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SYSTEM IDENTIFICATION AND EXPLICIT PREDICTIVE CONTROL OF CANTILEVER LATERAL VIBRATIONS

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Abstract: The aim of this study is to investigate the efficiency of a explicit predictive control technique in the numerical simulation and experimental study of system identification of piezoelectric smart structure. A complete active vibration control system comprising the cantilever plate, the piezoelectric actuators, the laser sensor and the digital signal processor board is set up to conduct the experimental system identification. Based on the structure responses determined by measurement, an explicit first mode state space model of the equivalent linear system is developed by employing subspace identification approach. The multiparametric programming algorithm is employed for controller design. The control law is incorporated into the finite state space partitions to perform as closed loop controller. The control law performance is further evaluated in the context of a simulation.

Keywords: piezoelectric sensor/actuator, explicit predictive controller, system identification of vibration

1. INTRODUCTION

Monitoring and control of vibrations has become important for the aims of many engineering systems, as for example automotive industry, airspace industry or precise mechanics. Advances in smart materials have shown an increased interesting applications for passive and active attenuation. The advanced Technologies of smart materials lead to relatively small and light actuators and sensors with good physical integration, i.e. in-building into materials. The vibration control is historically from its beginning highly topical which has led to many methods and approaches to solution.

In general, the efficiency of passive damping materials in suppressing of mechanical vibrations is insufficient for the range of low frequencies. What is more, the passive damping materials considerably add on to the mass of the structure they damp. At the same time, they manipulate with the stiffness of the structure. The resulting strong vibrations can damage or totally destroy the structure. In the case of machining devices, the undesired vibrations lead to decreased precision of products. To avoid the disadvantages of passive damping elements, piezoelectric materials have come to wider use that can be well controlled in a wide range of frequency, without adding great amount of mass to the structure. The actuators that are built in the controlled structure produce force on a given object. The signal that controls the actuator arises from the control system obtaining feedback from sensors that can be also built in the controlled structure. The research of piezoelectric materials is fast gaining attention and it is expected that this technology will upgrade the quality of production in engineering industry. The control of vibration that utilizes piezoelectric materials can be performed passively (with shunt circuits) (Hagood and Flotow, 1991; Wu, 1998; Granier et al., 2001; Niederberger, 2005) or actively. In the shunt circuit techniques the main task of piezoelectric materials is to absorb the energy from the structure.

In active control, an external power is applied to a piezoelectric material to produce a force in opposite direction to that produced by vibrating structure at a particular position. The opposite forces will annul each other and thus reduce the vibration of the structure. In the literature many papers focuss on active vibration techniques. Good comparative study of some control techniques can be found in (Kumar et al., 2006). The predictive control was applied with its on-line computation in Wills et al. (2008); Takács and Rohaľ-Ilkiv (2009).

In this paper a single mode explicit predictive controller for vibration attenuation will be studied. The explicit technique is particulary suitable for this fast dynamic application because of its low online computation load.

2. SYSTEM DESCRIPTION AND IDENTIFICATION

The vibration of cantilever is from the point of view of mathematical-physical modeling called problem of vibration of continuum, for which it must be considered that the cantilever has theoretically infinite number of its own frequencies (modes). For detailed description of a vibrating cantilever, the vibration of which is exited by external sources, generally speaking one would need to know the differential equation of infinite order for its every single point that would describe the vibration of that particular point. Not every own frequency has the same effect on the overall vibration. That is the reason why on describing the behavior of the vibrating cantilever, it is necessary to take into account the amount of energy (and in consequence the amount of displacement) that is related to the given own frequency. From the point of damping of vibrations, it is very often sufficient to damp the first n modes, where n depends on the actual aim of the control. The first own frequency is related to the greatest vibration of the cantilever in a given point.

In the area of physical mechanics several approaches are known that are based on rigid body mass simplification. They build on the simplified understanding and modeling of only the first n modes of vibration. Generally, these approaches arise from nonlinear differential equations.

