Slovak University of Technology in Bratislava
Institute of Information Engineering, Automation, and Mathematics

PROCEEDINGS

of the 18th International Conference on Process Control

Hotel Titris, Tatranská Lomnica, Slovakia, June 14 – 17, 2011

ISBN 978-80-227-3517-9

http://www.kirp.chtf.stuba.sk/pc11

Editors: M. Fikar and M. Kvasnica

Kopačka, M., Šimončič, P., Csambal, J., Honek, M., Wojnar, S., Polóni, T., Rohal'-Ilkiv, B.: Real-time Air/Fuel Ratio Model Predictive Control of a Spark Ignition Engine, Editors: Fikar, M., Kvasnica, M., In *Proceedings of the 18th International Conference on Process Control*, Tatranská Lomnica, Slovakia, 457–462, 2011.

Full paper online: http://www.kirp.chtf.stuba.sk/pc11/data/abstracts/061.html

Real-time Air/Fuel Ratio Model Predictive Control of a Spark Ignition Engine

Matúš Kopačka, Peter Šimončič, Jozef Csambál, Marek Honek, Sławomir Wojnar, Tomáš Polóni, Boris Rohaľ-Ilkiv

Faculty of Mechanical Engineering, Slovak University of Technology, Bratislava, Slovakia
(e-mail: matus.kopacka@stuba.sk)

Abstract: The following paper describes the control of air/fuel ratio (AFR) of a spark ignition engine utilizing the analytical model predictive controller based on the multi-model approach. The multi-model approach employs the autoregressive model (ARX) network, using the weighting of local models, coming from the sugeno-type fuzzy logic. The weighted ARX models are identified in the particular working points and are creating a global engine model, covering its nonlinearity. Awaited improvement of a proper air/fuel mixture combusted in a cylinder is mostly gained in the transient working regimes of an engine. In these regimes, the traditional control approach looses its quality, compared to steady state working regimes of an engine. This leads to higher fuel consumption and level of emissions from an engine. Presented results of the air/fuel ratio control are acquired from the real-time control of the VW Polo $1390cm^3$ engine, at which the original electronic control unit (ECU) has been replaced by a dSpace system executing the model predictive controller. It has been proven, that the proposed controller is suitable for the air/fuel ratio control giving sufficiently good and steady system output.

Keywords: model predictive control, analytical solution, air/fuel ratio, SI engine, ARX models

1. INTRODUCTION

A run of a spark ignition engine (SI) is highly dependent on the mixture of the sucked air and injected fuel present in the cylinder, waiting to be ignited by the spark. Incorrect ratio of this two components may lead to the poor engine power, ineffective functionality of the catalytic converter resulting in higher level of emissions polluting the environment and in the extreme case this can lead to the engine stoppage. Due to this reason it is crucial to keep the air/fuel ratio (AFR) at the stoichiometric level, which means, that both, the air and the fuel are completely combusted. Due to above mentioned reasons and all the time tightening emission standards the car producers are improving the control of the air/fuel ratio.

Traditional control of air/fuel ratio is based on a feedforward control using predefined tables determining how much fuel has to be injected into a cylinder, based on the information from the mass air flow meter. This fuel amount is subsequently corrected using the information from the lambda probe, so the stoichiometric mixture can be reached. Due to a lambda probe position (at the engine exhaust) a delay arises, causing an improper feedback correction at the unstable engine regimes, like acceleration, or deceleration. On the other side, this kind of control guarantees stability and robustness at all conditions and therefore is still preferred by car producers, despite its disadvantages in control.

The academic field have started to publish other kinds of air/fuel control, mostly model-based ones. The model-based approaches are bringing good quality of control, but are also more sensitive to the model precision and issues with stability and robustness appear. A survey through popular "mean value engine modeling" is described in Bengtsson et al. (2007). This analytical way of engine modeling is very clear, but requires exact knowledge of the system and the model error has to be taken into account explicitly. Other ways of a model acquisition are based on the experimental identification (black box modeling). Works of Zhai et al. (2010), Zhai and Yu (2009) and Hou (2007) are specialized in employment of neural networks, while Mao et al. (2009) uses for engine modeling CARIMA models.

