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## **Robust Decentralized Controller Design for Performance**

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**Abstract:** The paper presents an innovation of the robust decentralized controller design for multivariable uncertain systems within the setting of the Equivalent Subsystems Method (ESM). The aim of the proposed design procedure is to guarantee robust stability and plant-wide nominal performance in terms of maximum overshoot achieved through phase margins specified for equivalent subsystems. The developed design procedure is illustrated by an example.

Keywords: Decentralized controller, Frequency domain control, Nominal performance, Robust stability

## 1. INTRODUCTION

When designing decentralized control (DC), performance objectives can be of two basic types: a) achieving required performance in different subsystems; or b) achieving plantwide desired performance. The Nyquist-based frequency domain decentralized controller design technique for performance called "Equivalent Subsystems Method" (ESM) (Kozáková et al., 2009a, b) belongs to the latter group. According to it, the DC design for plants described by transfer function matrices is performed through independent designs for equivalent subsystems that are actually Nyquist plots of decoupled subsystems shaped by a selected characteristic locus of the interactions matrix. It has been proved that local controllers independently tuned for stability and specified feasible performance in terms of degree of stability in equivalent subsystems provide a decentralized controller guaranteeing the very degree of stability of the full system. In (Kozáková et al., 2010), the ESM design technique has been used to design digital decentralized controller for specified phase margin thus guaranteeing plantwide maximum overshoot by applying discrete Bode plots of equivalent subsystems,.

Application of the ESM in the design for robust stability and nominal performance can be found in (Kozáková and Veselý, 2007; 2008; 2009), (Kozáková et al., 2009a) always in a twostage design methodology: first, the DC for nominal performance is designed according to ESM, and afterwards, fulfillment of the robust stability conditions is examined; if robust stability is not achieved either controller parameters are to be modified, or the redesign is to be carried out with modified performance requirements.

This paper presents a robust DC design methodology based on direct integrating of robust stability conditions in the ESM. In this way, local controllers of equivalent subsystems are designed with regard to robust stability, and nominal performance in terms of maximum peak of the complementary sensitivity (or sensitivity, depending on uncertainty type) that provides information about the maximum overshoot and is transformable into lower bound for the phase margin of equivalent subsystems.

The paper is organized as follows: Preliminaries and problem formulation are in Section 2, principles of the Equivalent Subsystems Method (ESM) are revisited in Section 3. Section 4 presents the direct robust DC design procedure in the ESM setting. Theoretical results are demonstrated on an example in Section 5.

## 2. PRELIMINARIES AND PROBLEM FORMULATION

Consider a MIMO system described by a transfer function matrix  $G(s) \in \mathbb{R}^{m \times m}$ , and a controller  $\mathbb{R}(s) \in \mathbb{R}^{m \times m}$  in the standard feedback loop (Fig. 1); Necessary and sufficient closed-loop stability conditions are stipulated by the Generalized Nyquist Stability Theorem applied to the closed-loop characteristic polynomial (CLCP)

$$det F(s) = det[I + Q(s)]$$
(1)

where  $Q(s) = G(s)R(s) \in \mathbb{R}^{m \times m}$ .



Fig. 1. Standard feedback configuration

In the sequel, *D* denotes the standard Nyquist *D*-contour in the complex plane; Nyquist plot of g(s) is the image of the Nyquist contour under g(s); N[k, g(s)] is the number of anticlockwise encirclements of (k, j0) by the Nyquist plot of g(s). Characteristic functions of Q(s) are the set of *m* algebraic functions  $q_i(s)$ , i = 1,...,m given as

$$det[q_i(s)I_m - Q(s)] = 0 \qquad i = 1,...,m$$
(2)

Characteristic loci (CL) are the set of loci in the complex plane traced out by the characteristic functions of  $Q(s), \forall s \in D$ . The CLCP (1) expressed in terms of characteristic functions of Q(s) reads as follows

$$det \ F(s) = det[\ I + Q(s)] = \prod_{i=1}^{m} [\ I + q_i(s)]$$
(3)

*Theorem 1.* (Generalized Nyquist Stability Theorem) The closed-loop system in Fig. 1 is stable if and only if

a. det  $F(s) \neq 0$   $\forall s \in D$ 

b. 
$$N[0, det F(s)] = \sum_{i=1}^{m} N\{0, [1 + q_i(s)]\} = n_q$$
 (4)

where F(s) = (I + Q(s)) and  $n_q$  is the number of unstable poles of Q(s).

