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MODAL SENSORS PLACEMENT OPTIMIZATION

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Abstract: A new approach to optimal placement of sensors in modal sensor sense is presented. In contrast to existing methods, the optimal sensor set selection is based on suppressing of the observation of unwanted modes (typically higher order modes), while simultaneously the observability of low frequency modes should be as high as possible. An efficient numerical algorithm is presented, developed from an existing routine based on the Fischer information matrix analysis. Performance of our approach is demonstrated be means of two simple textbook examples.

Keywords: Optimal Sensor Placement, Fisher Information Matrix, EFI.

1. INTRODUCTION

Optimal sensor placement (OSP) in mechanical systems and structures has become a popular and frequently discussed research topic during last ten years. Applications cover modeling, identification, fault detection, and active control of such systems as bridges [1] [2], rail wagons [3], large space structures [4]. The goal is to tell the designers of the whole mechanical system where displacement, force, inertial acceleration, or other sensors are to be installed so that they are as informative as possible.

Various approaches have been developed. We will mention two in brief. The former, information based approach, is based on the analysis of the output shape matrix. An iterative elimination algorithm, denoted as EFI (for "Effective Independence") has been developed that repeatedly deletes the lines of the initial, full output shape matrix with lowest amount of information, measured by either the trace or determinant of an underlying Fischer information matrix. See [7] for more detailed treatments and [1] [2] [4] for some case studies.

An alternative approach is based on the idea of maximizing the energy of the underlying modes in the optimally placed sensors. Related procedures lead to optimization problems over output Gramians of the system. References: [5].

Both these approaches are applied on pre-selected modes of interest. For instance, in an active damping application for a transport vehicle, see a recent report [3], the bandwidth and thus implied modes are defined according to some comfort standards, and considerations of impact of particular modes on the loads induced in the structure. Typically, a few lower modes are selected as a result of such analysis. Resulting optimal sensors selection is subsequently called, with only those pre-selected modes in mind.

However, also those not-considered, typically mid- or high-frequency modes are still present in the process and, if excited by disturbances or the control action, they can influence the active damping system behavior in an unexpected manner.

This phenomenon, denoted as spillover, cannot be captured directly by the two existing approaches mentioned above. Although some procedures have been developed that address these issues, see e.g. [6], they are based on advanced signal processing (filtering) of the measured signals and do not suggest how to modify the sensors positions themselves accordingly.

And it is exactly the problem that this paper is focused on. The aforementioned information approach is taken as the starting point. The underlying criterion is modified so that the influence of desirable modes is maximized, and those unwanted modes are minimized in the observations at the same time, see section 2. The result is a compromise where suitably chosen weights serve as a tuning knob for the designer. A related numerical procedure is then developed, based on the EFI existing results, in section 3. Two examples are presented in section 4 where one can appreciate the intuitively expected placements and study the influence of tuning. Conclusions and suggestions for further research then follow in section ??.

2. THE EFFECTIVE INDEPENDENCE METHOD (EFI)

Optimal sensors placement techniques are extensively discussed in papers [1] [2]. A short overview of the EFI method follows in this section 2, adopted from [1] [2].

The aim of the EFI method is to select measurement positions that make the mode shapes of interest as linearly independent as possible while containing sufficient information about the target modal responses in the measurements. The method originates from estimation theory by sensitivity analysis of the parameters to be estimated, and then it arrives at, so to say, maximization of the Fisher information matrix, measured by the determinant or the trace, which in fact is equivalent minimization of the condition number of the information matrix. Technically, it is reflected in a coefficients' covariance matrix (the covariance matrix of the estimate error of the modal coordinates is minimized). The number of sensors is reduced from an initially large candidate set in an iterative manner by removing sensors from those places which contribute least among all the candidate sensors to the linear independence of the target modes. In the end, it preserves the required necessary candidate sensors as the optimal sensor set. As a useful guideline toward the selection of suitable number of sensors, the determinant of the Fisher information matrix can be plotted with respect to the number of sensors; if a considerable drop is identified, further reduction should be considered with care.