On the other hand rather elegant approaches for obtaining system model of vibrating cantilever arise from the principles of system identification, such principles are based mainly on measured input output data. The method of subspace identification (Overschee and Moor, 1996) is an effective method to determine matrices A, B, C of the general dynamic system described by deterministic state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

To obtain the matrices of the state space model, an identification experiment was performed on the laboratory equipment shown in Fig. 1. The cantilever was excited by a piezoelectric actuator fixed to the cantilever. The input variable $u(k)$ was the voltage at amplifier. The output measured variable $y(k)$ was the end displacement of the cantilever (beam) that was measured with laser sensor. The excitation signal was set as swept sine signal with varying frequency, in the range of $0.1 - 15Hz$. The output signal had a significant gain of amplitude at the resonance frequency $8.126Hz$. Both the input and the output signal with resonance are shown in Fig. 2. By applying the FFT (Fast Fourier Transform) algorithm, the data from the time domain were transformed to the frequency domain. With appropriate software toolbox (Ljung, 2008) the matrices of the scaled system of the vibrating cantilever were determined with the following numerical values

$$x(k+1) = \begin{bmatrix} 0.857 & 1.114 \\ -0.212 & 0.874 \end{bmatrix} x(k) + \begin{bmatrix} -1.386 \\ -0.548 \end{bmatrix} u(k) \tag{2}$$

$$y(k) = \begin{bmatrix} -0.562 & 0.699 \end{bmatrix} x(k) \tag{3}$$

A comparison of agreement between the measured data and the model is in the Fig. 3. The system in the range of its first frequency showed linear behavior, which corresponds to a good agreement with the data measured. The sampling period of the model is $0.01s$

3. EXPLICIT PREDICTIVE CONTROLLER

The aim of the explicit predictive control was to demonstrate the damping effect at the end of a cantilever by minimization of its displacement from its equilibrium state. The task of the control law is to drive piezoelectric actuators in such a way that the control objective is attained in

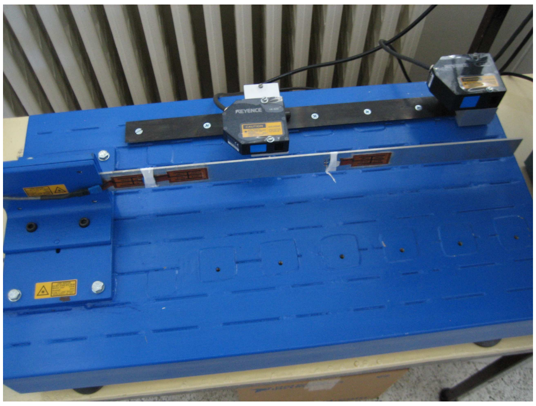

Fig. 1. Cantilever laboratory setup

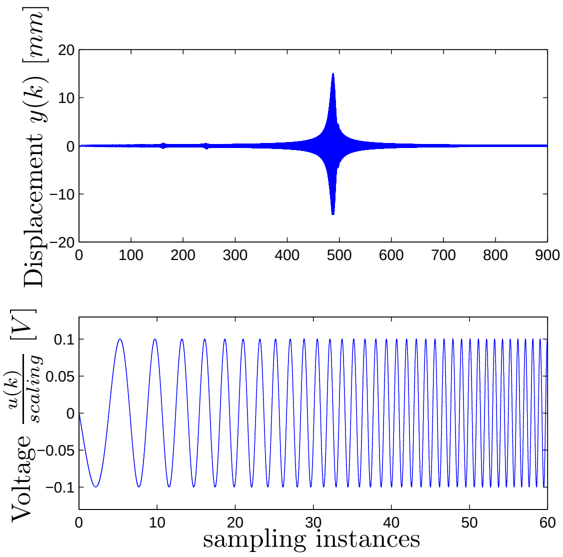

Fig. 2. Input/Output signals for system identification ($scaling = 1000$)

