Dual multi-stage NMPC using sigma point principles

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- Optimal operation of the process
- Model based controllers (e.g. Model predictive control)

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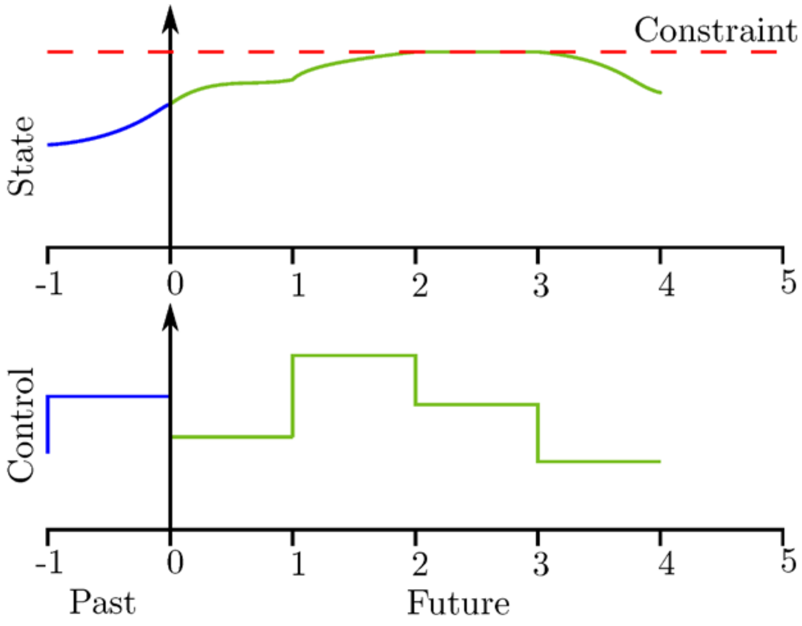

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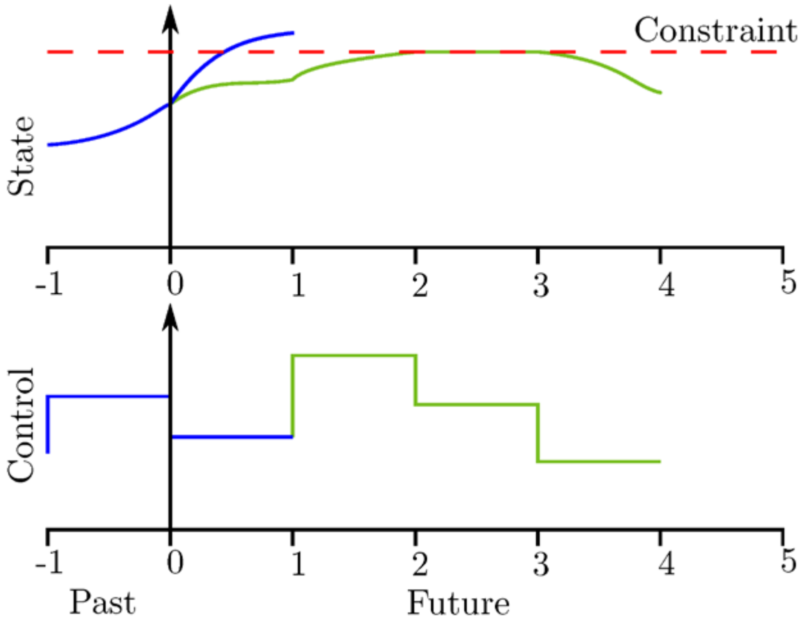
  - Constraint violation
  - Performance degradation and instability



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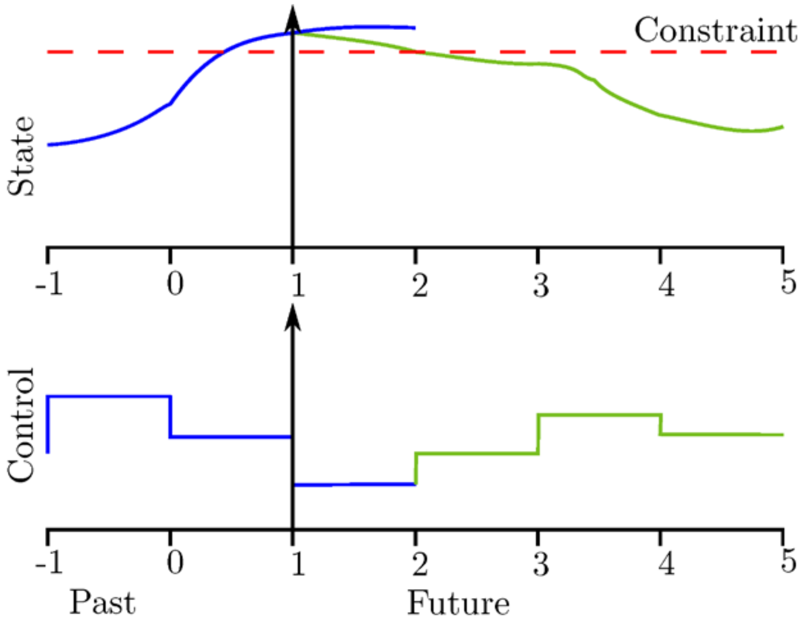

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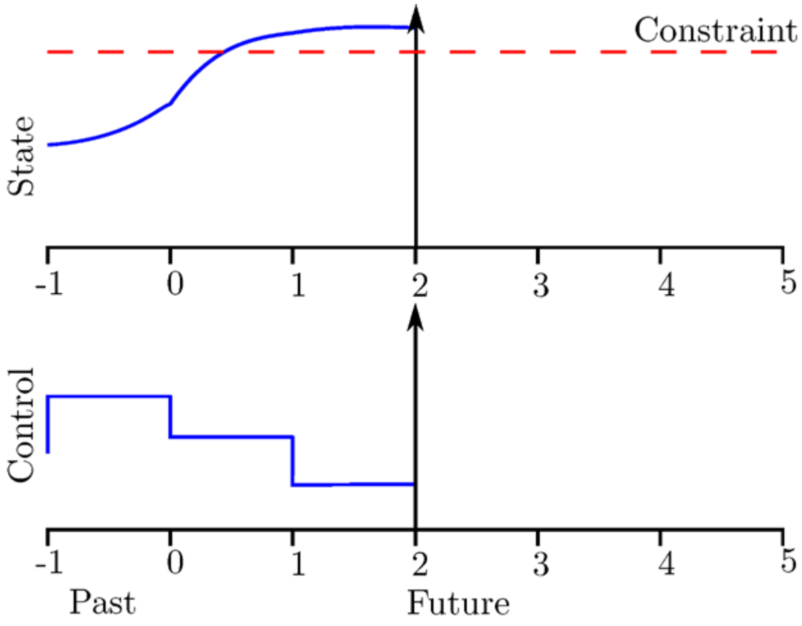

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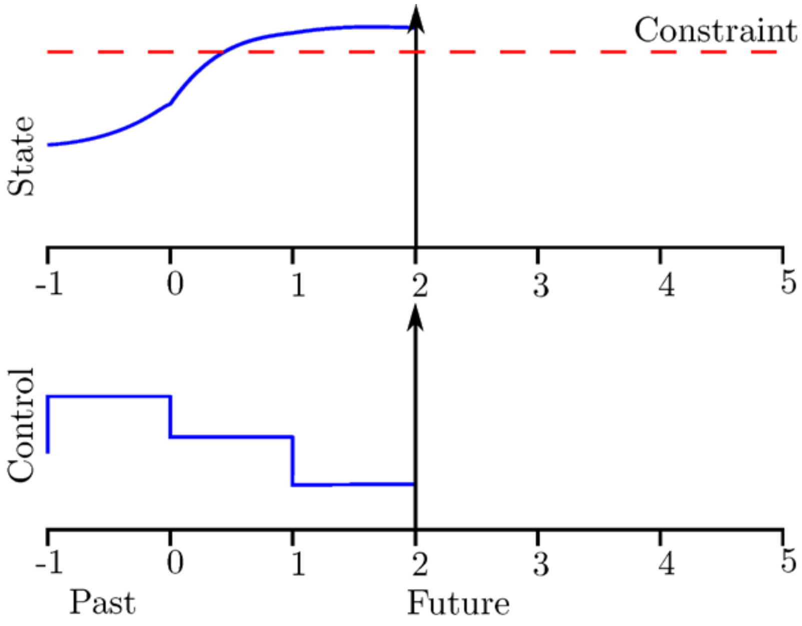

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Nonlinear Model Predictive Control (NMPC)

