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PULSE-STEP MODEL PREDICTIVE CONTROLLER FOR TITO SYSTEM

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Abstract: This paper describes a novel model based predictive controller with manipulated value constraints for TITO systems. Both amplitude and rate constraints are considered. It is assumed that the controlled system is stable, linear and t-invariant FIR system. Four discrete step response sequences are used as the process model. Alternatively it is possible to use three-parameter models. To make the open-loop optimization easier the set of admissible control sequences is restricted to stepwise pulse-step sequences. The optimization procedure is then executable in reasonable time.

Keywords: predictive control, step function responses, constraints, multi-variable systems, quadratic performance indices.

1. INTRODUCTION

Many process control plants are complex and multi-variable, which means they have a number of actuators (inputs) and sensors (outputs) and the couplings between the different inputs and outputs are often complicated. It is not easy to design a controller for such multi-input multi-output (MIMO) system because the routine methods used for single-input single-output (SISO) control systems cannot be used directly. These methods can by no means take into account the cross-coupling effects. In order to use the SISO methods, a decoupler must be designed first, which decomposes a MIMO system into a group of SISO systems by suppressing the cross-couplings (e.g. Nordfeldt and Hägglund (2006)).

But there are also multi-variable techniques available. One of the up-to-date approaches is the Model Predictive Control (MPC), see J.M. Maciejowski (2002); Huang et al. (2002). The main advantage of MPC is its general principle which can be used for both SISO and MIMO systems and also the possibility to include constraints directly into the design procedure. On the other hand, this generality brings several problems, especially the computational cost which makes the implementation of predictive control algorithms into compact controllers and PLCs almost impossible.

2. MULTIVARIABLE PREDICTIVE CONTROLLER

2.1 *The pulse-step control sequence*

A tough problem in predictive control with constraints is its complexity and computational cost. To lower the computational burden, it is possible to use some blocking strategy (Tondel and Johansen, 2002), for example constant manipulated value or constant manipulated value differences over time intervals of specified length. Another possibility is the so-called functional predictive control (Richalet et al., 1987), where the control sequence is restricted to a linear combination of suitable base functions.

Alternative approach to complexity restriction presented by Schlegel and Sobota (2008) is based

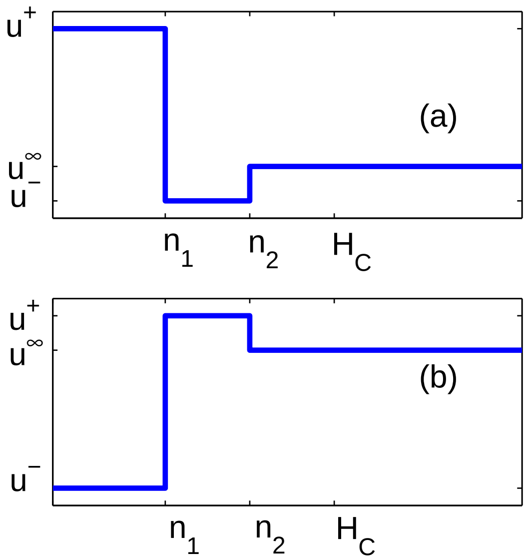

Fig. 1. Example of "pulse-step up" (a, $n_0 = 1$) and "pulse-step down" (b, $n_0 = 0$) control sequence

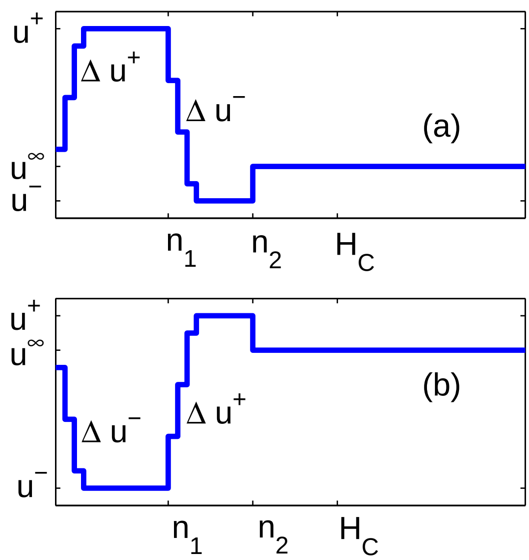

Fig. 2. Example of "pulse-step up" (a, $n_0 = 1$) and "pulse-step down" (b, $n_0 = 0$) control sequence with rate limiter

on the so-called pulse-step control, a well known aggressive technique used for manual control in industrial practice. The properties of SISO pulse-step predictive controller were compared to the classical PID feedback control. This paper shows how to incorporate the pulse-step control idea into multivariable MPC.