In the engine control itself became popular fuzzy logic (Hou (2007)), neural network control (Arsie et al. (2008)) and model predictive control (MPC) approaches (Lorini et al. (2006) and Muske and Jones (2006)). General topics on an issue of stability and robustness in MPC can be found in Mayne et al. (2000), or Zeman and Rohal-Ilkiv (2003).

Our approach, introduced in Polóni et al. (2007) is utilizing an analytical model predictive controller with a penalization of a terminal state. It uses a multi-model approach using a weighted net (sugeno-type fuzzy logic) of autoregressive models (ARX) as a system model. The ARX models were identified in the particular working points of the engine as black box models. This method of engine modeling offers an easy way of "global nonlinear system model" acquisition with subsequent utilization in the model based system control. The preliminary real-time predictive control results presented in this paper indicate that the proposed controller could be suitable alternative toward the air/fuel ratio control through the look-up tables.

2. AIR/FUEL RATIO

The model of the air/fuel ratio dynamics λ of a spark ignition engine is based on the mixture, defined as a mass ratio of the air and fuel present in a cylinder at a time instance k. Due to the fact, that the air mass flow is measured as an absolute value, it was necessary to integrate this amount during the particular time and express the air and fuel quantity as relative mass densities $(\frac{grams/cylinder}{grams/cylinder})$. Hence, the air/fuel ratio is defined, as:

$$\lambda(k) = \frac{m_a(k)}{m_f(k)}\frac{1}{L_{th}} \tag{1}$$

Where $m_a(k)$ and $m_f(k)$ are relative mass amounts of air and fuel in a cylinder and $L_{th} \approx 14.64$ is the theoretical amount of air necessary for the ideal combustion of a unit amount of fuel. The L_{th} constant normalizes the ideal value of λ to be 1.0.

3. SI ENGINE MODELING USING ARX MODELS

The engine modeling is based on the weighted linear local model with single input single output (SISO) structure (Polóni et al., 2008). The parameters of local linear ARX models with weighted validity (Murray-Smith and Johanssen, 1997) are identified to model the nonlinear dynamics of the AFR. The principle of this nonlinear modeling technique is in partitioning of the engine's working range into smaller working points.

A net of local ARX models weighted for a particular working point ϕ is defined, as:

$$\begin{gathered}\sum_{h=1}^{n_M} \rho_h(\phi(k))A_h(q)y(k) = \\ \sum_{h=1}^{n_M} \rho_h(\phi(k))B_h(q)u(k) + \sum_{h=1}^{n_M} \rho_h(\phi(k))c_h + e(k)\end{gathered} \tag{2}$$

defined by polynomials A_h and B_h:

$$\begin{aligned}A_h(q) &= 1 + a_{h,1}q^{-1} + \ldots + a_{h,n_y}q^{-ny} \\ B_h(q) &= b_{h,1+d_h}q^{-1-d_h} + \ldots + b_{h,n_u+d_h}q^{-nu-d_h}\end{aligned} \tag{3}$$

where symbolics q^{-i} denotes a sample delay, e.x. $q^{-i}y(k) = y(k-i)$, $a_{h,i}$ and $b_{h,(j+d_h)}$ are parameters of hth local function and d_h is its delay. Parameter n_M represents the number of local models.

The ρ_h denotes a weighting function of a particular ARX model (see Sec. 3.1) and the $e(k)$ is a stochastic term with a white noise properties. The engine working point itself is defined by engine revolutions n_{en} and the throttle valve position t_r, hence: $\phi(k) = [n_{en}(k), t_r(k)]^T$. The absolute term $\hat{c}_h$ of the equation is computed from the steady state values of the system output $y_{e,h}$ and the system input $u_{e,h}$, as:

$$\hat{c}_h = y_{e,h} + y_{e,h}\sum_{i=1}^{ny}\hat{a}_{h,i} - u_{e,h}\sum_{j=1}^{nu}\hat{b}_{h,j} \tag{4}$$