Let the uncertain plant be specified as a set  $\Pi$  of N transfer function matrices

$$\Pi = \{ G^{k}(s) \}, \ k = 1, 2, ..., N \text{ where } G^{k}(s) = \{ G^{k}_{ij}(s) \}_{m \times m}$$
(5)

The set of unstructured perturbations  $D_U$  is defined as follows

$$D_U := \{ E(j\omega) : \sigma_{max} [E(j\omega)] \le \ell(\omega), \\ \ell(\omega) = \max_k \sigma_{max} [E(j\omega)] \}$$
(6)

where  $\ell(\omega)$  is a scalar weight function on the norm-bounded perturbation  $\Delta(s) \in \mathbb{R}^{m \times m}$ ,  $\sigma_{max}[\Delta(j\omega)] \leq 1$  over the given frequency range,  $\sigma_{max}(\cdot)$  is the maximum singular value of (.); hence  $E(j\omega) = \ell(\omega)\Delta(j\omega)$ .

For unstructured uncertainty, the set  $\Pi$  can be generated by either additive ( $E_a$ ), multiplicative input ( $E_i$ ) or output ( $E_o$ ) uncertainties, or their inverse counterparts ( $E_{ia} E_{ii} E_{io}$ ) used for uncertainty associated with plant poles located in the closed right half-plane. Only the additive and inverse additive uncertainties will be addressed in detail; relations for other uncertainty forms can be derived by analogy.

Denote G(s) any member of  $\Pi$ ,  $G_0(s)$  the nominal model, and  $\ell_j(\omega)$  the scalar weight on a normalized perturbation. Individual uncertainty forms generate the related sets  $\Pi_j$  for j=a, ia.

Additive uncertainty:

$$\Pi_{a} := \{ G(s) : G(s) = G_{0}(s) + E_{a}(s), \\ E_{a}(j\omega) \leq \ell_{a}(\omega) \Delta(j\omega) \}$$

$$\ell_{a}(\omega) = \max_{k} \sigma_{max} [G^{k}(j\omega) - G_{0}(j\omega)], \ k = 1, 2, ..., N$$

$$(7)$$

Inverse additive uncertainty

$$\Pi_{ia} := \{ G(s) : G(s) = G_0(s) [I - E_{ia}(s)G_0(j\omega)]^{-1}, E_{ia}(j\omega) \le \ell_{ia}(\omega)\Delta(j\omega) \} \ell_{ia}(\omega) = \max_k \sigma_{max} \{ [G_0(j\omega)]^{-1} - [G^k(j\omega)]^{-1} \}, k = 1, 2, ..., N$$

$$(8)$$

The standard feedback loop with uncertain plant can be recast into the  $M - \Delta$  structure (Fig. 2) where  $\Delta(s) \in R^{m \times m}$  is the

norm-bounded complex perturbation. For the uncertainty forms (7), (8) the corresponding M(s) are respectively

$$M(s) = \ell_{a}(s)R(s)[I + G_{0}(s)R(s)]^{-1}$$
(9)

$$M(s) = \ell_{ia}(s) [I + G_0(s)R(s)]^{-1} G_0(s)$$
(10)