As shown in Figure 1, the pulse-step control sequence $u(k)$ begins with n_1 maximal (minimal) elements, followed by $n_2 - n_1$ minimal (maximal) elements according to the constraints $u^- \leq u(k) \leq u^+$. The remaining part of the control sequence is constant, $u(k) = u^\infty$ for $k \geq n_2$. The control horizon H_C determines the limit for n_1 and n_2, $0 \leq n_1 \leq n_2 \leq H_C - 1$ and of course u^∞ is subject to constraints $u^- \leq u^\infty \leq u^+$. So the whole control sequence is determined by only 4 variables n_0, n_1, n_2, and u^∞, where the parameter n_0 distinguishes between "pulse-step up" and "pulse-step down" control sequences.

It is also possible to incorporate the rate limiter into the idea of pulse-step control as shown in Figure 2. In that case, the pulse-step control sequence rises (drops) to the maximum u^+ (minimum u^-) as fast as possible with respect to the rate constraint Δu^+ (Δu^-). It starts to drop (rise) towards the minimum (maximum) at time n_1 and finally, at time n_2, it is changed for the last time and it remains constant for $k \geq n_2$.

2.2 *The process model*

The model based predictive control always employs some model of the controlled process. In

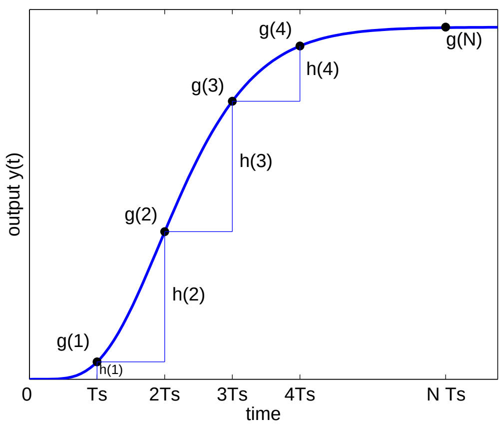

Fig. 3. Discrete step response

this approach, four discrete step responses of TITO system $g_{pq}(j)$, $j = 1, \ldots, N$ are used, where $p \in \{1, 2\}$ and $q \in \{1, 2\}$ denote the p-th input and q-th output respectively. Figure 3 shows how to obtain the discrete step response $g(j)$, $j = 0, 1, 2, \ldots$ and the discrete impulse response $h(j)$, $j = 0, 1, 2, \ldots$ with sampling period T_S from continuous step response. Note that $g(0) = h(0) = 0$, $h(j) = g(j) - g(j-1)$ for $j \geq 1$.

For stable, linear and t-invariant FIR systems with monotonous step responses it is also possible to use the moment model set approach (Schlegel and Večerek, 2005) and describe each input-output relation by only 3 characteristic numbers κ_{pq}, μ_{pq}, and σ^2_{pq}, which can be obtained easily from a very short and simple experiment. This identification technique has been widely accepted in industrial practice for PID controllers tuning purposes. The characteristic numbers κ, μ, and

σ^2 of the SISO system in the form

$$P(s) = \frac{K}{\prod_{i=1}^{l}(\tau_i s + 1)} \cdot e^{-Ds}$$

are defined as

$$\kappa = K, \quad \mu = D + \sum_{i=1}^{l} \tau_i, \quad \sigma^2 = \sum_{i=1}^{l} \tau_i^2. \tag{1}$$