The model output is computed from the equation:

$$\begin{gathered}y_s(k) = \sum_{h=1}^{n_M}\rho_h(\phi(k)) \\ \cdot\left(\sum_{i=1}^{ny}\hat{a}_{h,i}q^{-i}y_s(k) + \sum_{j=1}^{nu}\hat{b}_{h,(j+d_h)}q^{-j-d_h}u(k) + \hat{c}_h\right)\end{gathered} \tag{5}$$

which after the introduction of the estimated parameter vector $\hat{\theta}_h$ and the regression vector $\gamma(k)$, becomes:

$$y_s(k) = \gamma^T(k)\sum_{h=1}^{n_M}\rho_h(\phi(k))\hat{\theta}_h + \sum_{h=1}^{n_M}\rho_h(\phi(k))\hat{c}_h \tag{6}$$

3.1 Weighting functions

The full working range of the engine has been covered by a discrete amount of local linear models (LLMs), identified at particular working points. The LLMs are being weighted by a weighting functions defining validity of each local model according to an instantaneous working point of the engine. Due to a request of a smooth and continuous global engine model, design of those weighting functions was crucial.
There were designed particular interpolation functions for every LLM, assigning it 100% validity exactly at the belonging working point with a decreasing tendency in the directions of the deviation of the throttle valve opening Δt_r and the engine revolutions Δn_{en} from the particular working point. The "three dimensional" Gaussian functions:

$$\begin{gathered}\tilde{\rho}_h(\phi(k)) = \\ \exp\left[-\left[\,\Delta n_{en}(k)\ \ \Delta t_r(k)\,\right]\begin{bmatrix}\frac{1}{\sigma_{h,1}^2} & 0 \\ 0 & \frac{1}{\sigma_{h,2}^2}\end{bmatrix}\begin{bmatrix}\Delta n_{en}(k) \\ \Delta t_r(k)\end{bmatrix}\right]\end{gathered} \tag{7}$$

were used as the local weighting functions, due to their suitable shape fulfilling the approximation properties. The choice of tuning parameters $\sigma_{h,1} = 250$ and $\sigma_{h,2} = 0.8$ used in the weighting functions has been chosen experimentally, awaiting continuous and smooth output of the modeled system. At the same time the experiments have shown, that there can be used identical weighting functions for weighting of the air and fuel path parameters.
All the weighting functions were at the end normalized by creating normalized weighting functions:

$$\rho_h(\phi(k)) = \frac{\tilde{\rho_h}(\phi(k))}{\sum_{h=1}^{n_M}\tilde{\rho_h}(\phi(k))} \tag{8}$$

so the sum of values of all weighting functions belonging to a particular working point (Fig. 1), equals exactly one: $\sum_{h=1}^{n_M} \rho_h(\phi(k)) = 1$.

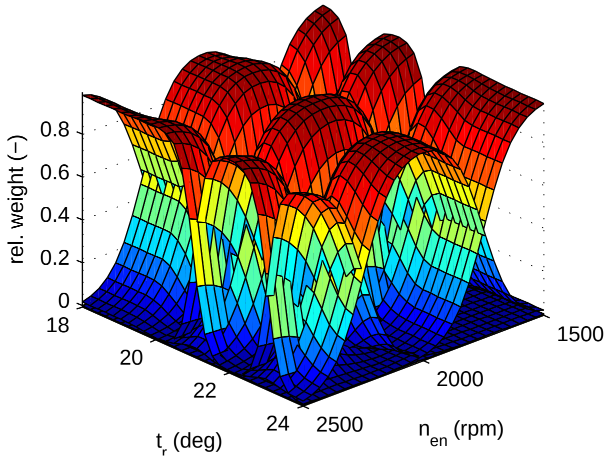

Fig. 1. Relative weighting Gaussian functions

3.2 Model identification

Considering the $\lambda(k)$ modeling, the engine has been divided into two subsystems with independent inputs, namely into:

air path with the air throttle position as the disturbance input, and
fuel path with the input of fuel injector opening time.

Another disturbance-like acting quantity in the air path were engine revolutions, implicitly included in the engine model, particularly for each working point.

Parameters of the local ARX models have been estimated from the data acquired from the exhaust gas oxygen sensor and an air flow sensor. The identification has been designed so, that the dynamics of the air path and fuel path stayed uncoupled, hence the dynamics of both paths were measured indirectly.