2.1 *Structural model*

The sensor placement problem can be investigated from uncoupled modal coordinates of governing structural equations as follows:

$$\ddot{q}_i+M_i^{-1}\cdot C_i\cdot\dot{q}_i+M_i^{-1}\cdot K_i\cdot q_i = M_i^{-1}\cdot\Phi^T\cdot B_0\cdot u \quad (1)$$

$$y = \Phi\cdot q+\epsilon = \sum_{i=1}^{N} q_i\cdot\Phi_i+\epsilon \quad (2)$$

where q_i is the i^{th} modal coordinate and is also the i^{th} element of the vector, q, in the 2^{nd} equation, M_i, K_i and C_i are the corresponding i^{th} modal mass, stiffness and damping matrix, respectively, Φ is the mode shape matrix with its i^{th} column as the i^{th} mass-normalized mode shape, B_0 is simply a location matrix formed by ones (corresponding to actuators) and zeros (no loadings), specifying the positions of the force vector u. y is a measurement column vector indicating which positions of the structure are measured, and ϵ is a stationary Gaussian white noise with zero mean and a variance of σ^2.

2.2 *Method principle*

From the output measurement the EFI analyzes the covariance matrix of the estimate error for an efficient unbiased estimator as follows:

$$E\left[(q-\hat{q})\cdot(q-\hat{q})^T\right] = \quad (3)$$

$$\left[\left(\frac{\partial y}{\partial q}\right)^T\cdot\left[\sigma^2\right]^{-1}\cdot\left(\frac{\partial y}{\partial q}\right)\right]^{-1} = Q^{-1}$$

in which Q is the Fisher information matrix, σ^2 represents the variance of the stationary Gaussian measurement white noise ϵ in (2), E denote the mean value, and $\hat{q}$ is the efficient unbiased estimator of q. Maximizing Q will result in the best state estimate of q. The EFI coefficients of the candidate sensors are computed by the following formation:

$$E_D = [\Phi\cdot\Psi]\otimes[\Phi\cdot\Psi]\cdot\lambda^{-1}\cdot 1 \quad (4)$$

in which $\otimes$ represents a term-by-term matrix multiplication, 1 is an $n\times 1$ column vector with all elements of 1. Ψ denote the eigenvectors matrix according to Eigenvalues on diagonal of λ matrix. E_D is the EFI indices, which evaluate the contribution of a candidate sensor location to the linear independent of the modal partitions . The selection procedure is to sort the elements of the E_D coefficients, and to remove the smallest one at a time. The E_D coefficients are then updated according to the new modal shape matrix, and the process is repeated iteratively until the number of remained sensors equals to a preset value. The remained DOFs serve as the measurement locations.

2.3 *Assumptions and limitations*

The main technical constraint for the EFI method is that the number of finally retained sensors must be greater or equal to the number of modes selected. For instance, for the first three modes being of interest, one can receive optimal 20, 5 or 3 locations, but no meaningful results can be achieved for 2 sensors. This limitation is due to evaluation of E_D vector coefficients, based on Eigenvalues and eigenvectors of the Fisher information matrix Q (singular values of target mode shape matrix). When we consider fewer measurements then modes, the Fisher information matrix becomes singular (some singular values become very close to zero).

3. THE EFFECTIVE INDEPENDENCE METHOD WITH MODIFIED CRITERION

We consider two requirements in the optimal sensor placement procedure, to find out configuration of sensors (measurements) for optimal estimation of desired modes. This is also aim of classical EFI algorithm. The second, often contending, claim is to select sensors configuration to minimize spillover of unwanted signals into useful signals. We decide to use information approach to OSP based on EFI method and modify the underlying criterion to meet both of our requirements (maximize useful signal and minimize spillover of unwanted signals).

3.1 *Method principle*

The modified criterion is based on the EFI reasoning presented above. Main task of the pure EFI is just to maximize information on desired modes through optimal configuration of sensors (measurements) expressed by Fisher information matrix (FIM), or its trace or determinant respectively. The modified criterion we propose reads:

$$J_{MEFI} = max_{[i,j,k]\in\Omega} \left[\alpha J_{EFI} + (1-\alpha) J_{SNR}\right] \tag{5}$$

where

$$J_{EFI} = trace\left(Q^m_{[i,j,k]}\right) \tag{6}$$

stands for the standard EFI part (maximize the information content for those desirable modes), and

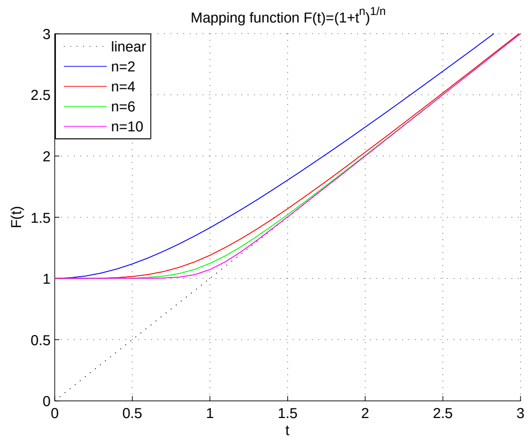

Fig. 1. Mapping function.