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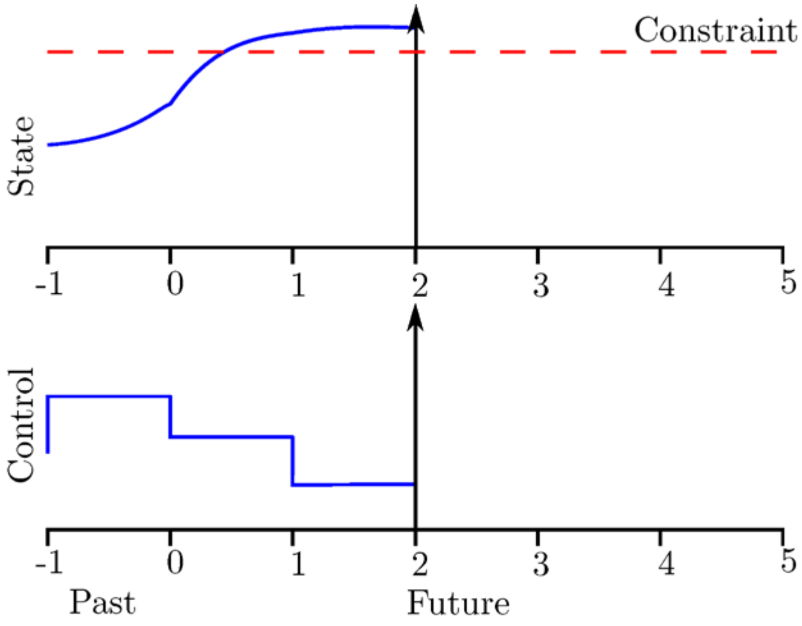
Robust MPC

- Robust MPC schemes are conservative when compared to the case where true information about the plant is available



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- In this talk, we focus on improving the performance of the robust multi-stage NMPC in the presence of an ellipsoidal uncertainty set using sigma point principles and dual control schemes

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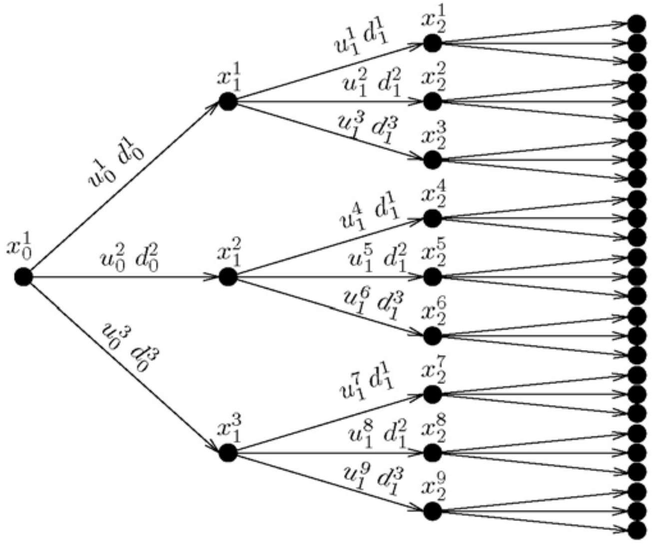

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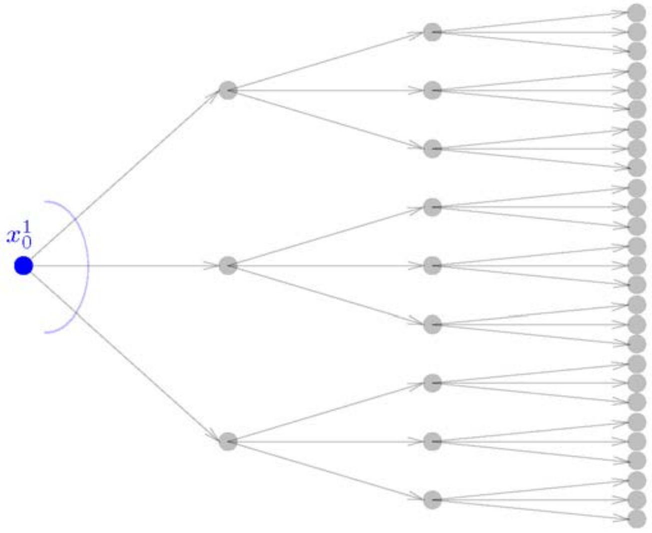

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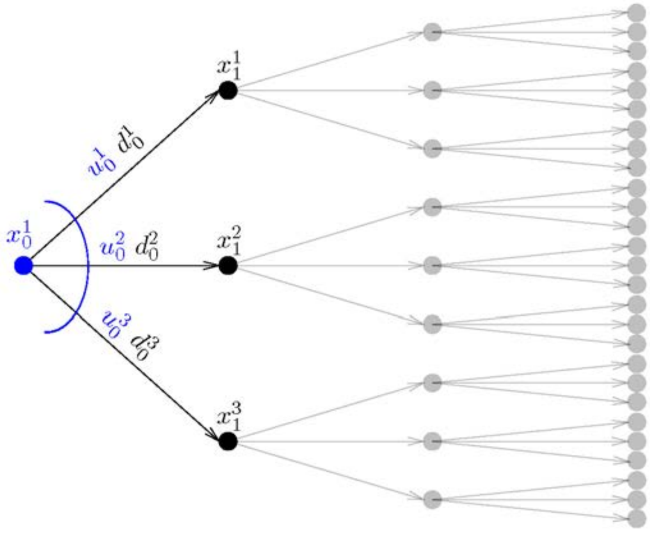

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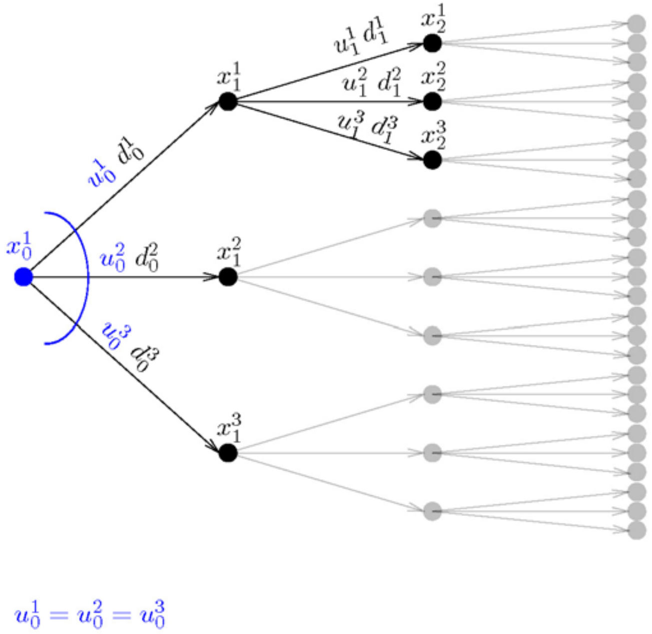

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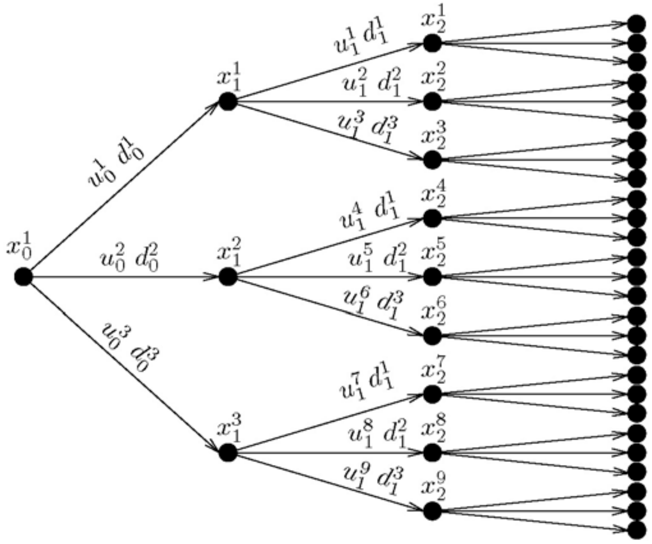

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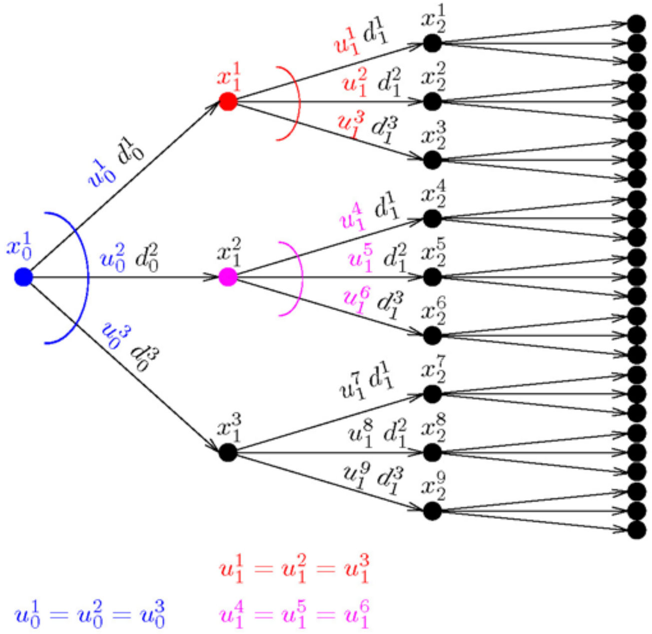