Air path identification The first experiment started at the stoichiometric value of λ_a in the operation point ϕ. To excite the air path dynamics, the throttle valve position was oscillating around its steady position according to a pseudo-random binary signal (PRBS), while the fuel injectors were delivering constant fuel mass $m_{f,e}$. The change in λ_a value has been recorded. During the experiment the engine had been braked at constant revolutions.

Fuel path identification The identification of the fuel path dynamics has been done similarly, but with the fixed throttle valve delivering a constant air mass $m_{a,e}$. The PRBS was varying the fuel injectors' opening time and the value of λ_f had been measured again.

In both experiments it was necessary to wisely propose a PRBS, so that the air/fuel mixture is always ignitable. The local ARX models can be subsequently determined from the measured values of instantaneous $\lambda_a(k)$ and $\lambda_f(k)$ belonging to the air path and fuel path, utilizing relative air and fuel mass densities:

$$m_a(k) = m_{a,e}(\phi)\lambda_a(k) \tag{9}$$

and

$$m_f(k) = \frac{m_{f,e}(\phi)}{\lambda_f(k)} \tag{10}$$

The final formula describing the aif/fuel ratio dynamics is built up of local linear ARX models of the air and fuel paths is in the form:

$$\lambda_s(k) = \frac{1}{L_{th}} \cdot \left[\frac{\gamma_a^T(k) \sum_{h=1}^{n_A} \rho_{a,h}(\phi(k))\hat{\theta}_{a,h} + \sum_{h=1}^{n_A} \rho_{a,h}(\phi(k))\hat{c}_{a,h}}{\gamma_f^T(k) \sum_{h=1}^{n_F} \rho_{f,h}(\phi(k))\hat{\theta}_{f,h} + \sum_{h=1}^{n_F} \rho_{f,h}(\phi(k))\hat{c}_{f,h}} \right] \tag{11}$$

Where:

γ	is the regression vector of system inputs and outputs
n_A	is the amount of working points
ρ	is the interpolation function
ϕ	is the vector of a working point
θ	is the vector of ARX parameters
c	is the absolute term of an ARX model

In accordance with the general model structure presented, the key variables are defined in the Table 1.

Table 1. Symbol connection between the general expression and the model

general symbol	air-path model	fuel-path model	operating point
$y(k)$	$m_a(k)$	$m_f(k)$	
$u(k)$	$t_r(k)$	$u_f(k)$	
$\gamma(k)$	$\gamma_a(k)$	$\gamma_f(k)$	
$\hat{\theta}_h$	$\hat{\theta}_{a,h}$	$\hat{\theta}_{f,h}$	
$\rho_h(\phi(k))$	$\rho_{a,h}(\phi(k))$	$\rho_{f,h}(\phi(k))$	
$\hat{c}_h$	$\hat{c}_{a,h}$	$\hat{c}_{f,h}$	
$\phi(k)$			$[n_e(k), t_r(k-\delta)]^T$

4. PREDICTIVE CONTROL

The strategy of an "exceeding oxygen amount" control using a predictive controller is based on a prediction of a controlled quantity λ and subsequent minimization of a chosen cost function on the horizon N_p expressed in a standard quadratic form. The value of λ is predicted by utilization of partially linear models of the air and fuel path. Through the independent air path model the proper amount of fuel is predicted and enters the cost function J. Hence, the target of the cost function minimization is to determine such a control law, that the measured system output λ is stoichiometric. The second modeled subsystem, the fuel-path, is an explicit component of the objective function where the amount of the fuel is the function of optimized control action (Polóni et al. (2008)).

4.1 Predictive model

The applied control strategy is based on the knowledge of the internal model (IM) of air-path, predicting the change of air flow through the exhaust pipe, and consequently, setting the profile of desired values of the objective function on the control horizon. In this case we will consider the state space (SS) formulation of the system and therefore it is necessary to express linear local ARX models in the SS structure with time varying parameters:

$$x_{(a,f)}(k+1) = A_{(a,f)}(\phi)x_{(a,f)}(k) + B_{(a,f)}(\phi)u_{(a,f)}(k)$$
$$m_{s,(a,f)}(k) = C_{(a,f)}x_{(a,f)}(k) \quad (12)$$

