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Solution of a Robust Stabilization Problem Using YALMIP and Robust Control Toolboxes

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Abstract: The aim of this paper is to compare two toolboxes used for solving the robust stabilization problem. Robust static output feedback controller was designed for a continuous stirred tank reactor (CSTR) in which two parallel exothermic reactions take place. The reactor is a system with parametric uncertainty and multiple steady states. The problem of robust controller design was converted to a problem of solution of linear matrix inequalities (LMIs) and computationally simple non-iterative and iterative algorithms can be used for controller tuning. The MATLAB–Simulink environment enables to compare the results of the YALMIP and the Robust Control toolboxes.

Keywords: chemical reactor, multiple steady states, robust stabilization, static output feedback

1. INTRODUCTION

Continuous-time stirred tank reactors (CSTRs) belong to the most important plants in chemical and food industries. From the control viewpoint, CSTRs are very interesting systems, because of their potential safety problems and the possibility of exotic behavior such as multiple steady states, see e.g. Molnár et al. (2002). Furthermore, operation of chemical reactors is corrupted by various uncertainties. Some of them arise from varying or not exactly known parameters, as e.g. reaction rate constants, reaction enthalpies, heat transfer coefficients, etc. Operating points of reactors change in other cases. All these uncertainties can cause poor performance or even instability of closed-loop control systems. Application of robust control approach is one way for overcoming all these problems, as it is shown e.g. in Alvarez-Ramirez and Femat (1999), Gerhard et al. (2004), Bakošová et al. (2005), Tlacuahuac et al. (2005) and others. From the viewpoint of safety operation or in the case when the unstable steady state coincides with the point that yields the maximum reaction rate at a prescribed temperature, it is necessary to control CSTRs into the prescribed open-loop unstable steady state (Bakošová and Oravec (2010), Bakošová et al. (2009), Bakošová et al. (2006), Puna et al. (2006)).

One of solved problems in robust control theory is the problem of robust static output feedback control (Dong and Yang (2007), Iwasaki et al. (1994), Syrmos et al. (1997) and references therein). This approach can be successfully used for solving the problem of robust stabilization of CSTRs. For obtaining robust stabilizing controllers, the non-iterative and iterative algorithms can be applied (Veselý (2002)).

In this paper, the problem of robust stabilization of a CSTR is solved. The conditions for robust stabilization are formulated in the form of linear matrix inequalities (LMIs). Solution of LMIs represents a convex optimization problem that has been solved in the MATLAB environment by Robust Control toolbox (Balas et al. (2006)) and YALMIP toolbox (Löfberg (2004), Kvasnica and Fikar (2010)) with solver SeDuMi (Henrion and Lasserre (2003)).

2. CONTROLLED CSTR

The controlled reactor is a continuous-time stirred tank reactor with two first order irreversible parallel exothermic reactions according to the scheme A $\xrightarrow{k_1}$B, A $\xrightarrow{k_2}$ C, where B is the main product and C is the side product. Chemical reactions are performed in a reaction vessel and reaction heat is removed from the reactor by coolant in a reactor jacket. Because of the exponential dependency of reactant concentrations on the temperature of the reaction mixture known as the Arrhenius equation (Molnár et al. (2002)), it is supposed that it is not necessary to control directly concentrations. The multivariable controller is used in order to achieve control of the reaction mixture temperature in the reaction vessel and the coolant temperature in the jacket. Control inputs are flow rates of reaction mixture and coolant. Parameters and inputs of the considered CSTR (Bakošová et al. (2006)) are shown in Table 1 and Table 2.

Model uncertainties of the over described reactor follow from the fact that there are four only approximately known physical parameters in the reactor, which values are shown in Table 3. Here, $\Delta_r H_1$, $\Delta_r H_2$ are reaction enthalpies of the chemical reactions and $k_{\infty 1}$, $k_{\infty 2}$ are

Table 1. Parameters of CSTR

Variable	Value	Unit
V	0.23	m^3
V_C	0.21	m^3
ρ	1020	kg m^{-3}
ρ_C	998	kg m^{-3}
c_P	4.02	kJ $kg^{-1}K^{-1}$
c_{P_C}	4.182	kJ $kg^{-1}K^{-1}$
A	1.51	m^2
α	42.8	kJ $min^{-1}m^{-2}K^{-1}$
$g_1 = E_1/R$	9850	K
$g_2 = E_2/R$	22019	K