$$J_{SNR} = \left[\frac{trace\left(Q^m_{[i,j,k]}\right)}{trace\left(Q^n_{[i,j,k]}\right)}\right] \tag{7}$$

is a newly added term to penalize the unwanted mode shapes. Ω is the set of all candidate triples of sensors (we consider three sensors to be selected, for simplicity). $Q^m_{[i,j,k]}$ is the Fisher information matrix (see (3)) for m^{th} modes (those to be captured), wheres $Q^n_{[i,j,k]}$ is the Fisher information matrix for the unwanted modes. The coefficient $\alpha \in (0,1)$ serves as a tuning parameter and defines the relative importance of each part of the criterion.

The ratio part in J_{SNR} however becomes problematic as both terms in

$$\frac{trace\left(Q^m_{[i,j,k]}\right)}{trace\left(Q^n_{[i,j,k]}\right)}$$

approach zero (near the nodes of both desirable and unwanted mode shapes) which leads to irrelevant results. This unintended behavior is suppressed by applying a suitable mapping function on $trace\left(Q^m_{[i,j,k]}\right)$ and $trace\left(Q^n_{[i,j,k]}\right)$ to assure for reasonably high information content (those degenerated, almost $\frac{0}{0}$ candidates, are effectively discriminated). A suitable mapping function can take the following form, for example:

$$f(t) = \sqrt[n]{(1+t^n)}. \tag{8}$$

3.2 *Modified EFI algorithm*

Now we have an accordingly modified criterion. Next task is to modify the EFI heuristic in a very similar manner manner. Critical part of EFI method is in evaluation of E_D vector (see (4)), modify evaluation takes the following shape:

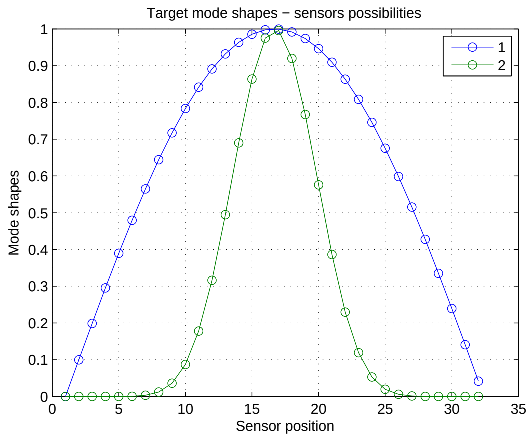

Fig. 2. Mode shapes and possibilities of sensors positions (modes 1 and 2).

$$E_{DM} = \alpha E_D + (1-\alpha)E_{DSNR}$$
$$E_D = [\Phi \cdot \Psi] \otimes [\Phi \cdot \Psi] \cdot \lambda^{-1} \cdot 1 \qquad (9)$$
$$E_{DSNR} = \frac{[\Phi^m \cdot \Psi^m] \otimes [\Phi^m \cdot \Psi^m] \cdot \lambda^{m-1} \cdot 1}{[\Phi^n \cdot \Psi^n] \otimes [\Phi^n \cdot \Psi^n] \cdot \lambda^{n-1} \cdot 1}.$$

Note that potential numerical issues near the nodes points are also covered by the mapping function (8) applied on E_D and E_{DSNR} vector.

4. EXPERIMENTS AND RESULTS

Previous theoretical formulations were applied on two examples to demonstrate properties of the original EFI algorithm and our modification. The first one is a rather artificial example of a system with two modes, see the target mode shape plotted in Fig. 2. This example was generated because it is fairly intuitive to decide where sensors should be placed if we want to maximize measurement of the first mode and reduce the second one. One can see results of the classical EFI approach in Fig. 4 and the brute-force-calculated optimum in Fig. 3 for comparison. It is clear that EFI approach (and maximizing of FIM trace approach) gives rise to sensors configuration optimal to fit desired mode (first one), but spillover of the second one is huge. Measured energy of both modes (required E_{RQ} and not required E_{NOTRQ}) is plotted upward each figure. Signal to noise ratio (SNR defined in dB units) coefficient was evaluated for each method to represent spillover. SNR is defined by following form:

$$SNR = 20 \cdot log_{10}\left(\frac{E_{RQ}}{E_{NOTRQ}}\right). \qquad (10)$$

Spillover minimization of unwanted modes into useful ones can be achieved by our modified criterion (see 3). One can see in Fig. 5 that spillover of the second mode into required first mode is reduced, and the measurement of the useful mode is still at a good level. The modified EFI algorithm leads to good results, see Fig. 6. Differences in sensors indexes are caused by target mode shape symmetry.