The weighted parameters of multi-ARX models are displayed in matrices $A_{a,f}$ and $B_{a,f}$ for both subsystems. This is a non-minimal SS representation whose advantage is, that no state observer is needed. The "fuel pulse width control" is tracking the air mass changing on a prediction horizon from IM of the air-path, by changing the amount of injected fuel mass. Due to tracking offset elimination, the SS model of the fuel-path (12) (index f), with its state space vector x_f, is written in augmented SS model form to incorporate the integral action:

$$\tilde{x}_f(k+1) = \tilde{A}_f(\phi)\tilde{x}_f(k) + \tilde{B}_f(\phi)\Delta u_f(k) \quad (13)$$

or

$$\begin{bmatrix} x_f(k+1) \\ u_f(k) \end{bmatrix} = \begin{bmatrix} A_f(\phi) & B_f(\phi) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_f(k) \\ u_f(k-1) \end{bmatrix} + + \begin{bmatrix} B_f(\phi) \\ 1 \end{bmatrix} \Delta u_f(k)$$

$$m_{s,f}(k) = \tilde{C}_f\tilde{x}_f(k) + D_f\Delta u_f(k) \quad (14)$$

or

$$m_{s,f}(k) = [\, C_f \;\; D_f \,]\, \tilde{x}_f(k) + D_f\Delta u_f(k)$$

The prediction of the air mass ($\underset{\rightarrow}{m}_a$) on the prediction horizon (N_p) is dependent on the throttle position ($\underset{\rightarrow}{t}_r$) and is computed as

$$\underset{\rightarrow}{m}_a(k) = \Gamma_a(\phi)x_a(k) + \Omega_a(\phi)\underset{\rightarrow}{t}_r(k-1) \quad (15)$$

where the x_a denotes the state space vector of the air path.

Due to the unprecise modeling (IM strategy), the biased predictions of the air mass future trajectory and consequently biased fuel mass might occur. This error is compensated incorporation the term $L[\hat{m}_f(k) - m_{s,f}(k)]$ into the fuel mass prediction equation:

$$\underset{\rightarrow}{m}_f(k) = \Gamma_f(\phi)\tilde{x}_f(k) + \Omega_f(\phi)\Delta\underset{\rightarrow}{u}_f(k-1) + + L[\hat{m}_f(k) - m_{s,f}(k)] \quad (16)$$

The matrices of free response Γ_a, Γ_f and forced response Ω_a, Ω_f are computed from the SS model (12), respectively (Maciejowski, 2000). Since there is only $\lambda(k)$ measurable in equation (1), the value of $m_a(k)$ needs to be substituted using IM of the air-path, then:

$$\hat{m}_f(k) = \frac{1}{L_{th}}\frac{m_{s,a}(k)}{\lambda(k)} \quad (17)$$

The estimate $\hat{m}_f(k)$ is used to compensate for possible bias errors of predicted $\underset{\rightarrow}{m}_f(k)$ in (16).

4.2 Analytical solution

The analytical solution is based on the cost function (18), encompassing deviations of predicted fuel mass amounts between the air and fuel path (based on (1)); a penalization of control increments r; and a penalization p of a deviation between a predicted and desired end state.

$$J_\lambda = \left\| \frac{\underset{\rightarrow}{m}_a(k)}{L_{th}} - \underset{\rightarrow}{m}_f(k) \right\|_2^2 + r\|\Delta\underset{\rightarrow}{u}_f(k-1)\|_2^2 + p\|\tilde{x}_f(N) - \tilde{x}_{f,r}(N)\|_2^2 \quad (18)$$

The chosen MPC approach utilizes the state space representation with an integral control for the correction of the prediction.
Due to a disturbance $d(k)$, the steady state values of u and x have to be adapted so, that the assumption $J = 0$ is valid. This problem solves an explicit inclusion of the disturbance into the model.
The fuel injectors are controlled by a fuel pulse width, what is at the same time the control u_f. The optimal injection time can be computed by minimization of a cost function (18), which has after expansion by the fuel path prediction equation, form:

$$J_\lambda = \left\| \frac{\underset{\rightarrow}{m}_a}{L_{th}} - \Gamma_f\tilde{x}_f(k) + \Omega_f\Delta\underset{\rightarrow}{u}_f(k-1) \right\|_2^2 + r\left\|\Delta\underset{\rightarrow}{u}_f(k-1)\right\|_2^2 + p\|\tilde{x}_f(N) - \tilde{x}_{f,r}(N)\|_2^2 \quad (19)$$

