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Robust Controller Design for a Laboratory Process with Uncertainties

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Abstract: The paper presents a method for design of robust PI controllers for systems with interval uncertainty. The method is based on plotting the stability boundary locus in the (k_p, k_i)-plane and sixteen plant theorem. The stability boundaries obtained for interval plants split the (k_p, k_i)-plane in stable and unstable regions. The parameters of robust PI controllers are chosen from the stable region. The designed robust PI controller is used for control of a laboratory chemical continuous stirred tank reactor (CSTR). The reactor is used for preparing of NaCl solution with desired concentration. The conductivity of the solution is the controlled variable and the volumetric flow rate of water is the control variable.

Keywords: robust control, PI controller, interval uncertainty, process control

1. INTRODUCTION

Chemical reactors are ones of the most important plants in chemical industry, see e.g. Mikleš and Fikar (2007). Their operation, however, is corrupted with various uncertainties. Some of them arise from varying or not exactly known parameters, as e.g. reaction rate constants, reaction enthalpies or heat transfer coefficients. In other cases, operating points of reactors vary or reactor dynamics is affected by various changes of parameters of inlet streams. All these uncertainties can cause poor performance or even instability of closed-loop control systems. Application of robust control approach can be one of ways for overcoming all these problems, which may seriously influence control design for chemical reactors and other chemical processes, see e.g. Alvarez-Ramirez and Femat (1999), Gerhard et al. (2004).

In this paper, a simple method for design of robust PI controllers is presented (Tan and Kaya (2003)). The method is based on plotting the stability boundary locus in the (k_p, k_i)-plane and then parameters of a stabilizing PI controller are determined from the stability region. The PI controller stabilizes a controlled system with interval parametric uncertainties, when the stability region is found for sufficient number of Kharitonov plants (Barmish (1994)).

The approach is used for design of a robust PI controller for a laboratory continuous stirred tank reactor, which can be modelled in the form of a transfer function with parametric interval uncertainty. The reactor serves for preparing of the NaCl solution with required concentration. Composition of the solution is determined by measurement of the solution conductivity and the conductivity is the controlled variable. The volumetric flow rate of water which is used for adulterating of NaCl solution, is the control variable.

2. DESCRIPTION OF THE LABORATORY CSTR

Multifunctional process control teaching system - The Armfield PCT40 (Armfield (2005), Vojtešek et al. (2007)) is the system which enables to test a wide class of technological processes, as a tank, a heat exchanger, a continuous stirred tank reactor and their combinations (Armfield (2006a), Armfield (2006b)).

PCT40 unit consists of two process vessels, several pumps, sensors and connection to the computer. Additional equipments PCT41 and PCT42 represent a chemical reactor with a stirrer and a cooling/heating coil.

Inlet streams of reactants can be injected into the reactor via a normally closed solenoid valve or by a proportional solenoid valve (PSV). The third possibility for feeding water into the reactor is using one of two peristaltic pumps. The technological parameters of the reactor are shown in Table 1.

Table 1. Technological parameters of the reactor

Parameter	Value
Vessel diameter	0.153 m
Maximum vessel depth	0.108 m
Maximum operation volume	2 l
Minimum vessel depth	0.054 m
Minimum operation volume	1 l

The connection to the computer is realized via an I/O connector, which is connected to the PCL card. The card used is the MF624 multifunction I/O card from Humusoft. This card has 8 inputs and 8 outputs. The whole system provides 9 inputs and 17 outputs, hence two MF624 cards were used. This connection enables use of Matlab Real-time Toolbox and Simulink or data entry from the Matlab command window.

NaCl solution with the concentration 0.8555 mol/dm^3 is fed into the reactor by a peristatic pump. The performance of the pump may be theoretically set in the range $0-100\%$. But for the pump performance less than 30%, revolutions of the rotor are very small and the produced force is not high enough to transport the fluid from the barrel. The volumetric flow rate of the NaCl solution for all measurements was 0.00175 dm^3/s, which represents the pumpe performance 40%.

The water was dosed into the reactor by the PSV. Application of the PSV allowed flow measurements by the adjoint flowmeter. The PSV opening could be again done in the range $0 - 100\%$, but the volumetric flow rate of water for the PSV opening in the range $0 - 30\%$ was negligible.