Thus the controlled system can be approximated by first order plus dead-time system

$$P_{FOPDT}(s) = \frac{K}{\tau s + 1} \cdot e^{-Ds}, \quad \kappa = K,\ \mu = \tau + D,\ \sigma^2 = \tau^2 \tag{2}$$

or second order plus dead-time system

$$P_{SOPDT}(s) = \frac{K}{(\tau s + 1)^2} \cdot e^{-Ds}, \quad \kappa = K,\ \mu = 2\tau + D,\ \sigma^2 = 2\tau^2 \tag{3}$$

with the same characteristic numbers. The discrete step responses of these approximate SISO systems are then used to model the corresponding subsystems of the TITO system. The approximate step responses are depicted in Figure 4.

As shown in Figure 5, the characteristic numbers have a clear physical meaning for the systems (2) and (3), so it is also possible to adjust them manually to fit the step response of the real system. The characteristic number κ is static gain, the number μ has the character of time delay (known also as resident time constant, it shifts the step response along the time axis) and the parameter σ^2 changes the slope of the step response.

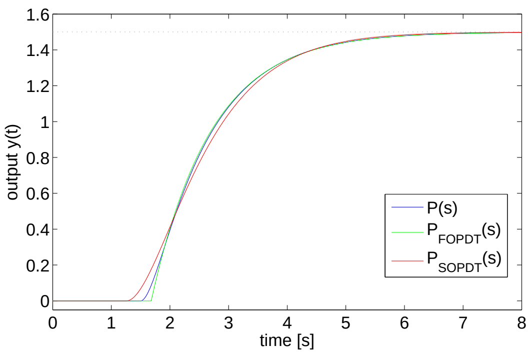

Fig. 4. Step responses of the 1st input - 1st output subsystem of the system (14) and its approximations

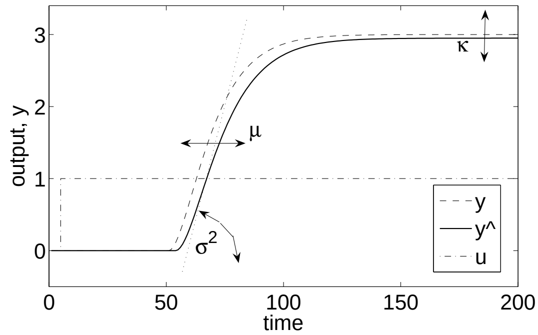

Fig. 5. Physical meaning of the characteristic numbers κ, μ, and σ^2

2.3 *Computing the control sequence*

Consider the controlled TITO system described by four discrete step responses $g_{pq}(j)$, $j = 1, \ldots, N$, $p \in \{1, 2\}$ and $q \in \{1, 2\}$ obtained either directly from the measurements on the real system or from three-parameter models (2) or (3) described in the previous section. The q-th output $y_q(k)$ of linear discrete TITO system is related to the inputs by well known convolution

$$y_q(k) = \sum_{p=1}^{2} \sum_{j=0}^{\infty} h_{pq}(j) u_p(k-j) \approxeq \approxeq \sum_{p=1}^{2} \sum_{j=1}^{N} h_{pq}(j) u_p(k-j), \tag{4}$$

where $h_{pq}(j)$, $j = 1, \ldots, N$, is the discrete impulse response of the p-th input - q-th output subsystem and N is suitable natural number ($h_{pq}(j) \approxeq 0$ for $j = N+1, \ldots, \infty$). From (4) we can obtain another relation which will be used further. It holds

$$\begin{aligned}
y_q(k) &= \sum_{p=1}^{2} \sum_{j=1}^{N} h_{pq}(j) u_p(k-j) = \\
&= \sum_{p=1}^{2} \sum_{j=1}^{N} \left[g_{pq}(j) - g_{pq}(j-1)\right] u_p(k-j) = \\
&= \sum_{p=1}^{2} [g_{pq}(1)u_p(k-1) + g_{pq}(2)u_p(k-2) + \ldots + \\
&+ g_{pq}(N-1)u_p(k-N+1) + g_{pq}(N)u_p(k-N) - \\
&- g_{pq}(0)u_p(k-1) - g_{pq}(1)u_p(k-2) - \ldots - \\
&- g_{pq}(N-2)u_p(k-N+1) - \\
&- g_{pq}(N-1)u_p(k-N)] = \\
&= \sum_{p=1}^{2} \left[\sum_{j=1}^{N} g_{pq}(j)\left[u_p(k-j) - u_p(k-j-1)\right] + \right. \\
&\left. + g_{pq}(N)u_p(k-N-1) \right].
\end{aligned} \tag{5}$$