An analytical solution of $\frac{dJ_\lambda}{\Delta \underset{\rightarrow}{u}} = 0$ of (19) without constraints leads to an expression determining the change of "fuel injector opening time" in a step (k), as:

$$\Delta u = \left(\Omega^T\Omega + Ir + p\Omega_{xN}^T\Omega_{xN}\right)^{-1} \cdot \left[\Omega^T[w(k) - \Gamma\tilde{x}(k)] - p\Omega_{xN}^T A^N\tilde{x}(k) + p\Omega_{xN}^T\tilde{x}_{f,r}(N)\right] \quad (20)$$

Hence, the absolute value of the control action in a step k is given by a sum of a newly computed increment in a control (20) and an absolute value of the control in a step ($k-1$):

$$u_f(k) = u_f(k-1) + \Delta u_f(k) \quad (21)$$

5. RAPID CONTROL PROTOTYPING SYSTEM

The computational unit necessary for the real-time implementation of the MPC control is based on a powerful and freely programmable control system based on *dSpace* and *RapidPro* units; or "Rapid Control Prototyping System" (RCP), (Fig. 2, dSPACE GmbH. (2009)). It is built-up on the processor board *ds1005* and hardware-in-loop platform *dS2202 HIL*. The RCP ensures sufficient headroom for the real-time execution of complex algorithms (Arsie et al. (2008)) and lets all engine tasks to be controlled directly. Also, the customized variants of the controller can be performed immediately.

Typical RCP system consists of:

- A math modeling program (prepared in Simulink)
- Symbolic input/output blocks
- A real-time target computer (embedded computer with an analog and digital I/O)

- A host PC with communication links to target computer
- A graphical user interface (GUI) which enables to control the real time process

The RCP system enables to use a support in the form of embedded functions which make the preparation of algorithms easy and fast. It is a great help, because one can then concentrate on significant problems (development and debugging of algorithms) without the spending time on not so important tasks (how to handle features of RCP system at low level programming).

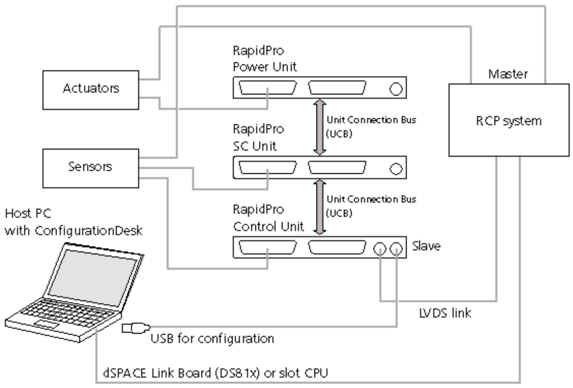

Fig. 2. Rapid control prototyping scheme

6. REAL-TIME APPLICATION OF A PREDICTIVE CONTROL

The ability to control the mixture concentration at stoichiometric level using MPC is demonstrated through the real-time SI engine control (Fig. 3). This has been

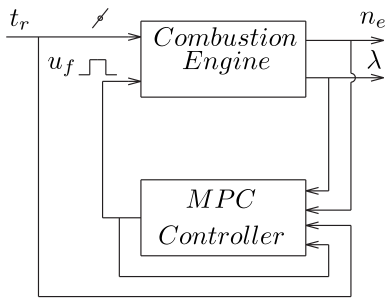

Fig. 3. Control scheme

performed using the AFR predictive control strategy described in the previous section, designed in *Matlab/Simulink* environment and compiled as a real-time application for a *dSpace* platform. It has been applied to the VW Polo engine (Fig. 4), 1390 cm^3 with 55kW@5000 rpm, not equipped with a turbocharger or an exhaust gas recirculation system. The control period was 0.2s. The result of an identification are 9 local linear models (LLM) for each, air and fuel path, dependent on a throttle valve opening and engine revolutions.