Fig. 2.  $M - \Delta$  structure

According to (Skogestad and Postlethwaite, 2005) if M(s) is stable (nominal stability) and the perturbation  $\Delta(s)$  is stable, then the  $M - \Delta$  system is stable for all  $\Delta(s) : \sigma_{max}(\Delta) \le 1$  if and only if

$$\sigma_{max}[M(j\omega)] < l , \ \forall \omega \tag{11}$$

Conservatism of the robust stability condition (19) can be relaxed by "structuring" the additive uncertainty to yield the additive affine-type uncertainty  $E_{af}(s)$  (Kozáková and Veselý, 2007; 2008)

$$E_{af}(s) = \sum_{i=1}^{p} G_i(s) q_i$$
(12)

where  $G_i(s) \in \mathbb{R}^{m \times m}$ , i=0,1, ..., p are stable matrices, p is the number of uncertainties defining  $2^p$  polytope vertices that correspond to individual perturbed models;  $q_i$  are polytope parameters. The related  $\Pi_{af}$  is

$$\Pi_{af} := \{ G(s) : G(s) = G_{0}(s) + E_{af}, \\ E_{af} = \sum_{i=1}^{p} G_{i}(s) q_{i}, \\ q_{i} \in \langle q_{i\min}, q_{i\max} \rangle, q_{i\min} + q_{i\max} = 0 \}$$
(13)

where  $G_0(s)$  is the "affine" nominal model. In the matrix form, individual plants from the set  $\Pi_{af}$  can be expressed as follows

$$G(s) = G_0(s) + QG_u(s) \tag{14}$$

where  $Q = [I_{q_1} \dots I_{q_p}]^T \in \mathbb{R}^{m \times (m \times p)}, I_{q_i} = q_i I_{m \times m},$  $G_u(s) = [G_1 \dots G_p]^T \in \mathbb{R}^{(m \times p) \times m}$ 

Similarly to previous uncertainty forms, the feedback loop with uncertain plant modeled using the additive affine type uncertainty in Fig. 3, can be recast into the  $M_{af} - Q$  structure with  $M_{af} = G_u R (I + G_0 R)^{-1} = G_u (I + RG_0)^{-1} R$ . Stability condition for the  $M_{af} - Q$  system is

$$\sigma_{max}(M_{af}Q) < 1, \tag{15}$$

under the assumption  $q_0 = |q_{imin}| = |q_{imax}|$ , (14) can further be modified to yield

$$\sigma_{max}(M_{af})q_0\sqrt{p} < 1 \tag{16}$$

$$w \rightarrow e \rightarrow R(s) \rightarrow G_0(s) \rightarrow y_4$$

$$u_4 \qquad y \rightarrow G_0(s) \rightarrow y \rightarrow g_1(s)$$

Fig. 3. Standard feedback loop with additive affine-type additive uncertainty

## 2.1 Problem Formulation

Consider an uncertain system with *m* subsystems given as a set of *N* transfer function matrices obtained as a result of identification in *N* working points of the plant operation range. Assume that the uncertain system be described by a nominal model  $G_0(s)$  and any unstructured uncertainty form (7), (8) or (12) where  $G_0(s)$  can be split as follows

$$G_0(s) = G_d(s) + G_m(s)$$
 (17)

where  $\forall s \in D$ 

$$G_d(s) = diag\{G_i(s)\}_{m \times m}, det G_d(s) \neq 0$$
  

$$G_m(s) = G_0(s) - G_d(s)$$

A decentralized controller (DC)

$$R(s) = diag\{R_i(s)\}_{m \times m}, \ det R(s) \neq 0$$
(18)

is to be designed to guarantee stability over the whole operating range of the plant specified by either (7), (8) or (13) (robust stability) and a specified performance of the nominal model (nominal performance), Fig. 4.

To solve the this problem, a frequency domain robust decentralized controller design technique has been developed (Kozáková and Veselý, 2009; Kozáková et. al., 2009b); the core of it is the Equivalent Subsystems Method (ESM).

## 3. EQUIVALENT SUBSYSTEMS METHOD

The Equivalent Subsystems Method (ESM) is a Nyquistbased DC design method for stability and guaranteed performance of the full system. According to it, independent local controller designs are carried out for the so-called equivalent subsystems that are actually Nyquist plots of decoupled subsystems, shaped by one selected characteristic locus of the interactions matrix. If local controllers of equivalent subsystems are independently tuned for stability specified feasible performance, the and resulting decentralized controller guarantees for the full system the same performance as specified for equivalent subsystems. ESM used in the design for robustness (Kozáková et al., 2009b) allows to consider the *full* nominal model, thus reducing conservatism of resulting robust stability conditions.