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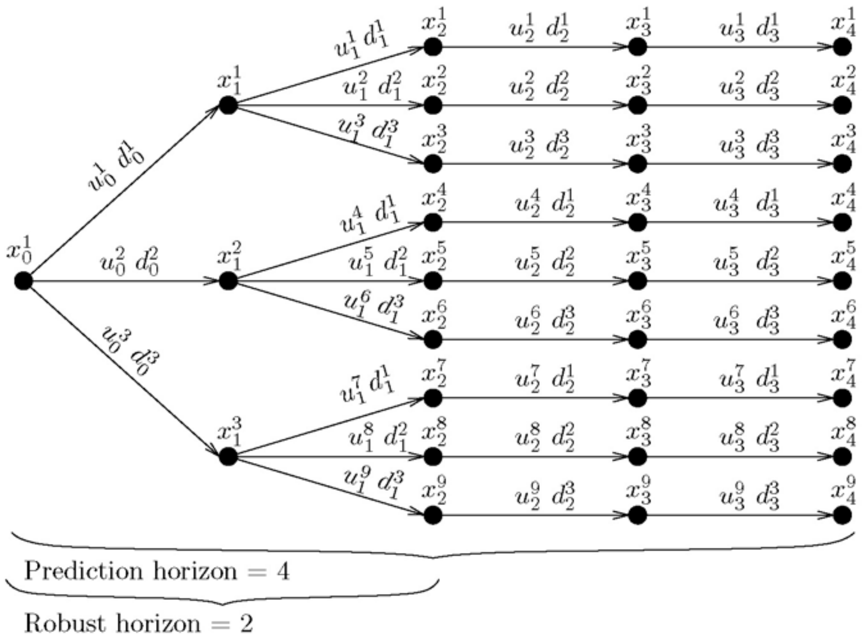

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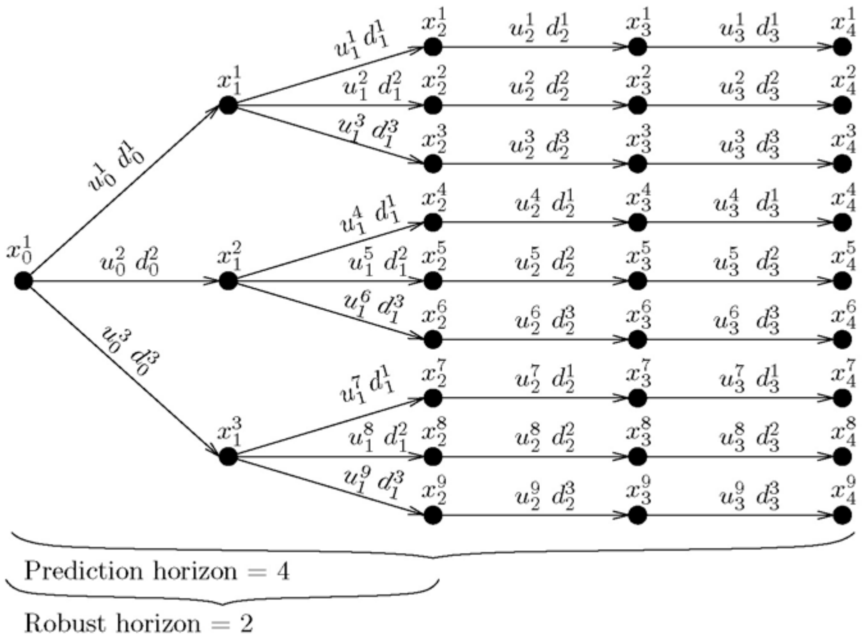
- In the nonlinear case, the scenario tree is rigorously valid in the presence of discrete valued uncertainty but it is an approximation in the presence of continuously valued uncertainty



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Building the scenario tree of robust multi-stage NMPC

- Plant model:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}),\ \forall \mathbf{d} \in \mathbb{D}(\mathbf{d}_0, \mathbf{P}_0)$$

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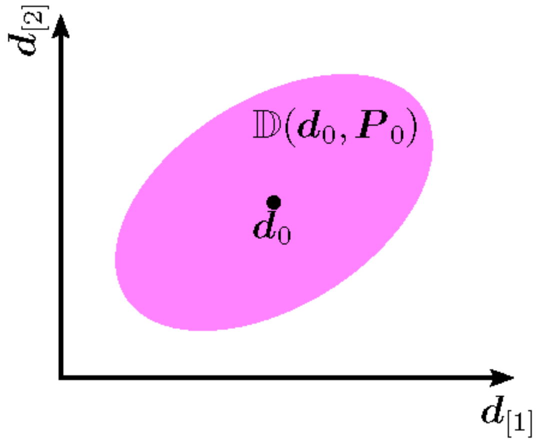

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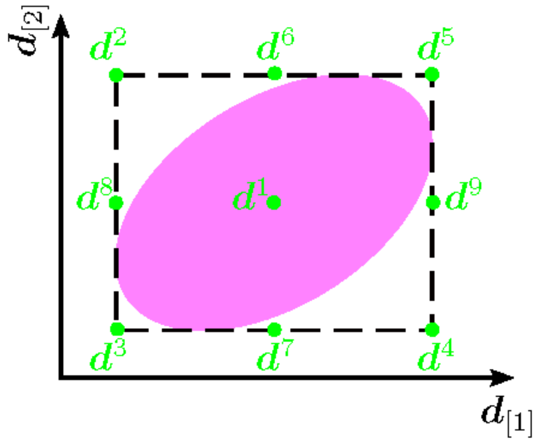
- In practice, the scenario tree built for all combinations of minimal, nominal and maximal values of uncertainty provides very good results [2]
- 3^{n_d} branches to be considered at each node in the scenario tree
- The robust constraint satisfaction is guaranteed if the parametric monotonic property of the nonlinear model is satisfied [3]



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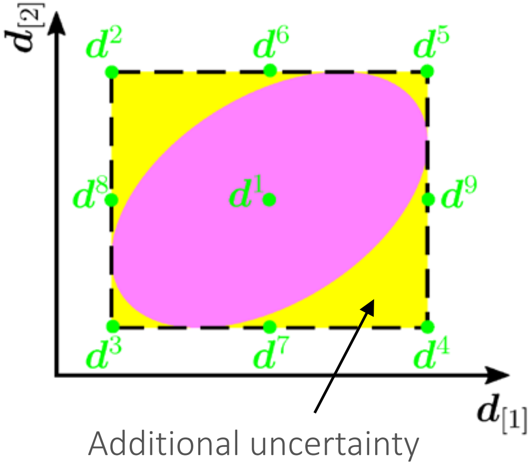
- The additional uncertainty considered in the scenario tree reduces the performance of the multi-stage NMPC



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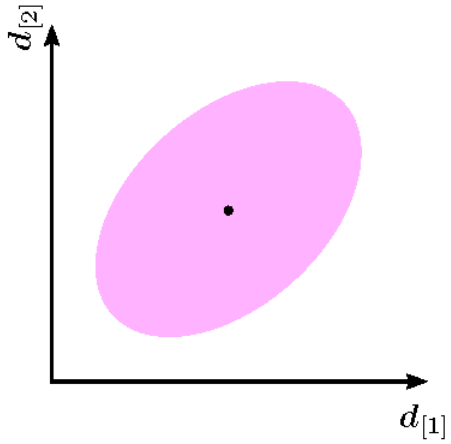

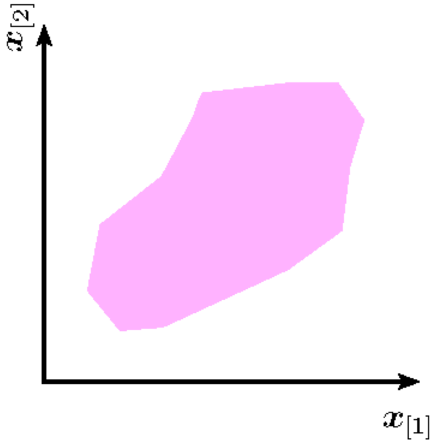

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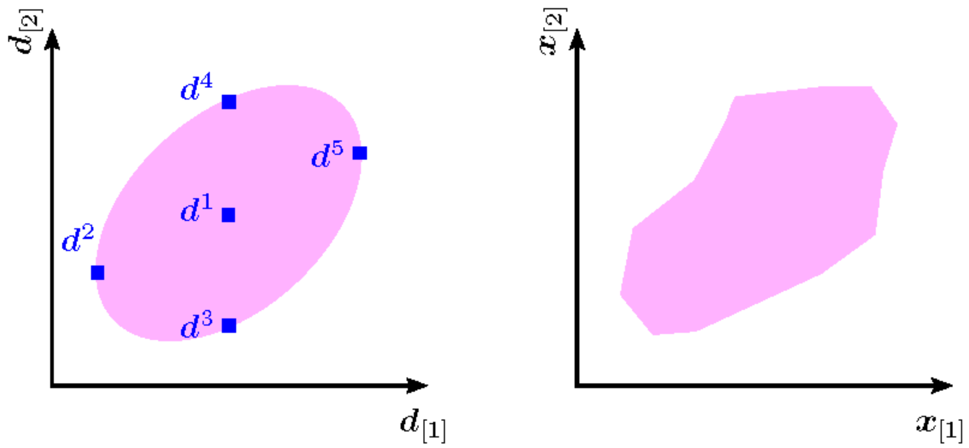