Table 2. Steady-state inputs of CSTR

Variable	Value	Unit
$c_{A,0}$	0.0824	kmol m^{-3}
$c_{B,0}$	0	kmol m^{-3}
T_0	310	K
$T_{C,0}$	288	K
q^s	0.015	$m^3 min^{-1}$
q_C^s	0.004	$m^3 min^{-1}$

pre-exponential factors in the reaction rate constants. The nominal values of uncertain parameters are considered to be mean values of given intervals. These uncertainties represent parametric uncertainties.

Table 3. Uncertain parameters of CSTR

Variable	Minimal value	Maximal value	Unit
$\Delta_r H_1$	-8.8×10^4	-8.4×10^4	kJ $kmol^{-1}$ min^{-1}
$\Delta_r H_2$	-5.7×10^4	-5.3×10^4	kJ $kmol^{-1}$ min^{-1}
$k_{\infty 1}$	1.5×10^{11}	1.6×10^{11}	min^{-1}
$k_{\infty 2}$	4.95×10^{26}	12.15×10^{26}	min^{-1}

It follows from the steady-state analysis that the reactor has three steady states, two of them are stable and one is unstable. The situation for the nominal model is shown in Figure 1, where the curve Q_{GEN} (red line) represents the heat generated by the reactions and the line Q_{OUT} (blue line) represents the heat withdrawn from the reactor. The steady-state operating points of the reactor are points, where the curve and the line intersect. The steady states are stable if the slope of the cooling line is higher then the slope of the heat generated curve. This condition is satisfied in the steady states at the temperatures $T = 308.4$ K and $T = 352.6$ K, and it is not satisfied in the steady state at $T = 338.4$ K. The steady-state behavior of the chemical reactor is similar for all vertex systems, which are obtained for all combinations of minimal and maximal values of uncertain parameters. The maximal concentration of the main product B (red line) is always obtained when the reactor operates in the unstable steady state as it is shown in Figure 2 (Bakošová et al. (2006)).

Linearized mathematical model of the reactor has been derived under the assumption that the control inputs are the reactant flow rate q and the coolant flow rate q_C, the controlled outputs are the reaction mixture temperature T and the coolant temperature T_C and the operating point of the reactor is its open-loop unstable steady state. Then the linearized model of the CSTR is in the form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t), \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0$$
$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \qquad (1)$$

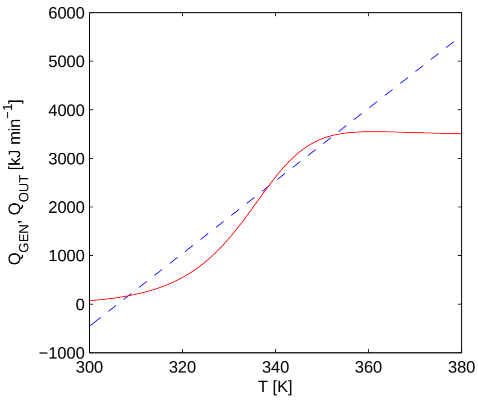

Fig. 1. Steady states of CSTR with nominal values of uncertain parameters

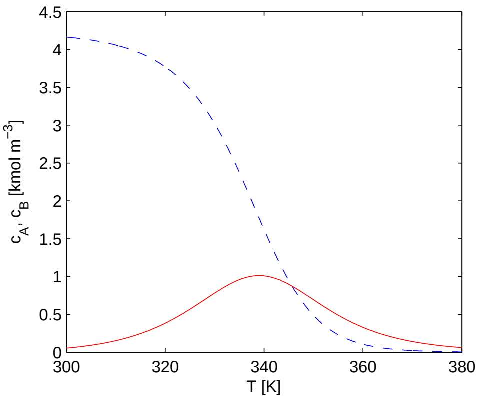

Fig. 2. Concentration of components A and B in dependence on the reaction mixture temperature – nominal model

where again $\boldsymbol{x}(t) \in R^n$ is the state, $\boldsymbol{u}(t) \in R^m$ the control, $\boldsymbol{y}(t) \in R^r$ the output. Matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are in the form

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & 0 & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix} \qquad (2)$$

$$\boldsymbol{B} = \begin{pmatrix} b_{11} & 0 \\ b_{21} & 0 \\ b_{31} & 0 \\ 0 & b_{42} \end{pmatrix} \qquad (3)$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (4)$$

Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ have varying coefficients as according to the values of uncertain parameters steady states of the reactor vary. For coefficients of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ see Table 4.