The key idea behind the method is factorization of the CLCP (1) in terms of the split nominal system (17) under the decentralized controller (18)

$$det F(s) = det[R^{-1}(s) + G_d(s) + G_m(s)] det R(s)$$
(19)

Denote the sum of diagonal matrices in the first bracketed term as follows

$$R^{-1}(s) + G_d(s) = P(s)$$
(20)

where  $P(s) = diag\{ p_i(s) \}_{m \times m}$ .



Fig. 4. Standard feedback loop under decentralized controller

If choosing the diagonal matrix  $P(s) = diag\{p_k(s)\}_{m \times m}$ with identical entries so as to "counterbalance" interactions  $G_m(s)$  then, according to (2), the characteristic equation corresponding to the first r.h.s. term in (19) defines the *k*-th of the *m* characteristic functions of  $[-G_m(s)]$  denoted  $g_i(s), i = 1, 2, ..., m$ ; thus

$$det[P(s) + G_m(s)] = det[p_k I + G_m] =$$

$$= \prod_{i=1}^{m} [-g_k(s) + g_i(s)] = 0, \quad k = 1, 2, ...m$$
(21)

According to the Cayley-Hamilton theorem from the viewpoint of stability, the interactions matrix  $G_m(s)$  can be replaced by [-P(s)] yielding the important relationship

$$det[I + G(s)R(s)] = det\{I + [G_d(s) + G_m(s)]R(s)\} = = det[R^{-1}(s) + G_d(s) - P(s)] det R(s) = (22) = det[I + G^{eq}(s)R(s)]$$

where

$$G^{eq}(s) = diag\{G_i^{eq}(s)\}_{m \times m}$$
(23)

is a diagonal matrix of *m* equivalent subsystems

$$G_i^{eq}(s) = G_i(s) + g_k(s), \quad i = 1, 2, ..., m;$$
 (24)

As all matrices are diagonal, on subsystems level (22) breaks down into *m* equivalent characteristic polynomials

$$CLCP_i^{eq}(s) = 1 + R_i(s)G_i^{eq}(s) \qquad i = 1, 2, ..., m$$
 (25)

Considering (21)-(25), stability conditions of Theorem 1 modify as follows:

## Corollary 1.

The closed-loop in Fig. 4 comprising the system (17) and the decentralized controller (18) is stable if and only if there exists a diagonal matrix  $P(s) = diag\{ p_i(s) \}_{m \times m}$  such that 1.  $det[ p_k(s) I + G_m ] = 0$ , for fixed  $k \in \{1, ..., m\}$ ;

2. all equivalent characteristic polynomials (25) have roots with  $Re\{s\} < 0$ ;

3. 
$$N[0, det F(s)] = n_q$$

where det F(s) = I + G(s)R(s) and  $n_q$  is number of open

loop poles with  $Re\{s\} > 0$ .

The design technique resulting from *Corollary 1* enables to design the decentralized controller through designing local controllers for independent equivalent subsystems using any SISO frequency-domain design method, e.g. the Neymark D-partition method (Kozáková et al. 2009b), standard Bode diagram design (Bucz et al., 2010) etc.

In the originally developed ESM version (Kozáková et. al. 2009a; 2009b) it was proved that local controllers independently tuned for stability and a specified feasible degree of stability of equivalent subsystems constitute the decentralized controller guaranteeing the same degree of stability plant-wide. In (Kozáková et al. 2010) the performance specification applied in ESM was based on the relationship between phase margins of equivalent subsystems and the maximum overshoot. This performance specification is further developed towards robust stability.

## 4. ROBUST DECENTRALIZED CONTROLLER DESIGN

This section deals with implementation of the ESM in the decentralized controller design for robust stability and nominal performance applicable for uncertain systems described as a set of transfer function matrices. The nominal model can be calculated either as the mean value parameter model (Skogestad and Postlethwaite, 2005), or the "affine" model, obtained within the procedure for calculating the affine-type additive uncertainty (Kozáková and Veselý, 2007; 2008). Unlike the standard robust approach to DC design in which the diagonal model as the nominal one (interactions are included in the uncertainty), the ESM method applied in the design for nominal performance allows to consider the *full* nominal model. Model uncertainty is described by any unstructured uncertainty form (7), (8) or (13).