The primary target of a control (Fig. 5) was to hold the air/fuel ratio in a stoichiometric region ($\lambda = 1$), in the worst case to keep the mixture ignitable ($0.7 \leq \lambda \leq 1.2$). During the experiment, the change in throttle valve opening, between 21 and 22 degrees (Fig. 5, variable t_r) and

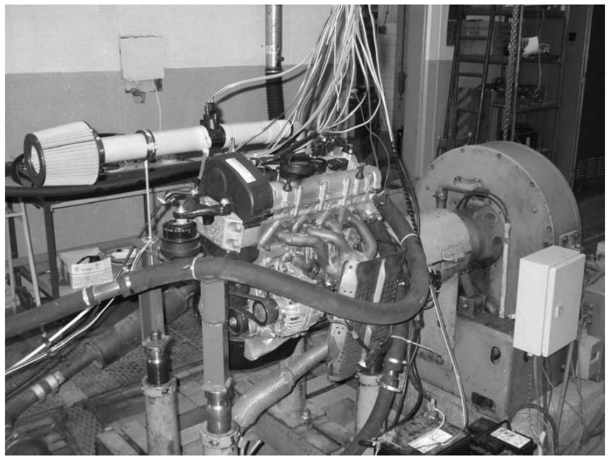

Fig. 4. Spark ignition engine VW Polo 1.4

the change of engine revolutions (Fig. 5, variable n_{en}), has been performed several times. These changes simulate varying working regimes of an engine, which is adapting its run to a daily traffic. Changes in t_r and n_{en} quantities are determining the engine load, at the same time, ensuring, that the engine passes through several working points during its operation. As mentioned in Section 3, the engine revolutions are not included among explicit variables of local models, but they build together with a delayed throttle valve position a vector of an working point $\phi(k)$.
The quality of control is sufficient (Fig. 5, variable λ), with exceptional acceptable overshoots in both directions. These overshoots of the controlled variable λ have been caused by smaller model precision, due to its distance from the working point, at which the system identification has been performed. This effect is caused by the approximation of a particular model from the other working points' models.
The corresponding control (fuel injection time) computed by the controller is shown in (Fig. 5, variable t_{inj}).

The initial engine warm-up (to 80 °C) eliminated model-plant mismatch caused by temperature dependent behavior of the engine.

The control has been performed by choosing the penalization $r = 0.1$. Utilizing the member $p\|\tilde{x}_f(N) - \tilde{x}_{f,r}(N)\|_2^2$ of a cost function by setting $p = 1.0$ allowed us to shorten the control horizon to $N_p = 20$ what significantly unloaded the computational unit and stabilized the controlled output of the engine on this shortened horizon, as well. The best control has been achieved in the neighborhood of working points, what is logically connected to the most precise engine model at those points. In other working points the control is still good enough, with small deviations from the stoichiometric mixture.

7. CONCLUSION

Considering the preliminary results from the real-time experiments at the engine, it can be concluded, that the idea of the AFR model predictive control based on local ARX models is suitable and applicable for the SI engine control. The proposed flexible design of a predictive controller offers easy tuning possibilities and a potential for