For control purposes, the laboratory continuous stirred tank reactor is a SISO system. The control variable is the volumetric flow rate of water (F) and the controlled variable is the conductivity of the NaCl solution (G) inside the reactor. Used water was cold water from the standard water distribution. The volume of the solution in the reactor was kept constant with the value 1 dm^3 during all experiments.

3. PROCESS IDENTIFICATION

Identification of the controlled laboratory reactor was done from measured step responses. The constant flow rate 0.00175 dm^3/s of NaCl solution dosed into the reactor was assured by the peristaltic pump with performance 40% in all experiments. Fourteen various step changes of water flow rate were realized between 0.0032 dm^3/s - 0.01145 dm^3/s which represented the PSV opening $50-100\%$. The step responses were measured repeatedly. The resultant transfer function of the laboratory reactor was identified in the form of a transfer function (1) with the parametric interval uncertainty. The software LDDIF (Čirka and Fikar (2007)) was used for identification, which is based on the least squares algorithm. The values of the uncertain parameters are shown in Table 2.

$$G(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \tag{1}$$

Table 2. Uncertain parameters

Parameter	Minimal value	Maximal value
b_1	0.0028	0.0428
b_0	-0.2776	-0.0156
a_2	1	1
a_1	0.6349	5.5024
a_0	0.2084	3.1351

4. DESIGN OF A ROBUST PI CONTROLLER

A simple method based on plotting the stability boundary locus in the (k_p, k_i)-plane and the sixteen plant theorem is used for robust PI controller design, Tan and Kaya (2003), Závacká et al. (2008), Barmish (1994). Parameters of a stabilizing PI controller are determined from the stability region of the (k_p, k_i)-plane. The PI controller stabilizes a controlled system with interval parametric uncertainties, when the stability region is found for sufficient number of Kharitonov plants.

For the controlled system in the form of the transfer function (1) with interval uncertainty (Table 2), the Kharitonov polynomials $N_i(s)$, $i = 1, 2, 3, 4$ for the numerator and $D_j(s)$, $j = 1, 2, 3, 4$ for the denominator can be created, as it is seen in (2), (3).

$$\begin{aligned} N_1(s) &= b_1^- s + b_0^- \\ N_2(s) &= b_1^+ s + b_0^+ \\ N_3(s) &= b_1^+ s + b_0^- \\ N_4(s) &= b_1^- s + b_0^+ \end{aligned} \tag{2}$$

$$\begin{aligned} D_1(s) &= a_2^- s^2 + a_1^- s + a_0^+ \\ D_2(s) &= a_2^+ s^2 + a_1^+ s + a_0^- \\ D_3(s) &= a_2^+ s^2 + a_1^- s + a_0^- \\ D_4(s) &= a_2^- s^2 + a_1^+ s + a_0^+ \end{aligned} \tag{3}$$

where b_k^- and b_k^+, $k = 0, 1$ are lower and upper bounds of the intervals of the numerator and a_l^- and a_l^+, $l = 0, 1, 2$, are lower and upper bounds of intervals of the denominator parameters. 16 Kharitonov systems (4) can be obtained using polynomials (2), (3)

$$G_{ij}(s) = \frac{N_i(s)}{D_j(s)} \tag{4}$$

Substituting $s = j\omega$ into (4) and decomposing the numerator and the denominator polynomials of (4) into their even and odd parts one obtains

$$G_{ij}(j\omega) = \frac{N_{ie}(-\omega^2) + j\omega N_{io}(-\omega^2)}{D_{je}(-\omega^2) + j\omega D_{jo}(-\omega^2)} \tag{5}$$

The closed loop characteristic polynomial is as follows

$$\begin{aligned} \Delta(j\omega) &= [k_i N_{ie}(-\omega^2) - k_p \omega^2 N_{io}(-\omega^2) - \\ &\quad -\omega^2 D_{jo}(-\omega^2)] + j[k_p \omega N_{ie}(-\omega^2) + \\ &\quad +k_i \omega N_{io}(-\omega^2) + \omega D_{je}(-\omega^2)] \end{aligned} \tag{6}$$