In (Kozáková and Veselý, 2008; 2009; Kozáková et al. 2009a) a two-stage robust DC design methodology was proposed based on ESM and fulfillment of the M- $\Delta$  structure stability conditions. The direct DC design for robust stability and nominal performance is the main result of this paper.

# 4.1 Direct decentralized controller design for robust stability and nominal performance

If the robust stability conditions (11) or (16) are directly integrated in the ESM, local controllers of equivalent subsystems are designed already with regard to robust stability. A suitable performance specification is the maximum peak of the complementary sensitivity  $M_T$  related to maximum overshoot in the full system; in equivalent subsystems it can be translated into lower bounds of phase margins according to (26) (Skogestad and Postlethwaite, 2005)

$$PM \ge 2 \arcsin\left(\frac{1}{2M_T}\right) \ge \frac{1}{M_T} [rad]$$
 (26)

where PM is the phase margin, and  $M_T$  is the maximum peak of the complementary sensitivity T(s)

$$T(s) = G(s)R(s)[I + G(s)R(s)]^{-1}$$
(27)

As for MIMO systems

$$M_T = \sigma_{max}(T), \tag{28}$$

the upper bound for the nominal complementary sensitivity  $T_0(s) = G_0(s)R(s)[I + G_0(s)R(s)]^{-1}$  can be derived by substituting into (1) the uncertain system model (additive uncertainty is considered in the following development) where  $G_0(s)$  is the nominal model:

$$det[I + (G_0 + \ell_a \Delta)R] =$$

$$= det(I + G_0 R) det[I + \ell_a \Delta R(I + G_0 R)^{-1}]$$
(29)

where the first term on the r.h.s. of is the CLCP of the nominal system that corresponds to the CLCP<sup>eq</sup> according to (22); condition for stability of the second term is determined using the small gain theorem. Hence the uncertain system is stable if and only if the nominal closed loop is stable and

$$\left\|\ell_a \Delta R (I + G_o R)^{-1}\right\| < 1 \tag{30}$$

Considering the spectral norm and the singular value properties, (30) can readily be manipulated to yield the final condition (33). Bounds for other uncertainty forms can be derived by analogy.

In case of inverse uncertainty forms, robustness bounds are obtained in terms of the maximum peak of the sensitivity  $M_S = \sigma_{max}(S)$  where

$$S(s) = [I + G(s)R(s)]^{-1}$$
(31)

and using the lower bounds for PM in the form (Skogestad and Postlethwaite, 2005)

$$PM \ge 2 \arcsin\left(\frac{1}{2M_S}\right) \ge \frac{1}{M_S} [rad]$$
(32)

Upper bounds for  $\sigma_{max}[T_0(j\omega)]$  or  $\sigma_{max}[S_0(j\omega)]$  for additive-type uncertainties are summarized below.

Additive uncertainty:

$$\sigma_{max}[T_0(j\omega)] < \frac{\sigma_{min}[G_0(j\omega)]}{|\ell_a(\omega)|} = L_A(\omega) \quad \forall \omega$$
(33)

Additive affine-type uncertainty:

$$\sigma_{max}[T_0(j\omega)] < \frac{1}{q_0\sqrt{p}} \frac{\sigma_{min}[G_0(j\omega)]}{\sigma_{max}[G_u(j\omega)]} = L_{AF}(\omega) \quad \forall \omega$$
(34)

Inverse additive uncertainty:

$$\sigma_{max}[S_{0}(j\omega)] < \frac{1}{\left|\ell_{ia}(\omega)\right| \sigma_{max}[G_{0}(j\omega)]} = L_{IAF}(\omega), \ \forall \omega$$
(35)

Any of the derived bounds (33), (34) or (35) for the nominal model can directly be implemented in the ESM due to the fact that performance achieved in equivalent subsystems is simultaneously guaranteed for the full system. The main benefit of this approach is the possibility to find the maximum overshoot of the full system in terms of  $\sigma_{max}(T_0)$  or  $\sigma_{max}(S_0)$  for which robust stability is guaranteed, translate it into corresponding minimum phase margins required in equivalent subsystems and finally design local controllers for individual single input – single output equivalent subsystems independently. In this case the recommended design method for the ESM setting is the Bode diagram design.