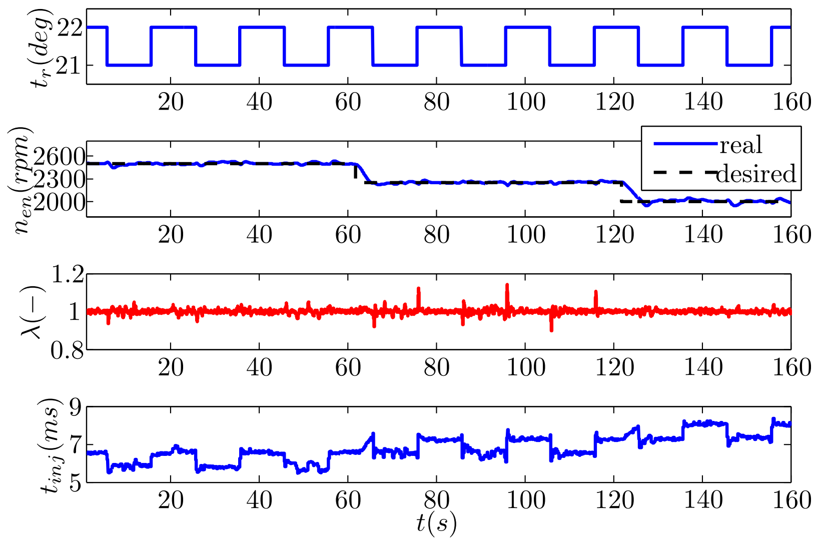

Fig. 5. Results of an AFR SI engine control

the model accuracy improvement by the extension of the global engine model to other working regimes of the engine. The next project step shall be the overshoot elimination in the λ - control by the identification of wider net of "local linear engine models" and implementation of constraints. Another task which has to be done is a comparison of the quality of control gained by the MPC controller with a baseline electronic control unit. This goal has been not yet achieved, as the original ECU has been replaced by the dSpace system running our controller.

ACKNOWLEDGMENTS

The work has been supported by the Slovak Research and Development Agency under grant LPP-0075-09, LPP-0118-09 and LPP-0096-07. This research is also supported by the grant from Norway through the EEA Financial Mechanism and the Norwegian Financial Mechanism. This project is also co-financed from the state budget of the Slovak Republic. This support is very gratefully acknowledged.

REFERENCES

Arsie, I., Iorio, S.D., Noschese, G., Pianese, C., and Sorrentino, M. (2008). Optimal air-fuel ratio. *dSpace Magazine*, (1), 20–23.

Bengtsson, J., Strandh, P., Johansson, R., Tunestal, P., and Johansson, B. (2007). Hybrid modeling of homogenous charge compression ignition (HCCI) engine dynamics -a survey. *International journal of control*, 80(11), 1814–1847.

dSPACE GmbH. (2009). *HelpDesk Application*.

Hou, Z. (2007). Air fuel ratio control for gasoline engine using neural network multi-step predictive model. 3rd international conference on intelligent computing, Qingdao, China.

Lorini, G., Miotti, A., and Scattolini, R. (2006). Modeling, simulation and predictive control of a spark ignition engine. In Predimot (ed.), *Predictive control of combustion engines*, 39–55. TRAUNER Druck GmbH & CoKG.

Maciejowski, J.M. (2000). *Predictive control with constraints*. University of Cambridge.

Mao, X., Wang, D., Xiao, W., Liu, Z., Wang, J., and Tang, H. (2009). Lean limit and emissions improvement for a spark-ignited natural gas engine using a generalized predictive control (GPC)-based air/fuel ratio controller. *Energy & Fuels*, (23), 6026–6032.

Mayne, D.Q., Rawlings, J.B., Rao, C.V., and Scokaert, P.O.M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, (36), 789–814.

Murray-Smith, R. and Johanssen, T.A. (1997). *Multiple model approaches to modelling and control*. Taylor & Francis.

Muske, K.R. and Jones, J.C.P. (2006). A model-based SI engine air fuel ratio controller. American Control Conference, Minneapolis, USA.

Polóni, T., Johansen, T.A., and Rohaľ-Ilkiv, B. (2008). Identification and modeling of air-fuel ratio dynamics of a gasoline combustion engine with weighted arx model network. *Transaction of the ASME (Journal of Dynamic Systems, Measurement, and Control)*, 130(6). 061009.

Polóni, T., Rohaľ-Ilkiv, B., and Johansen, T.A. (2007). Multiple ARX model-based air-fuel ratio predictive control for SI engines. In *IFAC Workshop on advanced fuzzy and neural control*. Valenciennes, France. Conference paper MO5-3.

Zeman, J. and Rohal-Ilkiv, B. (2003). Robust minmax model predictive control of linear systems with constraints. 930 – 935. IEEE International Conference on Industrial Technology.

Zhai, Y.J., Ding-WenYu, Hong-YuGuo, and D.L.Yu (2010). Robust air/fuel ratio control with adaptive DRNN model and AD tuning. *Engineering Applications of Artificial Intelligence*, (23), 283–289.

Zhai, Y.J. and Yu, D.L. (2009). Neural network model-based automotive engine air/fuel ratio control and robustness evaluation. *Engineering Applications of Artificial Intelligence*, (22), 171–180.