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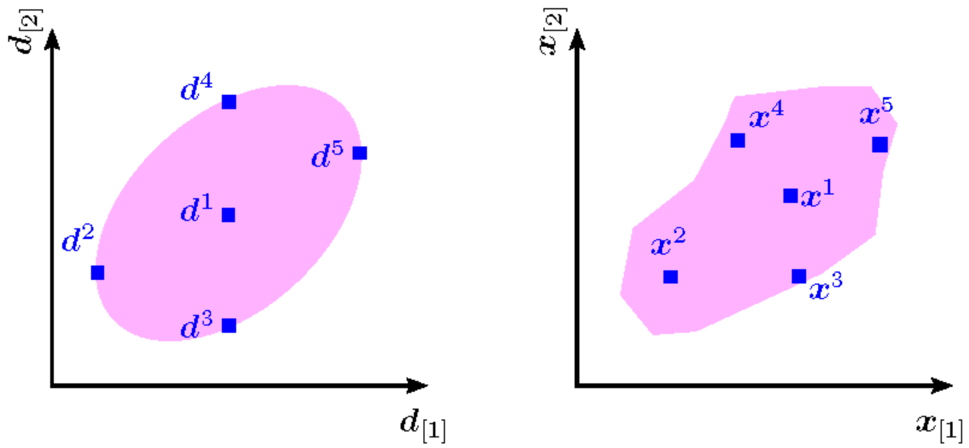

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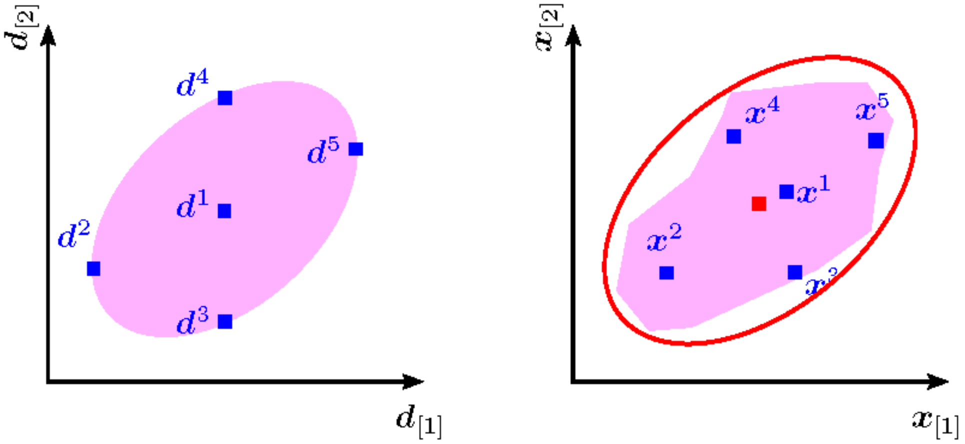

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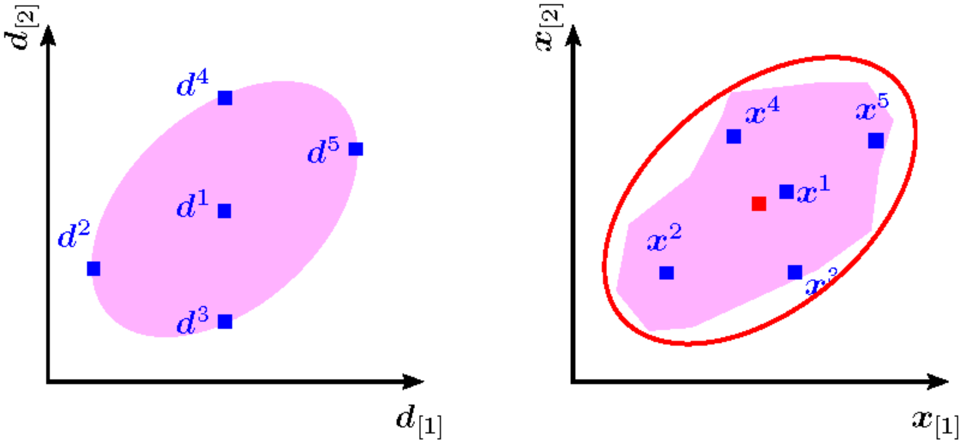
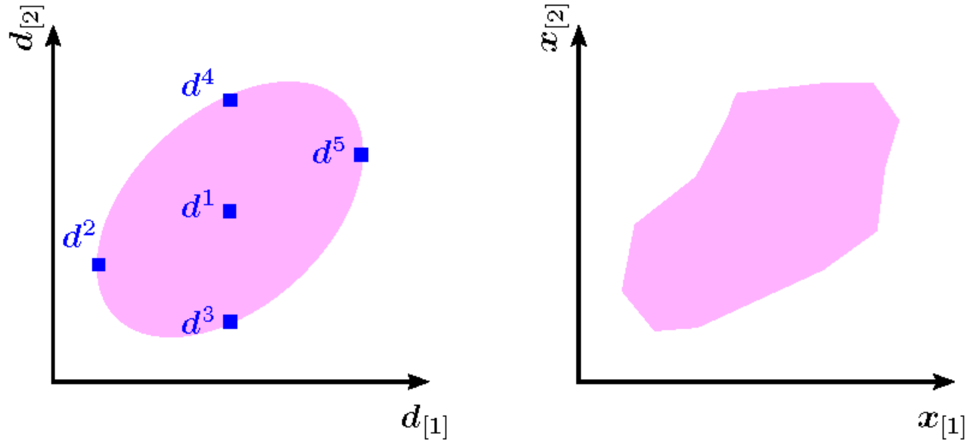
- Such state ellipsoid can be obtained using unscented transformation



Unscented Transformation [4]

- The $2n_d + 1$ sample points (sigma points) are propagated through the nonlinear model to compute state mean and state covariance matrix



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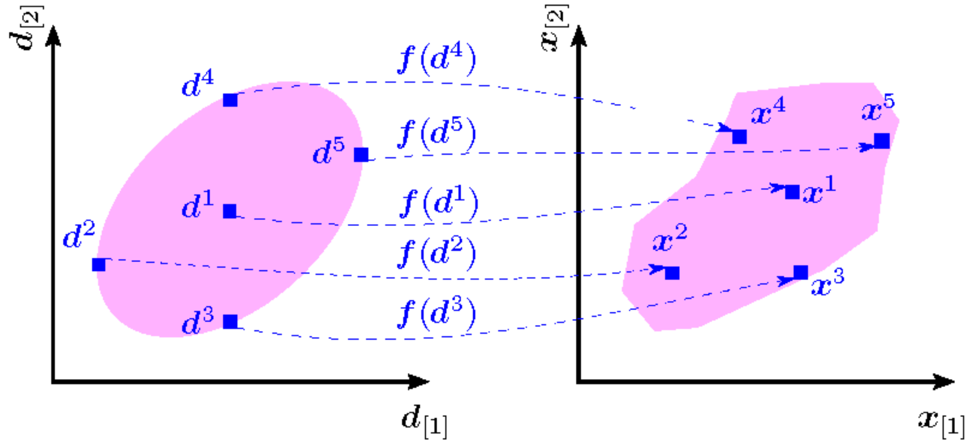

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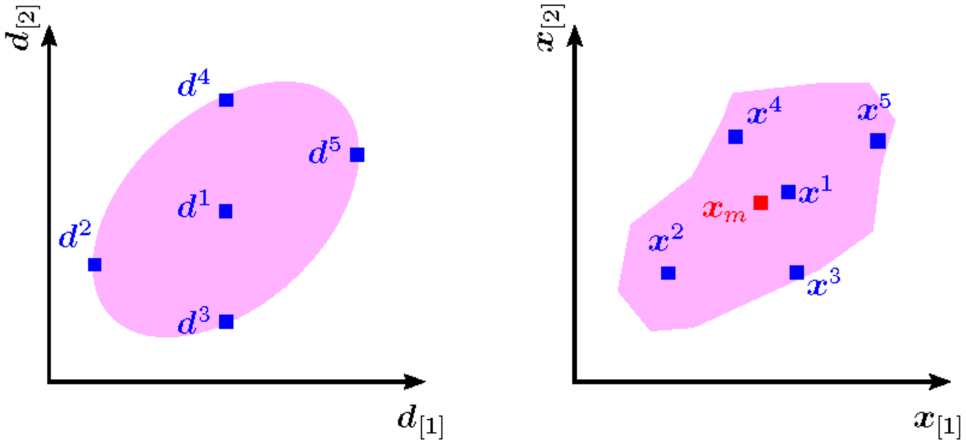

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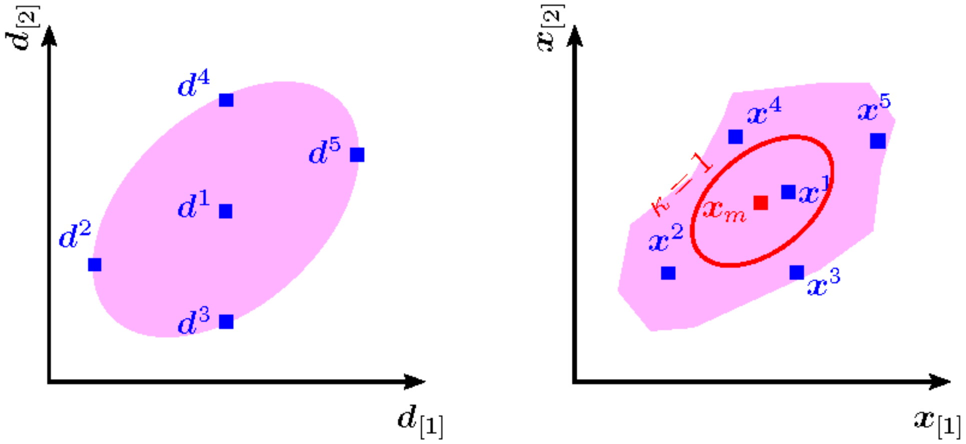