Considering performance just in terms of  $M_T$  or  $M_S$  is not sufficient, the speed of response has to be considered as well which leads to considering the bandwidth frequency of the closed-loop system as well. In general, a large bandwidth corresponds to a smaller rise time, since high frequency signals are more easily passed on to the outputs. If the bandwidth is small, the time response will generally be slow and the system will usually be more robust. The gain crossover frequency  $\omega_0$  is frequently used to define closedloop bandwidth.

The Bode plot design procedure with regard both to the required phase margin and the required bandwidth is demonstrated in the next section on a simple example of SISO robust PI(D) controller design with guaranteed overshoot and settling time of transients.

## 5. EXAMPLE

Consider a SISO plant given by 3 transfer functions corresponding to its three different working points:

$$G_1(s) = \frac{0.5s + 1.2}{50s^2 + 15s + 1} \qquad G_2(s) = \frac{0.5s + 1.3}{45s^2 + 15s + 1.3}$$
$$G_3(s) = \frac{0.5s + 0.7}{53s^2 + 15s + 0.8}$$

Next calculations include the nominal model (as a mean value parameter one), the additive uncertainty  $\ell_a(\omega)$  according to (7) and the upper bound for the nominal complementary sensitivity  $L_A(\omega)$  according to (33).



Fig. 5. Robustness bound  $L_A(\omega)$  for nominal complementary sensitivity

The least value  $min_{\omega}L_A(\omega) = M_T$  is chosen to generate the minimum required phase margin guaranteeing robust stability; in our case  $M_T = 2.66$  corresponds to  $PM_{min} = 21.6^0$ .



Fig. 6. Bode plot of the nominal system  $G_0(s)$ 

The PI(D) controller design is carried out with regard to both the required phase margin and the required bandwidth, the latter being related to the settling time according to the relation

$$\frac{\pi}{t_s} < \omega_0 < \frac{4\pi}{t_s} \tag{36}$$

The design philosophy is as follows:

After specifying the required  $PM_{req}$  and settling time  $t_S$ ,  $\omega_0$  is calculated from (36) and the  $PM(\omega_0)$  is read off. If  $PM(\omega_0) > PM_{req}$ , a PI controller is designed.

If  $PM(\omega_0) < PM_{req}$ , a PD controller  $G_{PD}(s) = l + K_D s$  is to be designed first to provide  $PM_{req}(\omega_0)$  and then a PI controller  $G_{PI}(s) = K_P + \frac{K_I}{s}$  is designed. The resulting PID controller is a combination of both  $G_{PID}(s) = (K_P + \frac{K_I}{s})(l + K_D s).$ 

Consider the required  $t_s = 60s$  which corresponds to  $\omega_0 = 0.1309s^{-1}$ . From the Bode plot of the uncompensated system in Fig. 6 and  $PM_{req}(\omega_0) = 49^0$  it is obvious that a PI controller will be sufficient, its resulting parameters are  $G_{PI} = 0.4602 + \frac{0.053}{s}$ . Bode plot of the compensated system in Fig. 7 proves achieving of the required parameters. Closed-loop step responses of the nominal model and models in individual working points are in Fig. 8 and Fig. 9, respectively. Stability robustness is verified in Fig. 10.



Fig. 7 Bode plot of the compensated system (PI controller)



Fig. 8 Closed-loop step response with the nominal model



Fig. 9 Closed-loop step responses in individual working points

## CONCLUSIONS

The paper deals with the decentralized PID controller design for robust stability and plant-wide nominal performance within the setting of the Equivalent Subsystems Method (ESM). The nominal performance for the full system specified in terms of maximum overshoot is achieved through phase margins specified for equivalent subsystems. The design methodology per se uses the Bode plots and is therefore applicable also for SISO systems. The design procedure is illustrated by an example.



Fig. 10 Robust stability verification

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