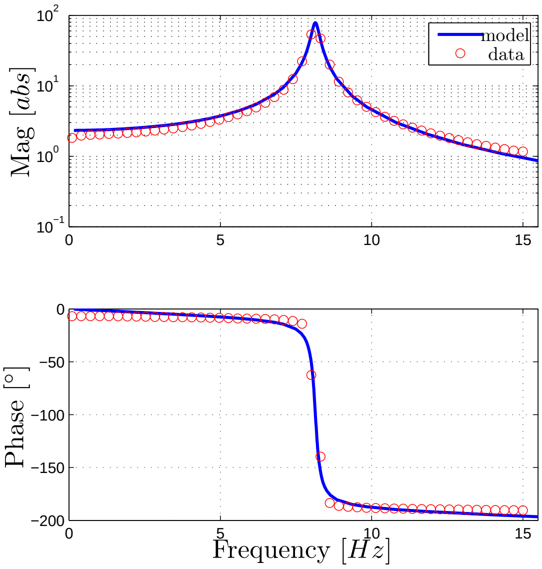

Fig. 3. Plots of measured frequency based data and identified model structure

agreement with the physical constraints and the stability of the system. In this case, the objective is to bring the state to its zero position.

To describe the state of the vibrating cantilever, the following equation is to be considered

$$x(k+1) = Ax(k) + Bu(k) \tag{4}$$

with the assumption that the state $x(k)$ always range in the polytope given

$$\mathcal{P}_i := \{x \in \mathbb{R}^n | H_i x \leq K_i\} \tag{5}$$

thus, $x \in \mathcal{P}_i$. The matrices H_i and K_i are the matrices of suitable dimensions. Defining the task of predictive control in the form

$$\min_u \sum_{k=0}^{N-1} [u^T(k)Ru(k) + x^T(k)Qx(k)] + \\ + x(k+N)^T Px(k+N)) \tag{6}$$

subject to

$$\begin{aligned} y_{min} \leq y(k+i) \leq y_{max}; i = 1, \ldots, N \quad (7) \\ u_{min} \leq u(k+i) \leq u_{max}; i = 0, 1, \ldots, N-1 \\ x(k+1) = f(x(k), u(k)), k \geq 0 \\ y(k) = Cx(k), k \geq 0 \end{aligned}$$

the following solution in the sense of multi-parametric programming (Bemporad et al., 2002; Grieder et al., 2005) can be obtained

$$u(k) = F_i x(k) + G_i \tag{8}$$

The matrices R, Q, P are the weighting matrices. The resultant solution leads to partitioning the state space to particular subspaces. For every subspace different calculated matrices F_i and G_i are valid. The task defined as minimization of objective function (6) with constraints under (7), was solved by Multi-parametric Toolbox (Kvasnica et al., 2006). The optimization task had the following input parameters: $y_{min} = -15$, $y_{max} = 15$, $u_{min} = -0.1$, $u_{max} = 0.1$, $Q = \begin{bmatrix} 10^{-12} & 0 \\ 0 & 0 \end{bmatrix}$, $R = 10^{-1}$, $P = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,with the prediction horizon $N = 3$. The explicit solution consists of 19 regions of partitioned state space, as shown in Fig. 4. The value of the control action as a function of the state is shown in Fig. 5. As can be seen from the values of all control actions for all the considered states, this procedure is the on-off type of control, which is logical, as the reshape of the end of the cantilever, caused by the action of the actuator, is negligible, compared to the displacement of the cantilever, caused by the overall vibration dynamics.