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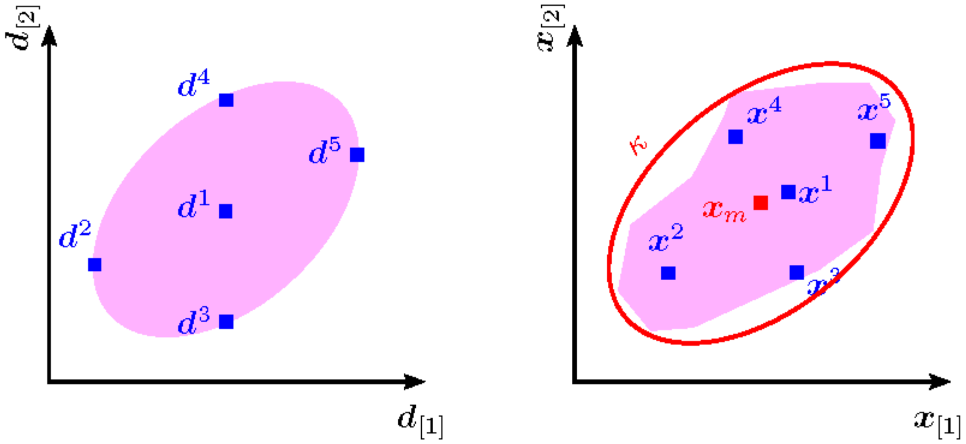

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- The state covariance matrix can be scaled using the scaling factor κ such that the state ellipsoid encloses the reachable set of the model which is a tuning parameter[5]

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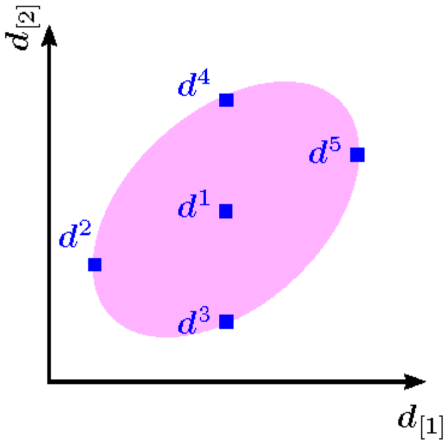

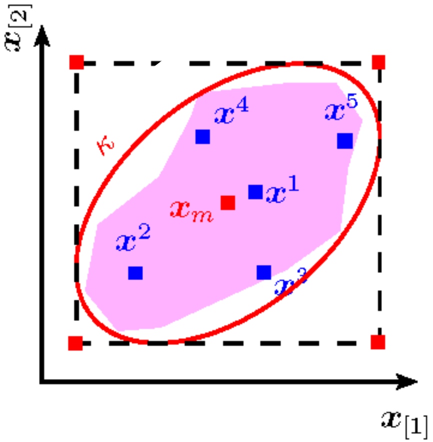

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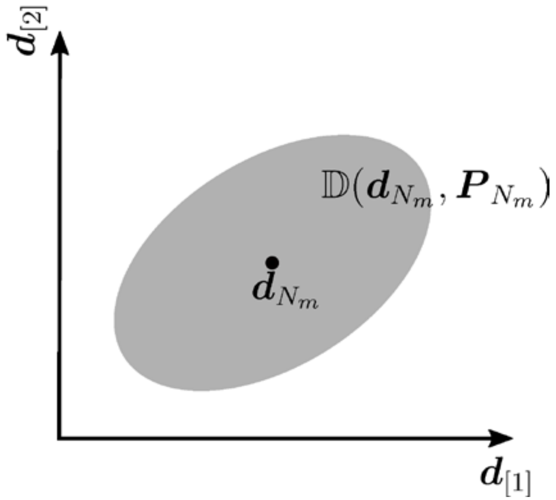

Confidence region using observed measurements

- An estimate of the uncertain parameters can be obtained using the observed measurements (e.g. least-squares estimates)
- A confidence region (CR) which encloses the true parameters can be obtained from the parameter covariance matrix
- The inverse of the Fisher information matrix gives the upper bound on the parameter covariance matrix

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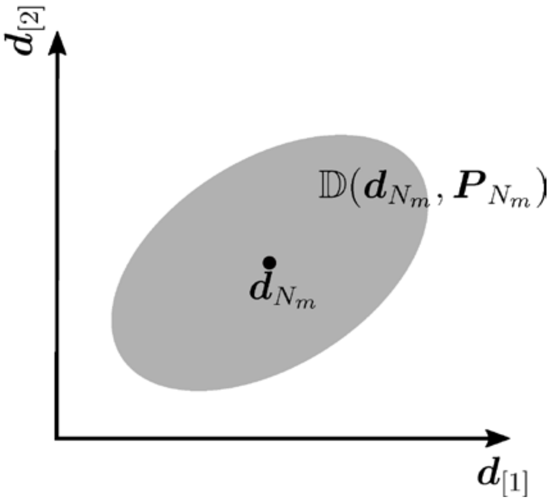

- The new confidence region is used to reduce the uncertainty in the parameters and update the scenario tree of the multi-stage NMPC

Updating the scenario of the adaptive multi-stage NMPC

- The information provided by the initial confidence region and the confidence region obtained using the observed measurements should be taken into account

[6] Thangavel, S., Paulen, R., and Engell, S. (2020), Adaptive multi-stage NMPC using sigma point principles, ECC 2020

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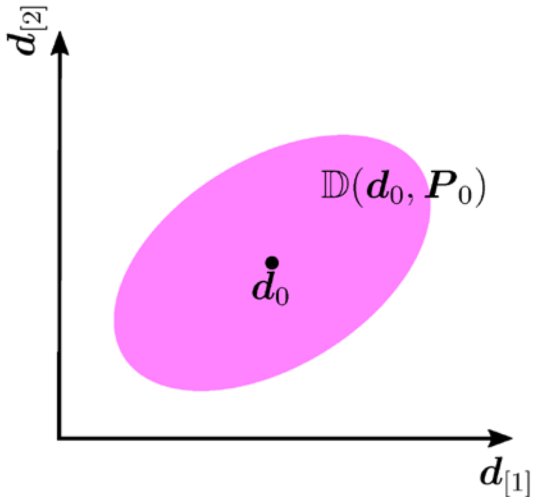

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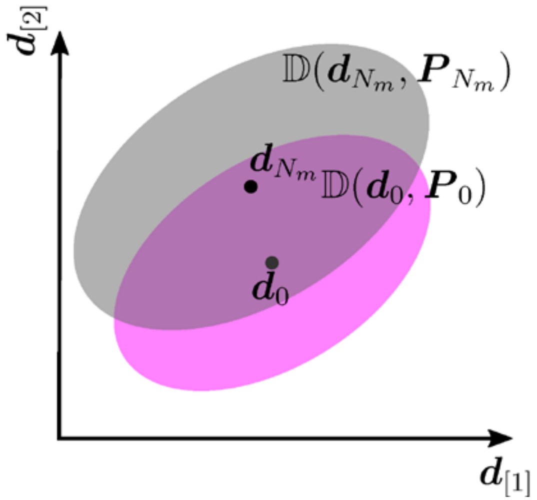

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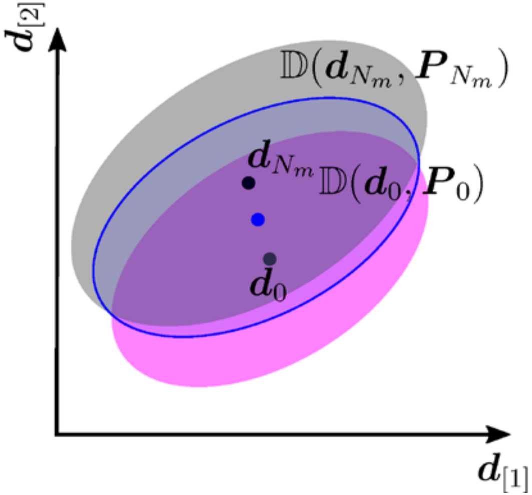

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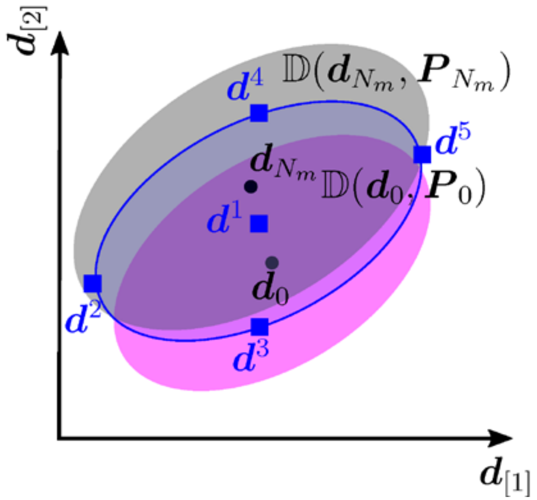