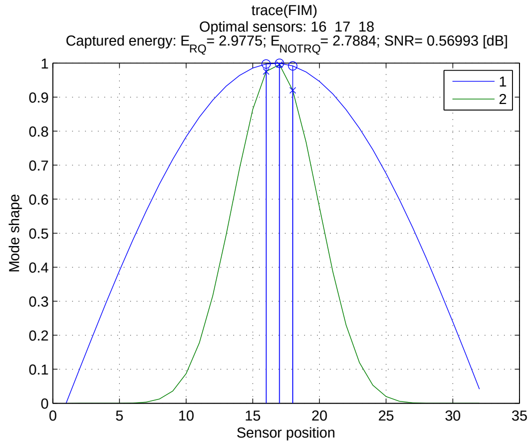

Fig. 3. OSP by maximization of FIM trace criteria (modes 1 and 2).

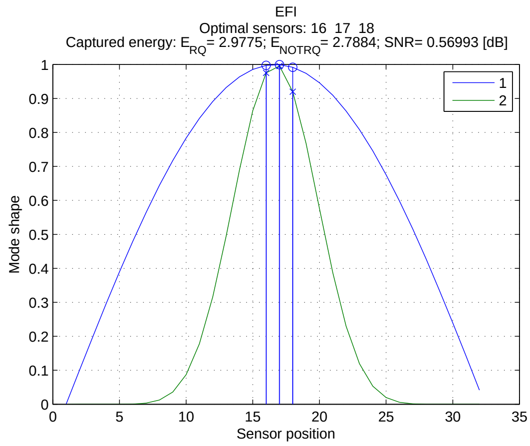

Fig. 4. OSP by classical EFI methodology (modes 1 and 2).

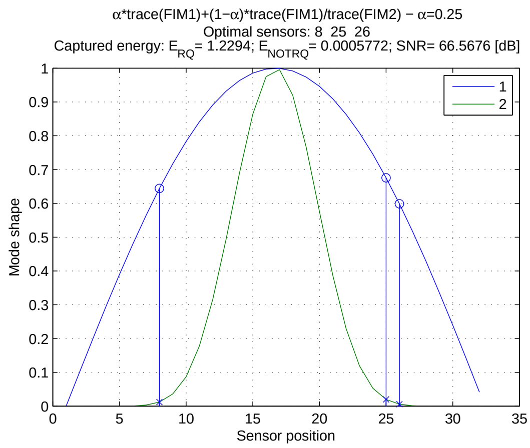

Fig. 5. OSP by maximization of FIM trace and SNR criteria (modes 1 and 2).

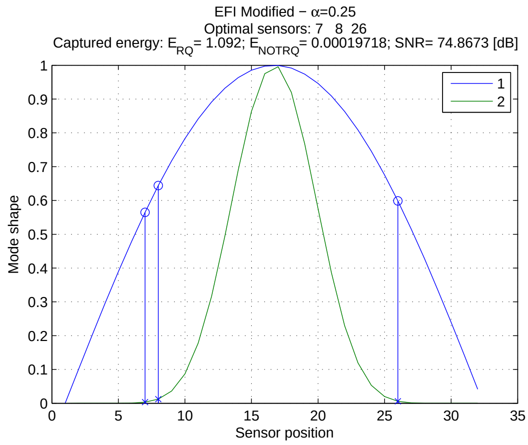

Fig. 6. OSP by modify EFI methodology (modes 1 and 2).

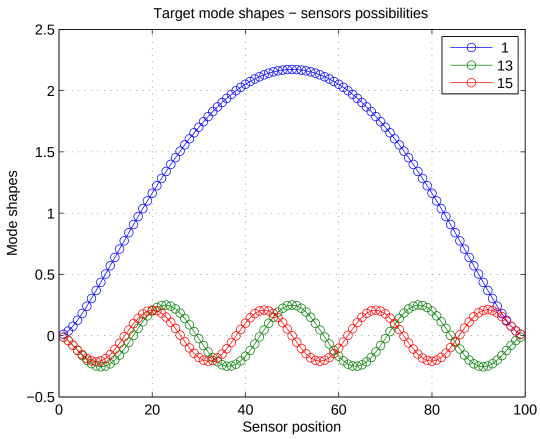

Fig. 7. Mode shapes and possibilities of sensors positions (required mode 1 and unwanted modes 13 and 15).