4. SIMULATION RESULTS

To simulate the control, the task of state transition from its initial value $x(0) = [-23, 0.5]^T$ to

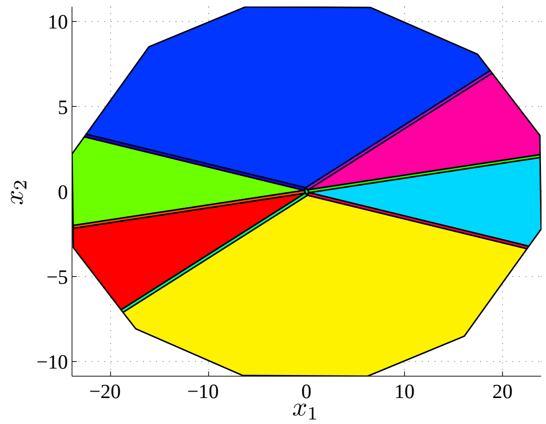

Fig. 4. Partitioning of the state space into 19 regions

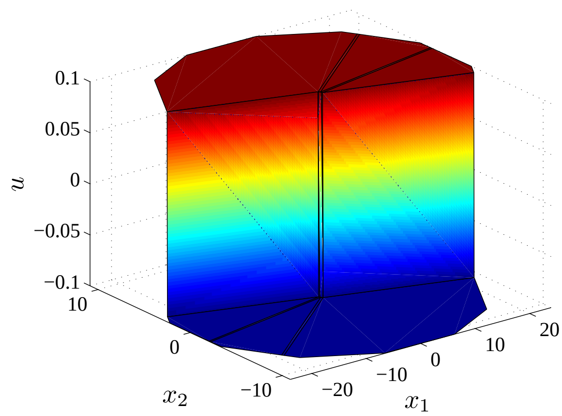

Fig. 5. Look-up table of the control moves

zero equilibrium of the identified model, given by equations (2) and (3) is to be considered. The closed loop control simulation is demonstrated in Fig. 6. From the course of the states and the output of the model, it can be seen that the control is able to damp the cantilever from a given initial state in $\approx 1s$. Fig. 7 shows the course of the free response and controlled response of the model of cantilever vibration from its given initial state.

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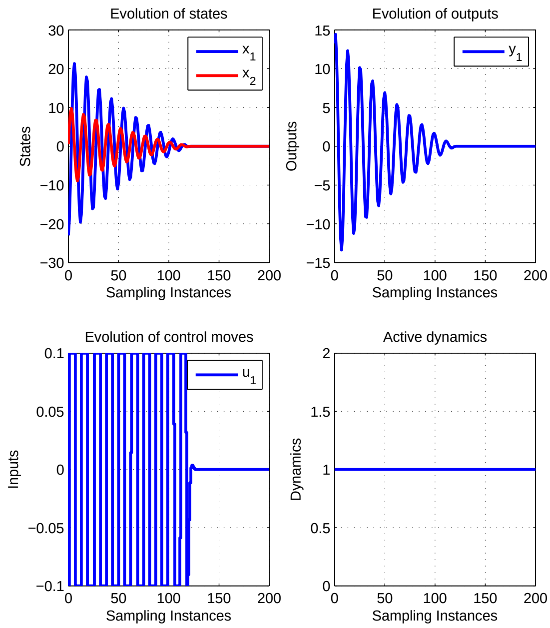

Fig. 6. Closed loop simulation

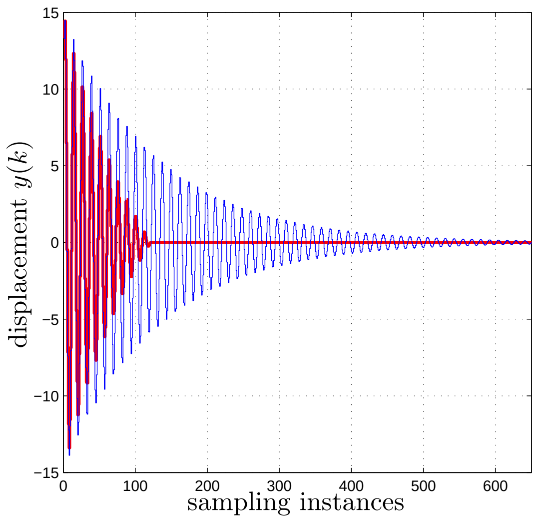

Fig. 7. Free and closed loop response of the system from initial condition $x(0) = [-23, 0.5]^T$

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