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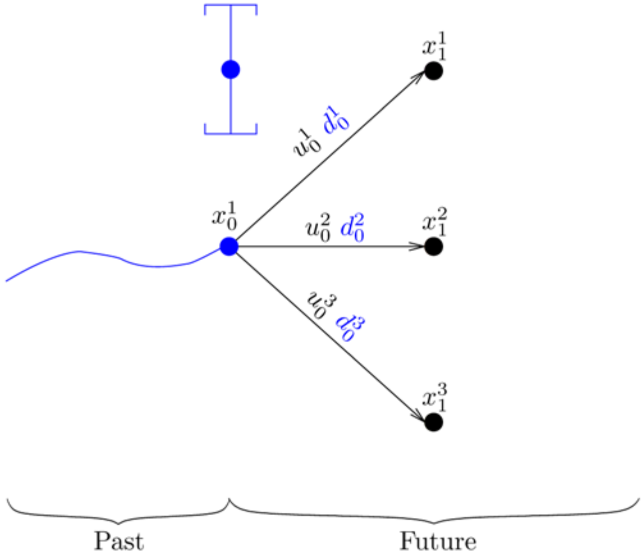
Proposed dual multi-stage NMPC using sigma point principles

- Dual NMPC considers the effect of the control input on the future reduction in uncertainty along the prediction horizon to improve the objective



Proposed dual multi-stage NMPC using sigma point principles

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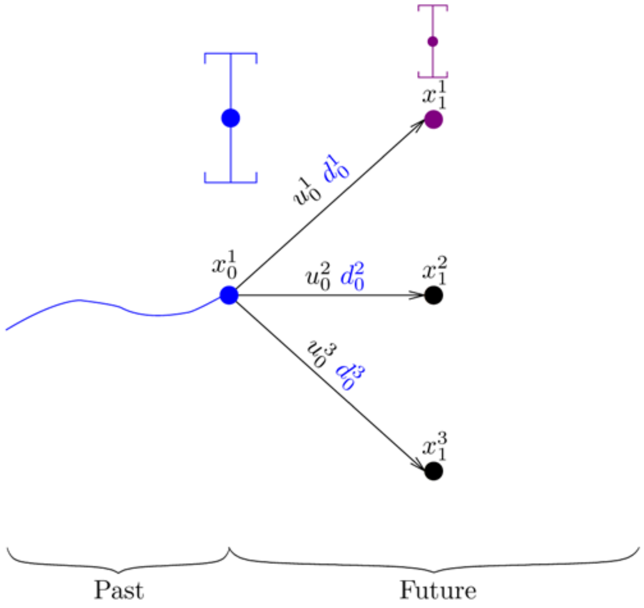

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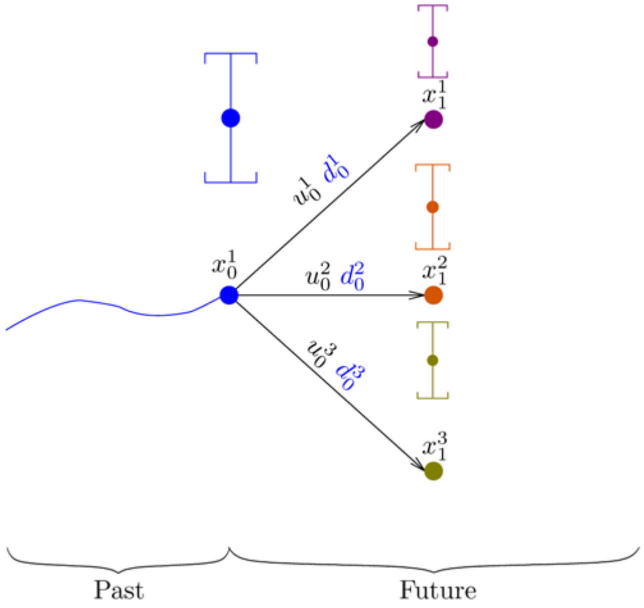

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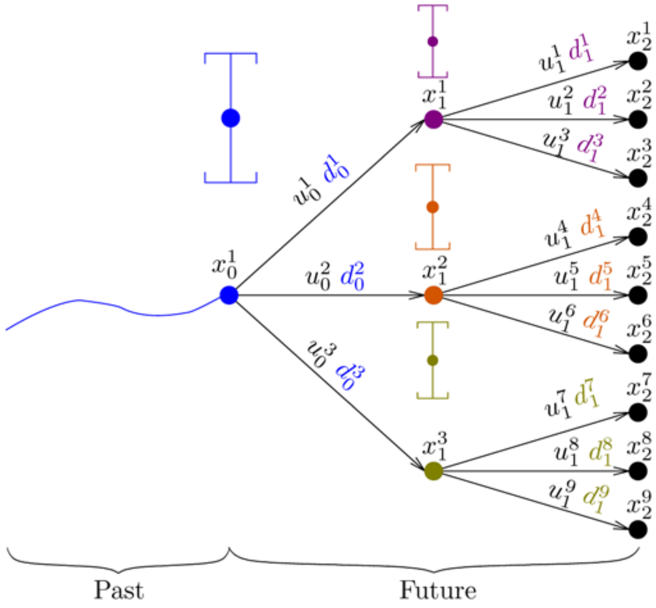

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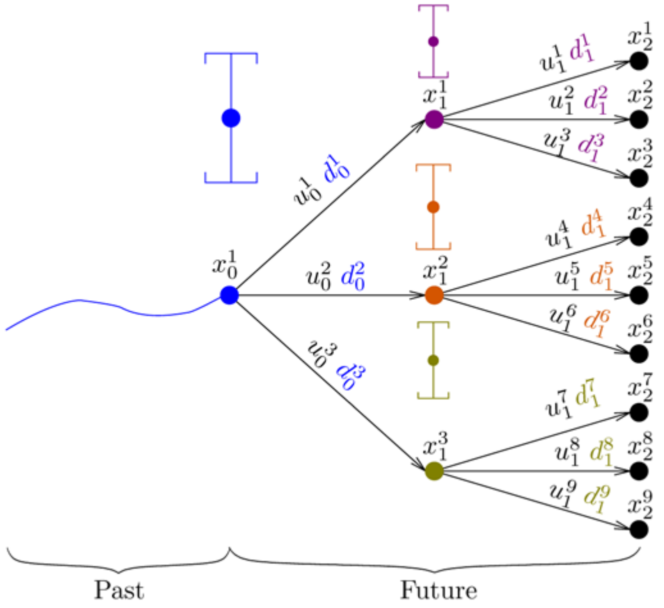
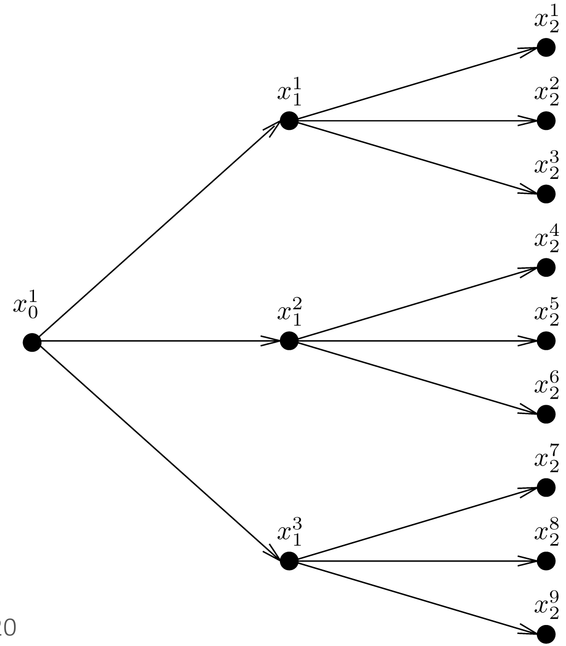
- The scenario tree is dual multi-stage NMPC is built based on the sigma points of the predicted future confidence region
- The scenario tree narrows down along the prediction horizon



Scenario tree of different multi-stage NMPC using sigma point principles

- Non-adaptive: sigma points of the **initial confidence region** of the uncertain parameters[5]



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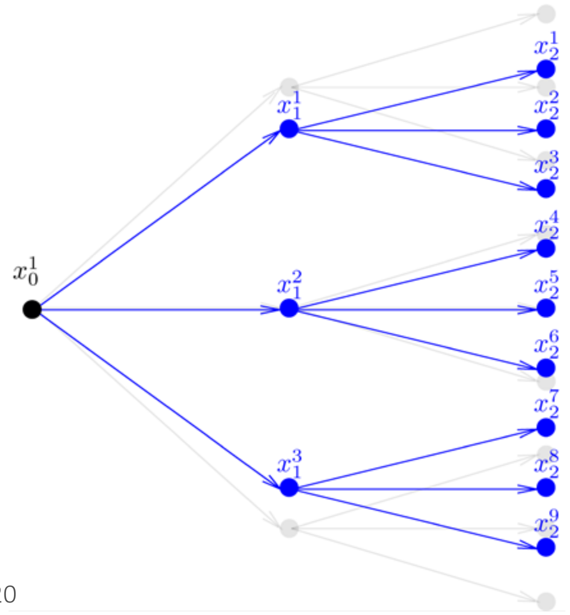