Then, equating the real and imaginary parts of $\Delta(j\omega)$ to zero, one obtains

$$\begin{aligned} &k_p(-\omega^2 N_{io}(-\omega^2)) + k_i(N_{ie}(-\omega^2)) \\ &= \omega^2 D_{jo}(-\omega^2) \end{aligned} \tag{7}$$

and

$$\begin{aligned} &k_p(N_{ie}(-\omega^2)) + k_i(N_{io}(-\omega^2)) \\ &= -D_{je}(-\omega^2) \end{aligned} \tag{8}$$

After denoting

$$\begin{aligned} F_i(\omega) &= -\omega^2 N_{io}(-\omega^2) \\ G_i(\omega) &= N_{ie}(-\omega^2) \\ H_i(\omega) &= N_{ie}(-\omega^2) \\ I_i(\omega) &= N_{io}(-\omega^2) \\ J_j(\omega) &= \omega^2 D_{jo}(-\omega^2) \\ K_j(\omega) &= -D_{je}(-\omega^2) \end{aligned} \tag{9}$$

(7), (8) and (9) can be written as

$$\begin{aligned} k_pF_i(\omega) + k_iG_i(\omega) &= J_j(\omega) \\ k_pH_i(\omega) + k_iI_i(\omega) &= K_j(\omega) \end{aligned} \qquad (10)$$

From these equations, parameters of the PI controller are expressed in the form

$$k_p = \frac{J_j(\omega)I_i(\omega) - K_j(\omega)G_i(\omega)}{F_i(\omega)I_i(\omega) - G_i(\omega)H_i(\omega)} \qquad (11)$$

and

$$k_i = \frac{K_j(\omega)F_i(\omega) - J_j(\omega)H_i(\omega)}{F_i(\omega)I_i(\omega) - G_i(\omega)H_i(\omega)} \qquad (12)$$

Consider one of the systems (4), where $i = 2$ and $j = 3$

$$G_{23}(s) = \frac{0.0428s - 0.0156}{s^2 + 0.6349s + 0.2084} \qquad (13)$$

Then

$$\begin{aligned} k_p &= \frac{a_2^+ b_0^+ \omega^2 - a_0^- b_0^+ - a_1^- b_1^+ \omega^2}{(b_1^+)^2\omega^2 + (b_0^+)^2} \\ k_i &= \frac{a_1^- \omega^2 + k_p b_1^+ \omega^2}{b_0^+} \end{aligned} \qquad (14)$$

The stability boundary of the closed loop with the system (13) in the (k_p,k_i)-plane for $\omega = [0, 0.6267]$ is plot in the Figure 1. Then parameters k_p and k_i of the stabilizing controller are chosen from the stable region.

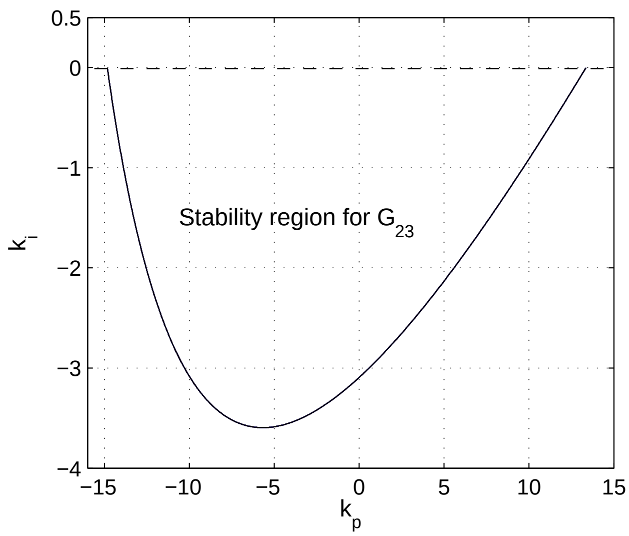

Fig. 1. Stability region of parameters k_p, k_i for the system G_{23}

Stable regions for all 16 Kharitonov systems are obtained alike. In the Figure 2 are shown stable regions for 16 Kharitonov systems (4). The controller which stabilizes all 16 Kharitonov systems has to be found in the intersection of all stable regions (the intersection lies in the red rectangle), which is in detail displayed in the Figure 3.