For all combinations of boundary values of 4 uncertain parameters, we have obtained $2^4 = 16$ linearized mathematical models with matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$. These systems represent vertices of an uncertain polytopic system. All these vertices are unstable systems as between the eigenvalues of $\boldsymbol{A}_i$, $i = 1, ..., 16$, are also positive eigenvalues. Unstable is also the linearized nominal model (Bakošová et al. (2006)).

Table 4. Matrices parameters

Parameter	Value
a_{11}	$-\left(\frac{q^s}{V} + k_1^s + k_2^s\right)$
a_{13}	$-\frac{c_A^s(k_1^s g_1 + k_2^s g_2)}{(T^s)^2}$
a_{21}	k_1^s
a_{22}	$-\frac{q^s}{V}$
a_{23}	$\frac{k_1^s \Delta_r H_1 + k_2^s \Delta_r H_1}{q^s c_P}$
a_{31}	$-\left(\frac{q^s}{V} + \frac{\alpha A}{V \rho c_p} + \frac{c_A^s(k_1^s g_1 \Delta_r H_1 + k_2^s g_2 \Delta_r H_1)}{\rho c_P (T^s)^2}\right)$
a_{34}	$\frac{\alpha A}{V \rho c_P}$
a_{43}	$\frac{\alpha A}{V_C \rho_C c_{P_C}}$
a_{44}	$-\left(\frac{q_C^s}{V_C} + \frac{\alpha A}{V_C \rho_C c_C}\right)$
b_{11}	$\frac{c_{A,0} - c_A^s}{V}$
b_{21}	$\frac{c_{B,0} - c_B^s}{V}$
b_{31}	$\frac{T_0 - T^s}{V}$
b_{42}	$\frac{T_{C,0} - T_C^s}{V}$

3. ROBUST STATIC OUTPUT FEEDBACK STABILIZATION OF CSTR

Design of a robust static output feedback controller is based on having a linear time-invariant state space model (1) of the controlled system. For the system (1), it is necessary to find a static output feedback $\boldsymbol{u}(t) = \boldsymbol{F}\boldsymbol{y}(t)$. Using this static output feedback we obtain an uncertain polytopic closed-loop system

$$\dot{\boldsymbol{x}}(t) = [\boldsymbol{A} + \boldsymbol{B}\boldsymbol{F}\boldsymbol{C}]\,\boldsymbol{x}(t) = \boldsymbol{A}_{CL}\boldsymbol{x}(t) \tag{5}$$

The system (1) is simultaneously static output feedback stabilizable with guaranteed cost

$$\int_0^\infty \left(\boldsymbol{x}(t)^T \boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}(t)^T \boldsymbol{R}\mathbf{u}(t)\right) dt \le \boldsymbol{x}_0^T \boldsymbol{P}\boldsymbol{x}_0 = J^*$$
$$\boldsymbol{P} > 0 \tag{6}$$

if there exist matrices $\boldsymbol{P} > 0, \boldsymbol{Q} > 0, \boldsymbol{R} > 0$ and a matrix $\boldsymbol{F}$ such that the following inequalities hold (Veselý (2002))

$$\boldsymbol{\Omega}_i^T \boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Omega}_i + \boldsymbol{Q} + \boldsymbol{C}_i^T \boldsymbol{F}^T \boldsymbol{R}\boldsymbol{F}\boldsymbol{C}_i < 0 \qquad i = 1, \ldots, N \tag{7}$$

where

$$\boldsymbol{\Omega}_i = \boldsymbol{A}_i + \boldsymbol{B}_i \boldsymbol{F}\boldsymbol{C}_i \tag{8}$$