In other form

$$y_q(k) = \sum_{p=1}^{2} \left[\sum_{j=1}^{N} g_{pq}(j)\Delta u_p(k-j) + \right. \\ \left. + g_{pq}(N)u_p(k-N-1) \right], \quad (6)$$

where $\Delta u_p(k) = u_p(k) - u_p(k-1)$.

Then the i-step ahead output prediction at time k, $1 \le i \le N$, is given by

$$\begin{aligned} \hat{y}_q(k+i|k) &= \sum_{p=1}^{2} \left[\sum_{j=1}^{N} g_{pq}(j)\Delta u_p(k+i-j) + \right. \\ &\left. + g_{pq}(N)u_p(k+i-N-1) \right] = \\ &= \sum_{p=1}^{2} \left[\sum_{j=i+1}^{N} g_{pq}(j)\Delta u_p(k+i-j) + \right. \\ &+ g_{pq}(N)u_p(k+i-N-1) + \\ &\left. + \sum_{j=1}^{i} g_{pq}(j)\Delta \hat{u}_p(k+i-j|k) \right] = \\ &= \hat{y}_{q_f}(k+i|k) + \\ &+ \sum_{p=1}^{2} \left[\sum_{j=1}^{i} g_{pq}(j)\Delta \hat{u}_p(k+i-j|k) \right], \end{aligned} \quad (7)$$

where the first term $\hat{y}_{q_f}$ is the response caused by the past inputs and the inner sum represents the response determined by future changes of the input signals $\Delta \hat{u}_p(k+i-j|k)$, $j = 1, \dots, i$, $p \in \{1, 2\}$. The disturbances d_q (prediction errors) are defined as

$$d_q \triangleq y_{q_m}(k) - \hat{y}_q(k|k-1), \quad (8)$$

where $y_{q_m}(k)$ is the real (measured) q-th output of the system at time k.

For the pulse-step control strategy described in section 2.1 we get from (7)

$$\hat{y}_q(k+i|k) = \hat{y}_{q_f}(k+i|k) + \\ + \sum_{p=1}^{2} \left[\sum_{j=0}^{n_{p2}} g_{pq}(i-j)\Delta \hat{u}_p(k+j|k) \right], \quad (9)$$

where the control signal increments $\Delta \hat{u}_p(\cdot)$ result from the pulse-step generator with regards to the parameters n_{p0}, n_{p1}, n_{p2} and u_p^∞ as described in section 2.1.

Now the requirement that the system outputs reach the desired values w_q, $q \in \{1, 2\}$ in N_{q_1} steps and stay steady until N_{q_2}-th step is formulated by

$$\hat{y}_q(k+N_{q_1}|k) + d_q = \dots = \\ = \hat{y}_q(k+N_{q_2}|k) + d_q = w_q, \quad (10)$$

where N_{q_1}, N_{q_2} are appropriate natural numbers defining the prediction horizon. Note that the disturbances d_q given by (8) are presumed to be constant over the whole time interval $0, \dots, N$ (btw. this presumption incorporates integrator into the structure of the controller, which ensures total compensation of arbitrary constant disturbance acting on the system). From equations (9) and (10) we obtain

$$\begin{aligned} w_1 = &\hat{y}_{1_f}(k+i|k) + \\ &+ \sum_{j=0}^{n_{12}} g_{11}(i-j)\Delta \hat{u}_1(k+j|k) + \\ &+ \sum_{j=0}^{n_{22}} g_{21}(i-j)\Delta \hat{u}_2(k+j|k) + d_1, \\ &i = N_{11}, \dots, N_{12} \end{aligned} \quad (11)$$

$$\begin{aligned} w_2 = &\hat{y}_{2_f}(k+i|k) + \\ &+ \sum_{j=0}^{n_{12}} g_{12}(i-j)\Delta \hat{u}_1(k+j|k) + \\ &+ \sum_{j=0}^{n_{22}} g_{22}(i-j)\Delta \hat{u}_2(k+j|k) + d_2, \\ &i = N_{21}, \dots, N_{22} \end{aligned} \quad (12)$$