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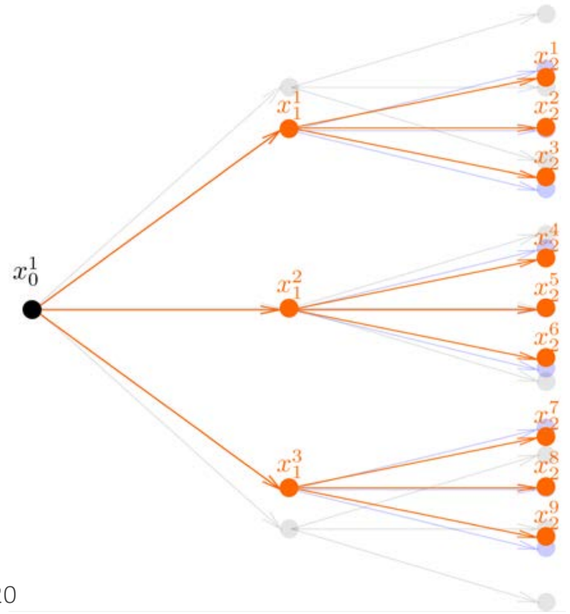

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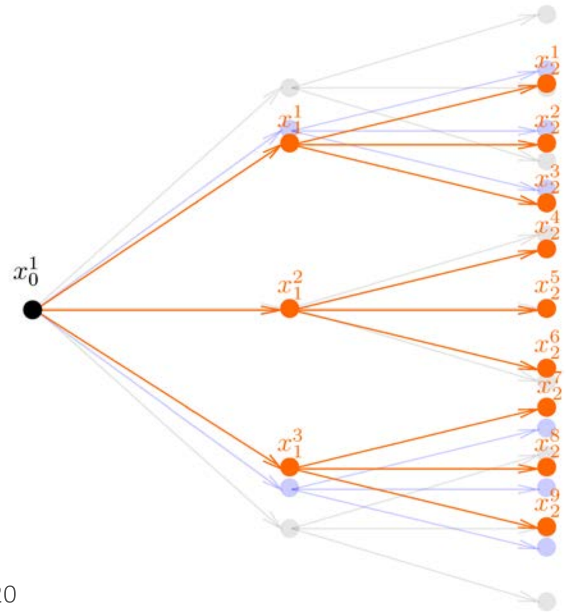
- The probing inputs will be applied to the plant if they result in a better performance along the prediction horizon



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[6] Thangavel, S., Paulen, R., and Engell, S. (2020), Adaptive multi-stage NMPC using sigma point principles, ECC 2020

Semi-batch reactor[5]

[5] Srinivasan, B., Palanki, S., and Bonvin, D., (2003) Dynamic optimization of batch processes: I. Characterization of the nominal solution, Computers & Chemical Engineering, Volume 27, Issue 1, Pages 1-26

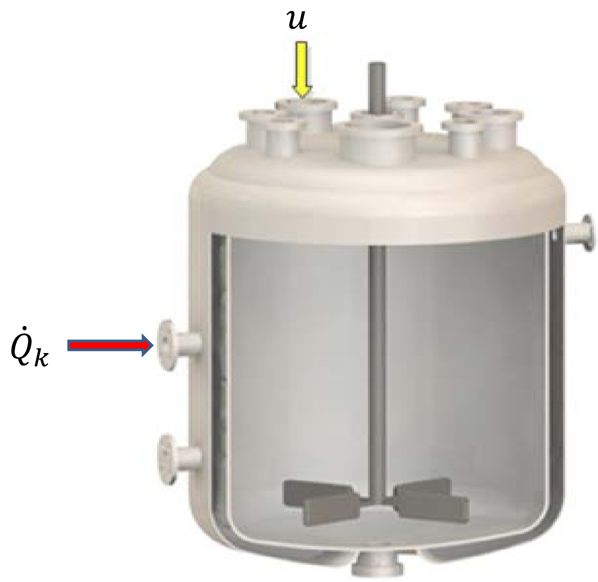
Reaction scheme

$$A + B \rightarrow C$$

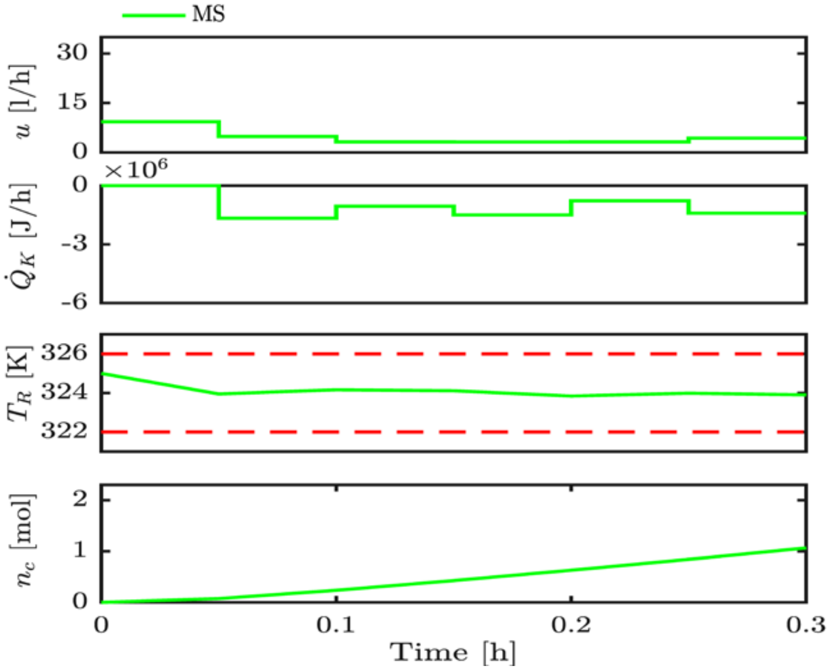
- The reaction is exothermic

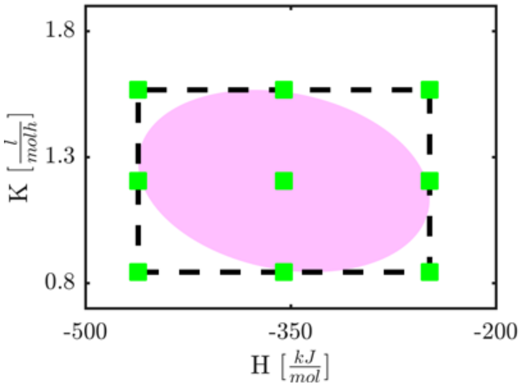
- **Objective function:** Maximize the amount of product C produced
- **Constraint function:** $322 \leq T_R \leq 326$

- **Manipulated variables:**
  - Feed rate of reactant (u)
  - Cooling energy ($\dot{Q}_k$)
- **Uncertain parameters:**
  - Reaction enthalpy (H)
  - Reaction rate constant (K)



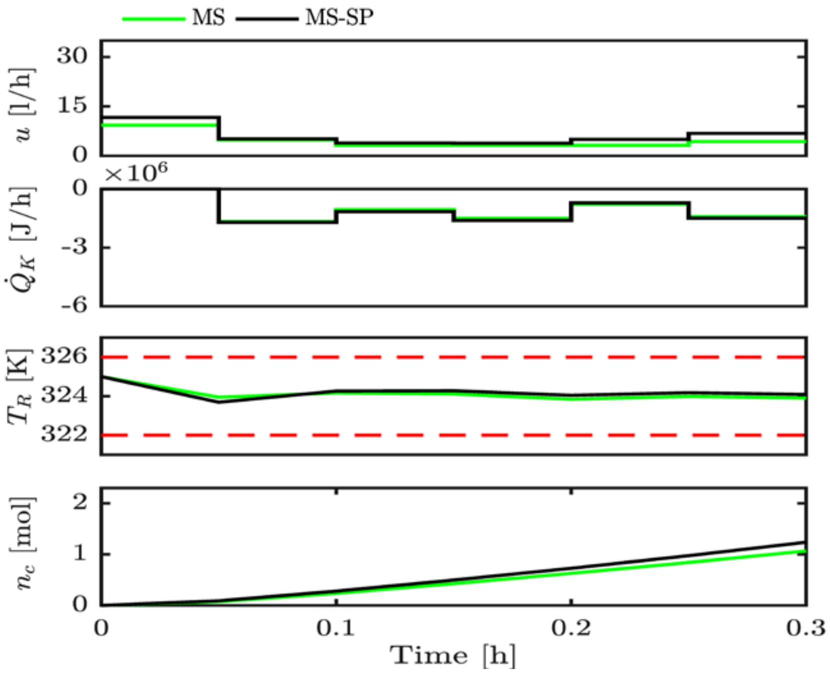
Simulation results

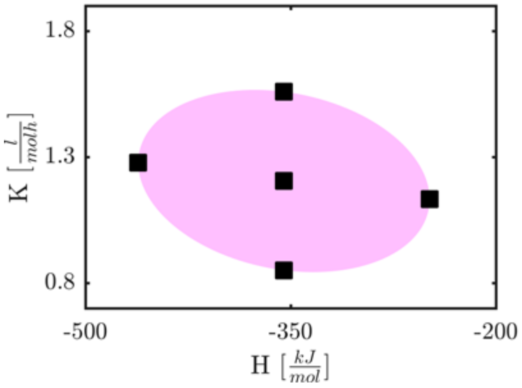




Confidence region described by the observed measurement information after the 2nd NMPC iteration