The parameters of the robust PI controller for control of the laboratory reactor (15) were chosen from the stable

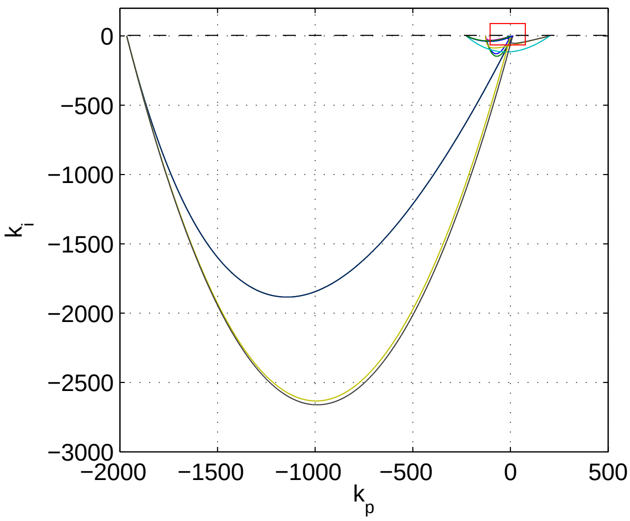

Fig. 2. Stability regions for 16 Kharitonov plants

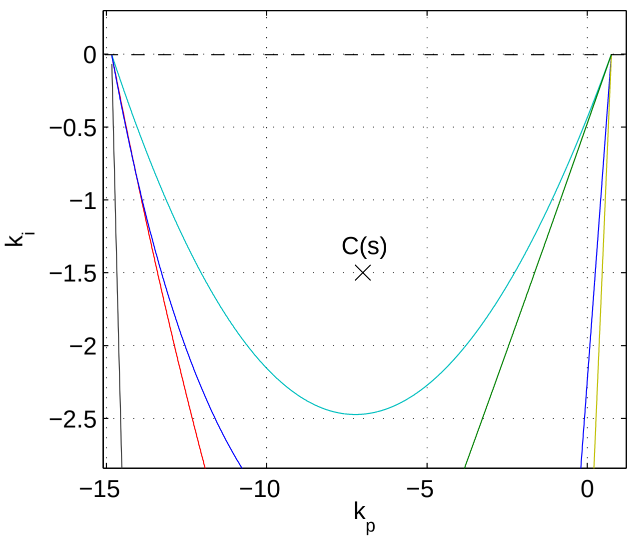

Fig. 3. Detail of the stability region for 16 Kharitonov plants

region of parameters k_p, k_i according to simulation results obtained for various choices of the PI controllers.

$$C(s) = \frac{k_ps + k_i}{s} = \frac{-7s - 1.5}{s} \qquad (15)$$

The designed PI controller was used for control of the laboratory reactor. The controlled variable $y(t)$ was the conductivity G [mS] of the NaCl solution, the control variable $u(t)$ was the water flow rate F [dm^3/s] and the reference $w(t)$ was the conductivity of the NaCl solution which corresponded to the required concentration of the NaCl solution.

Obtained experimental results are presented in the Figures 4 and 5. Robustness of the designed PI controller (15) was tested by setting the reference value in a wider area. Control responses of the reactor are shown in Figure 4 for $w \in [12; 32]$ mS and in the Figure 5 for $w \in [18; 30]$ mS.

