leq u_k \leq \overline{u}, && k = 0,\dots,N-1,\\
& \underline{y} \leq y_k \leq \overline{y}, && k = 0,\dots,N-1,\\
& x_0 = x(t),
\end{aligned}
$$

where x is the state vector, u is the control input. The advantage of the linear prediction form is that the MPC problem becomes a convex optimization problem, thus easily solvable using conventional optimization techniques provided the constraint sets $\mathcal{X}$, $\mathcal{U}$, $\mathcal{Y}$ are convex sets. If they are not, one can derive their respective inner convex approximations, e.g., by maximizing the volume of the inscribed ellipsoid, or by searching for a largest inscribed hyper-box.

Random Shooting

Random shooting method operates by investigating a (possible large) number of randomly generated control sequences. The algorithm of the method is described by



and it is repeated until the defined number of feasible sequences are generated. The new cost function J is enumerated by

$$J = \sum_{k=0}^{N-1} ((y_k - r)^\top Q (y_k - r) + u_k^\top R u_k),$$

where y is output of the system for given generated sequence and r is the final desired position. The random shooting method selects the feasible sequence that yields the smallest value of cost function.

Conclusions

This paper has shown how MPC can be applied to control the agile manoeuvres system. Two approaches were used. The random shooting method operates with nonlinear model of the system, but the provided result is only suboptimal. The other strategy was based on linearization of nonlinear dynamics, followed by solving the MPC problem as convex quadratic program. Both versions of the controller were implemented in real time with satisfactory results.

Acknowledgments

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