Simulation results





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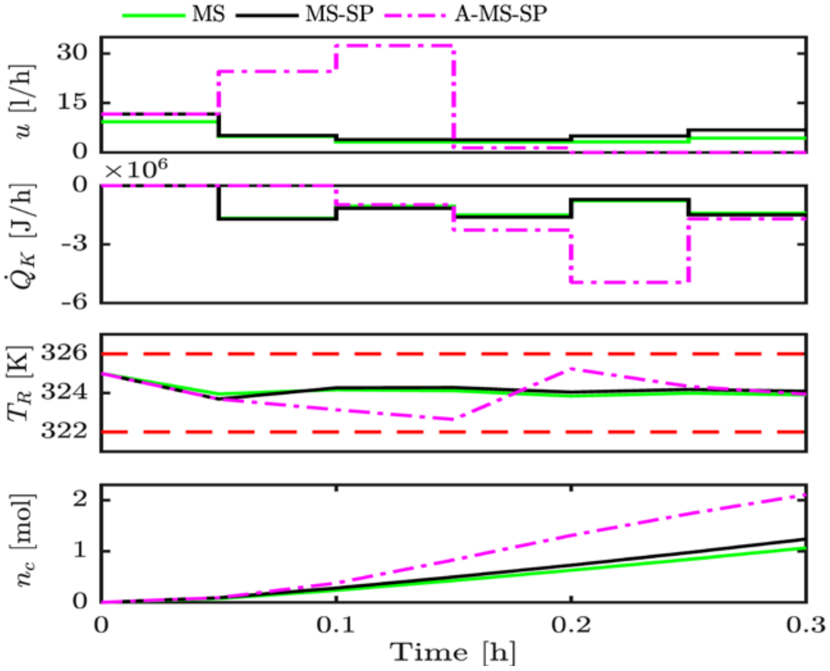

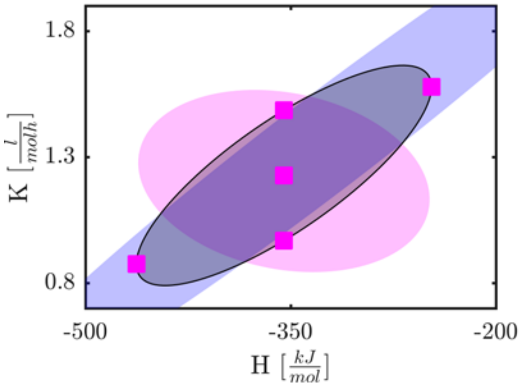

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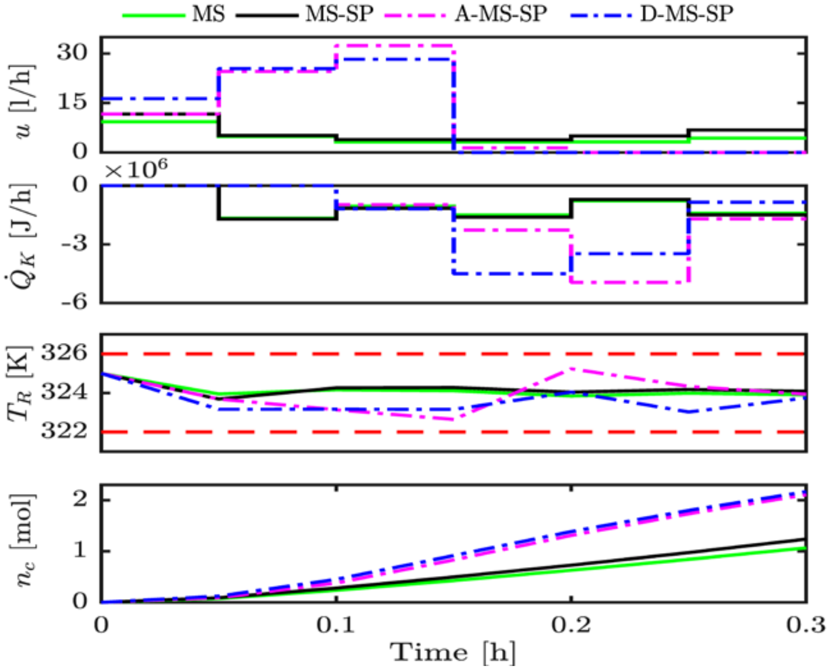

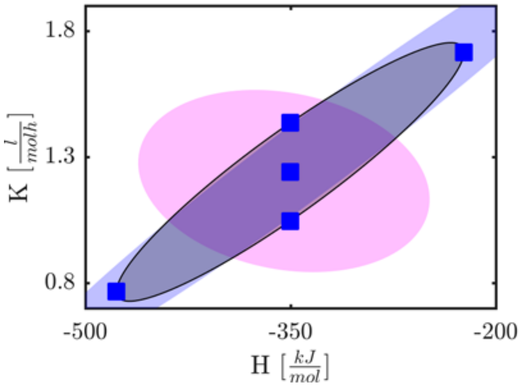

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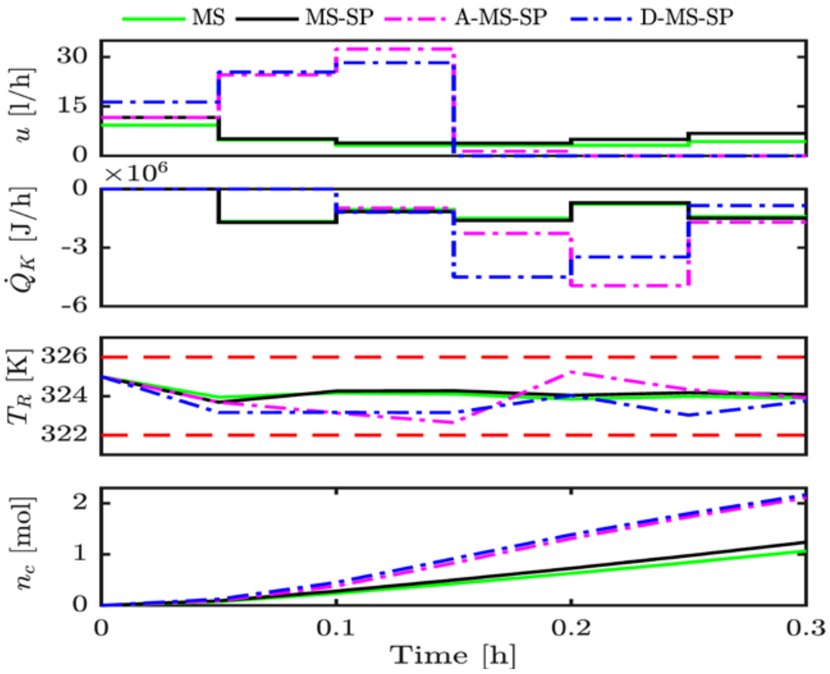

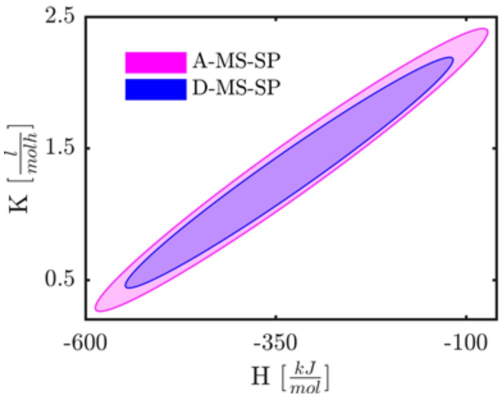

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Summary and Current work

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- The ellipsoidal over-approximation of the reachable set of the model is computed along the prediction horizon of the multi-stage NMPC based on sigma point principles
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Current work

- Extending the proposed dual control actions to compute the objective and constraint covariance matrix using the unscented transformation instead of the states

Relevant works

- Thangavel, S., Lucia, S., Paulen, R., Engell, S., (2018) Dual robust nonlinear model predictive control: A multi-stage approach, Journal of Process Control, Volume 72, pages 39-51
- Thangavel, S., Lucia, S., Paulen, R., Engell, S., (2015) Towards dual robust nonlinear model predictive control: A multi-stage approach, American Control Conference, pages 428-433
- Thangavel, S., Lucia, S., Paulen, R., Engell, S., (2017) Robust nonlinear model predictive control with reduction of uncertainty via dual control}, 21st International Conference on Process Control, pages 48-53
- Thangavel, S., Aboelnour, M., Lucia, S., Paulen, R., Engell, S., (2018) Robust Dual Multi-stage NMPC using Guaranteed Parameter Estimation, 6th IFAC Conference on Nonlinear Model Predictive Control, Volume 51, Issue 20, pages 72-77
- Thangavel, S., Paulen, R., Engell, S., (2020) Multi-stage NMPC using sigma point principles}, 2020 Advances in Control & Optimization of Dynamical Systems
- Thangavel, S., Paulen, R., Engell, S., (2020) Adaptive multi-stage NMPC using sigma point principles, 19th European Control Conference

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