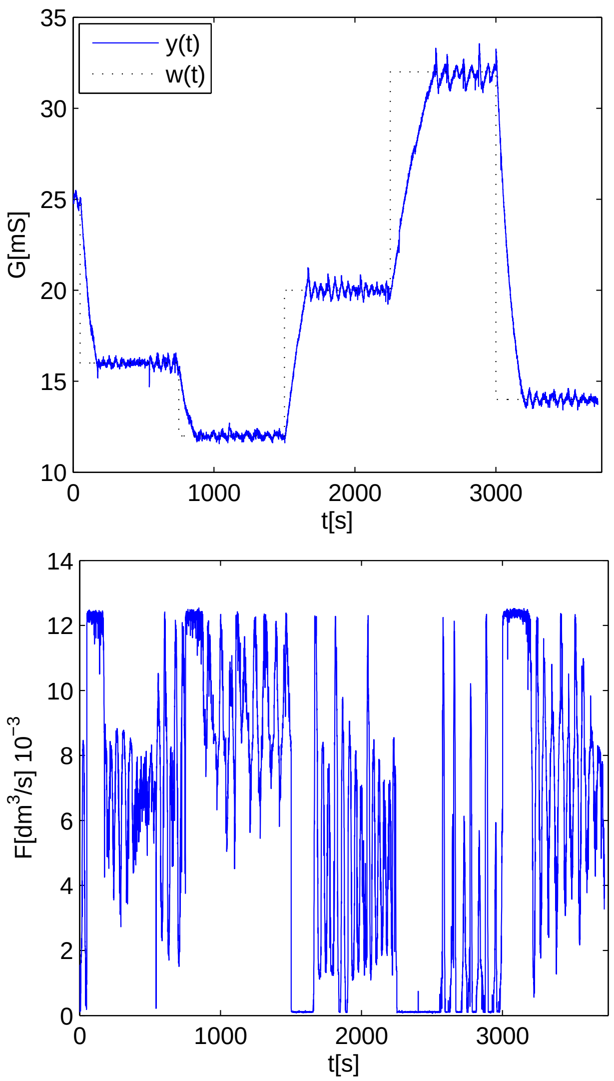

Fig. 4. Control of the reactor with robust PI controller

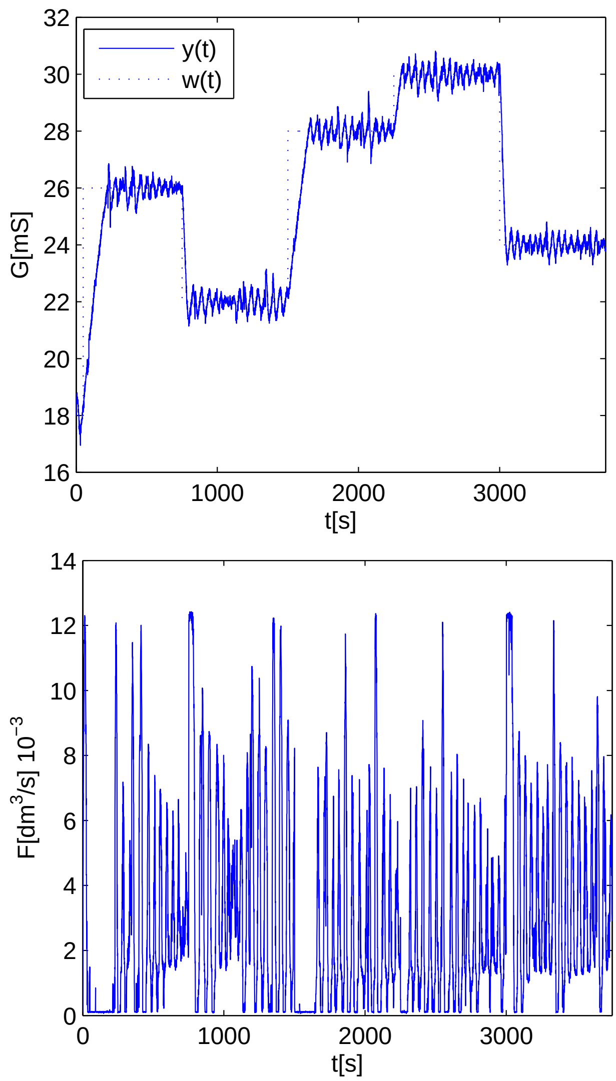

Fig. 5. Control of the reactor with robust PI controller

5. CONCLUSION

The robust PI controller was designed for control of the laboratory continuous stirred tank reactor. A simple robust synthesis was used which was based on plotting the stability boundary locus in the (k_p, k_i)-plane and the sixteen plant theorem. The robust PI controller was chosen from the stable region of the (k_p, k_i)-plane. The designed controller was tested experimentally by control of a laboratory reactor. Obtained experimental results confirmed that the designed robust PI controller successfully controlled the laboratory reactor where controlled variable - conductivity G [mS] of NaCl, was controlled by water flow rate F [dm^3/s]. The varying reference was always reached. The control responses were without overshoots and fast enough.

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