A more realistic model of a flexible beam with mode shapes plotted in Fig. 7 is considered next. Similarly to the previous case, classical EFI results are presented and confronted with the 'brute-force' found true FIM trace criterion minimizer, in Fig. 9 and 8. Results of the modified EFI algorithm and related true minimizer (again found by brute force, which is possible thanks to a limited number of potential sensor locations in all presented examples in this section) is plotted in Fig. 11 and 10.

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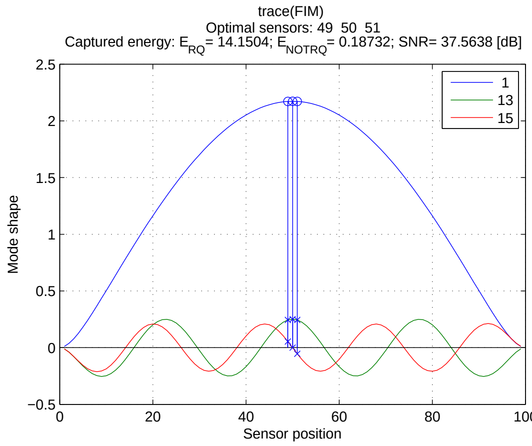

Fig. 8. OSP by maximization of FIM trace criteria (required mode 1 and unwanted modes 13 and 15).

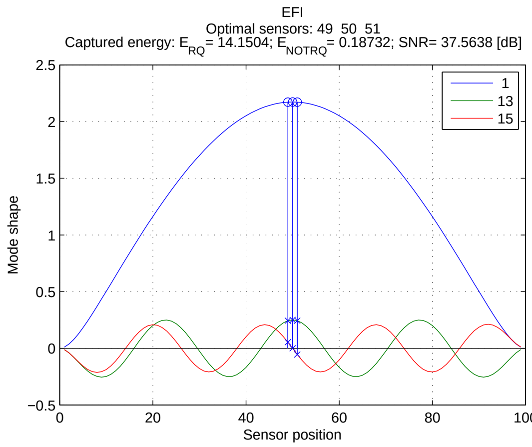

Fig. 9. OSP by classical EFI methodology (required mode 1 and unwanted modes 13 and 15).

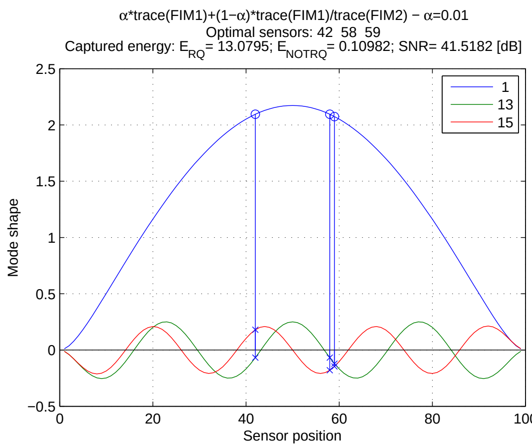

Fig. 10. OSP by maximization of FIM trace and SNR criteria (required mode 1 and unwanted modes 13 and 15).

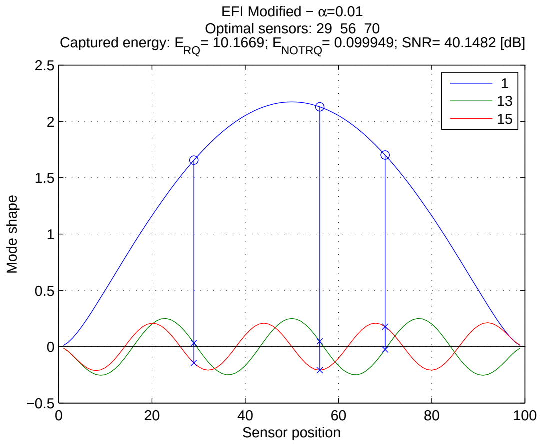

Fig. 11. OSP by modify EFI methodology (required mode 1 and unwanted modes 13 and 15).

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