Note that (11) together with (12) form a set of linear equations with only two variables $\Delta \hat{u}_1(k+n_{12}|k)$ and $\Delta \hat{u}_2(k+n_{22}|k)$ for fixed n_{10}, n_{11}, n_{12}, n_{20}, n_{21} and n_{22}. The coincidence condition (10) (or (11) and (12)) cannot be fulfilled exactly so it is necessary to define a quadratic performance index in the form

$$I = \sum_{q=1}^{2} \left[\gamma_q \sum_{i=N_{q1}}^{N_{q2}} \left(\hat{y}_q(k+i|k) + d_q - w_q\right)^2 \right] + \\ + \sum_{p=1}^{2} \left[\lambda_p \sum_{i=0}^{H_{C_p}-1} \Delta \hat{u}_p(k+i|k)^2 \right] \to \min, \quad (13)$$

where the optimized variables are n_{10}, n_{11}, n_{12}, n_{20}, n_{21}, n_{22} and $\Delta \hat{u}_1(k+n_{12}|k)$ (i.e. u_1^∞) and $\Delta \hat{u}_2(k+n_{22}|k)$ (i.e. u_2^∞).

It is important to mention that the parameters $N_{11}, N_{12}, H_{C_1}, \gamma_1, \lambda_1, N_{21}, N_{22}, H_{C_2}, \gamma_2, \lambda_2$ in the criterion (13) take the role of design parameters. The parameters N_{q1} and N_{q2} define the coincidence intervals (10) and strongly influence the resulting optimal control sequence. The standard choice is $N_{q1} = 1$ and $N_{q2} = N-1$. If dead-time D is present at the controlled system, it is reasonable to set $N_{q1} > D/T_S$, where T_S is the sampling period. The control horizon H_{C_p}, $1 \le H_{C_p} \le N-1$, influences the closed loop performance and mainly the complexity of the optimization procedure. In most cases the choice of $H_{C_p} \in \langle 5, 10 \rangle$ seems to be the best from the performance-to-complexity ratio point of view (supposing the sampling frequency is adequate with respect to the controlled system dynamics). In the case when $H_C = 1$, the nonlinear part of the control action cannot be applied and the only optimized variables are u_1^∞ and u_2^∞. The parameters λ_p penalize the changes in the control signals. The greater these parameters are, the less aggressive controller we get. Finally the γ_q parameters can be used for balancing the weight of the two outputs in the case when setpoints w_1 and w_2 differ, nevertheless one of the γ_q parameters is usually equal to 1.

The algorithm used for solving the optimization task (13) combines brute force and the least squares method. The values u_1^∞ and u_2^∞ are determined using the least squares method for all admissible combinations of n_{10}, n_{11}, n_{12}, n_{20}, n_{21} and n_{22} and the optimal control sequence is selected afterwards. The computational cost is proportional to ${H_{C_1}}^2 \cdot {H_{C_2}}^2$. The selected sequence in the pulse-step shape is optimal in the open-loop sense. To convert from open-loop to closed-loop control strategy, only the first elements of the computed control sequences are applied and the whole optimization procedure is repeated in the next sampling instant.

3. EXAMPLE

The properties of the model based predictive controller based on the algorithm described in section 2.3 will be illustrated here. Consider the controlled TITO system described by the transfer function

$$P(s) = \begin{pmatrix} \dfrac{7.5 \cdot e^{-1.5s}}{(s+1)(s+5)} & \dfrac{-2.8572 \cdot e^{-1.3s}}{(s+1)(s+10)} \\ \dfrac{-18.75}{(s+2)(s+15)} & \dfrac{7.143}{(s+2)(s+5)} \end{pmatrix} \tag{14}$$

and manipulated value constraints $u(k) \in \langle 0, 1 \rangle$, $\Delta u(k) \in \langle -0.2, 0.2 \rangle$. The sampling period of $T_S = 0.2$s will be used. It is of course necessary to work with the discretized model of the system (14) because the predictive control algorithm is discrete by its nature.

Figure 4 illustrates the step responses of the 1st input - 1st output subsystem of the system (14) and its approximations in the form (2) and (3) with the same characteristic numbers κ_{11}, μ_{11}, and σ_{11}^2, which will be used for prediction of the controlled system behavior. The approximations of other input-output relations are similar. Note that the most significant discrepancies occur at the beginning of the step responses, while the

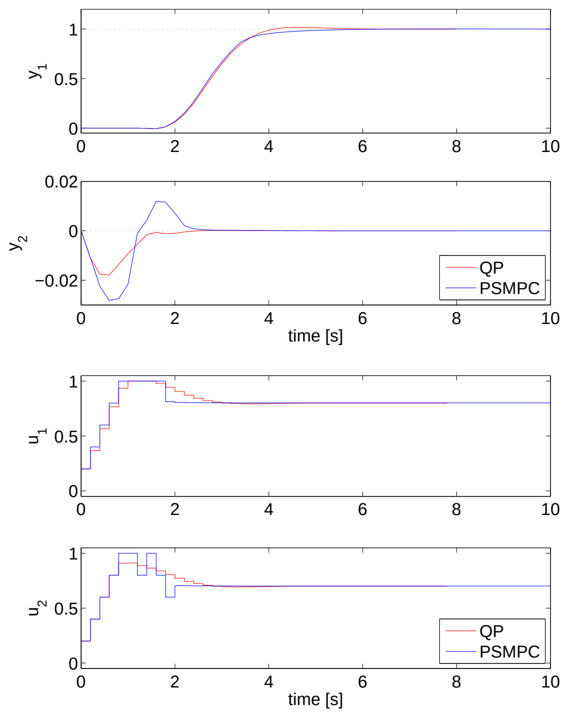

Fig. 6. Step change of w_1, exact model is used for prediction, $T_S = 0.2s$, $H_{C1} = H_{C2} = 5$, $N_{11} = N_{21} = 1$, $N_{12} = N_{22} = 39$, $\lambda_1 = 0.4678$, $\lambda_2 = 0.0921$, $\gamma_1 = 1$, $\gamma_2 = 10$, $N = 40$.

static gain of the real and approximate step response is the same.

Firstly the exact step responses are used for prediction. Figure 6 compares the behavior of the pulse-step predictive controller to a predictive controller based on quadratic programming (QP) in the case when step change in setpoint w_1 occurs.

The trajectories of the 1st output y_1 are almost the same, notice the pulse-step shape of the control sequences. The manipulated value constraints are kept and fully exploited when setpoint changes. But the cross-coupling effect on the 2nd output y_2 is more significant when pulse-step control is used. This is the result of restricting the control sequence to pulse-step shape, the control signal computed by QP has more flexibility, it can deal with setpoint change and decoupling at the same time much better. But it does not seem to be significant enough to pay for the increased computational cost and implementation complexity.

Figure 7 compares the performance of pulse-step predictive controller when exact model and FOPDT approximation of the real system are used for prediction. One can see that the performance deteriorate a little as a result of the inaccurate modelling. But on the other hand, there are many techniques for identifying FOPDT approx-

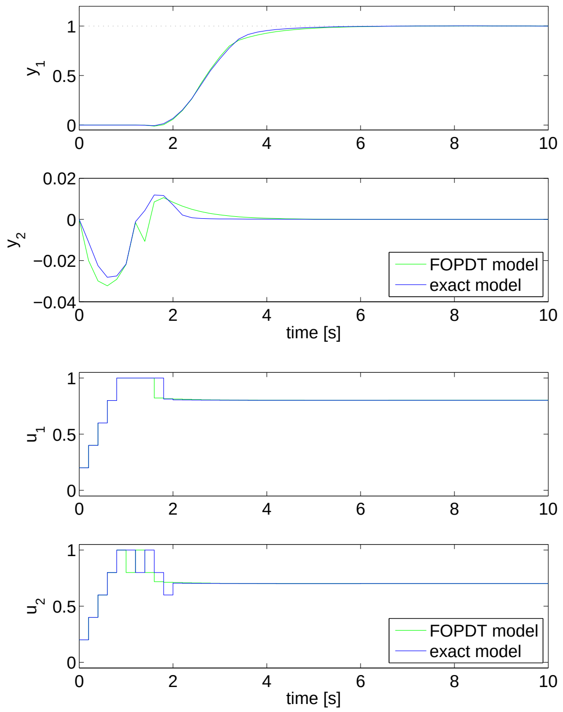

Fig. 7. Step change of w_1, $T_S = 0.2s$, $H_{C1} = H_{C2} = 5$, $N_{11} = N_{21} = 1$, $N_{12} = N_{22} = 40$, $\lambda_1 = 0.4678$, $\lambda_2 = 0.0921$, $\gamma_1 = 1$, $\gamma_2 = 10$, $N = 40$.

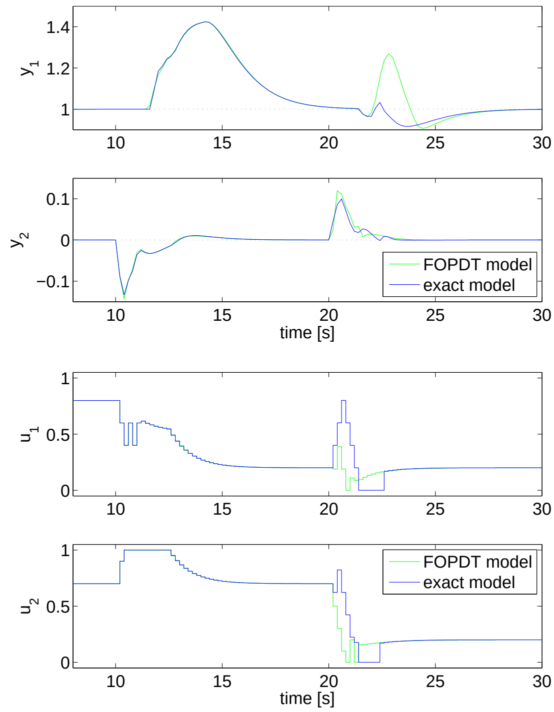

Fig. 8. Input disturbances on u_1 and u_2, $T_S = 0.2s$, $H_{C1} = H_{C2} = 5$, $N_{11} = N_{21} = 1$, $N_{12} = N_{22} = 40$, $\lambda_1 = 0.4678$, $\lambda_2 = 0.0921$, $\gamma_1 = 1$, $\gamma_2 = 10$, $N = 40$.

imations available, so this tradeoff is definitely worth it.

The inaccuracy of the model affects also the input disturbance rejection which is shown in Figure 8. At time $t = 10s$ a constant disturbance $l_1 = 0.6$ starts to act on the input u_1, followed by a disturbance $l_2 = 0.5$ which acts on the input u_2 from time $t = 20s$.

As a result to the model inaccuracy the effect of disturbance l_2 on output y_1 gains in significance. But only when talking about the absolute distance from setpoint w_1, the time needed for returning the system to the steady state is practically the same for both exact and FOPDT models.

4. CONCLUSION

The described pulse-step model predictive controller ensures acceptable quality behavior of the closed control loop. It keeps and exploits the manipulated value constraints while the computational cost is kept at a reasonable level. The tuning of the controller is very easy, as it has only few parameters (except the step responses). Most of them can be determined automatically with respect to the dynamics of the controlled system model. Only the weighting coefficients λ_1, λ_2, γ_1 and γ_2 are meant for manual tuning of the controller. It was illustrated that this controller can also deal with model uncertainties very well.

All this makes the PSMPC controller a suitable choice for TITO system control.

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