The system (1) is simultaneously static output feedback stabilizable with guaranteed cost (6) also if there exist matrices $\boldsymbol{P} > 0, \boldsymbol{Q} > 0, \boldsymbol{R} > 0$ and a matrix $\boldsymbol{F}$ such that the inequalities hold (Veselý (2002))

$$\boldsymbol{A}_i^T \boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i - \boldsymbol{P}\boldsymbol{B}_i \boldsymbol{R}^{-1} \boldsymbol{B}_i^T \boldsymbol{P} - \boldsymbol{\Theta}_i + \boldsymbol{Q} \le 0 \qquad i = 1, \ldots, N \tag{9}$$

where

$$\boldsymbol{\Theta}_i = \boldsymbol{C}_i^T \boldsymbol{F}^T \boldsymbol{R}\boldsymbol{F}\boldsymbol{C}_i \tag{10}$$

and also the inequalities hold

$$\lambda_i \phi_i^{-1} \lambda_i^T - \boldsymbol{R} \le 0 \qquad i = 1, \ldots, N \tag{11}$$

where

$$\lambda_i = \boldsymbol{B}_i^T \boldsymbol{P} + \boldsymbol{R}\boldsymbol{F}\boldsymbol{C}_i \tag{12}$$

$$\phi_i = -(\boldsymbol{A}_i^T \boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i - \boldsymbol{P}\boldsymbol{B}_i \boldsymbol{R}^{-1} \boldsymbol{B}_i^T \boldsymbol{P} - \boldsymbol{\Theta}_i + \boldsymbol{Q}) \tag{13}$$

The non-iterative and iterative procedures for simultaneous static output feedback stabilization of the system (1) with guaranteed cost (6) are based on statements formulated above (Veselý (2002)).

3.1 Non - iterative algorithm

Using the Schur complement formula and defining $\mathbf{S}$ is equal to $\boldsymbol{P}^{-1}$ and considering $\boldsymbol{\Theta}_i$ is equal zero, the inequality (9) is transformed to the following LMIs

$$\begin{bmatrix} \boldsymbol{S}\boldsymbol{A}_i^T + \boldsymbol{A}_i \boldsymbol{S} - \boldsymbol{B}_i \boldsymbol{R}^{-1} \boldsymbol{B}_i^T & \boldsymbol{S}\sqrt{\boldsymbol{Q}} \\ \sqrt{\boldsymbol{Q}}\boldsymbol{S} & -\boldsymbol{I} \end{bmatrix} \le 0$$
$$\gamma \boldsymbol{I} < \boldsymbol{S}, \quad i = 1, \ldots, N \tag{14}$$

where $\gamma > 0$ is any positive constant.

Using $\boldsymbol{P} = \boldsymbol{S}^{-1}$, the inequality (11) can be rewritten to the following LMIs

$$\begin{bmatrix} -\boldsymbol{R} & \boldsymbol{B}_i^T \boldsymbol{P} + \boldsymbol{R}\boldsymbol{F}\boldsymbol{C}_i \\ (\boldsymbol{B}_i^T \boldsymbol{P} + \boldsymbol{R}\boldsymbol{F}\boldsymbol{C}_i)^T & -\phi_i \end{bmatrix} \le 0 \qquad i = 1, \ldots, N \tag{15}$$

The non-iterative algorithm for static output simultaneous stabilization of the system (1) with the guaranteed cost (6) is following (Veselý (2002)).

(1) Set parameter γ and required values of the weight matrices $\boldsymbol{Q}, \boldsymbol{R}$ in the cost function (6).
(2) Compute $\boldsymbol{S} = \boldsymbol{S}^T > 0$ from the inequalities (14). If the solution of (14) is not feasible, the system (1) is not simultaneously stabilizable by static output feedback.
(3) Set $\boldsymbol{P} := \boldsymbol{S}^{-1}$.
(4) Compute $\boldsymbol{F}$ from the inequalities (15). If the solution of (15) is not feasible, the closed-loop system (5) is not quadratically stable with guaranteed cost. Then change $\boldsymbol{Q}, \boldsymbol{R}$ or γ in order to find feasible solutions.
(5) If the solutions of (14), (15) are feasible, then the system (1) is simultaneously stabilizable and the system (5) is quadratically stable with guaranteed cost control algorithm $\boldsymbol{u}^*(t) = \boldsymbol{F}\boldsymbol{y}(t)$ and $J^* = \boldsymbol{x}_0^T \boldsymbol{P}\boldsymbol{x}_0$ is the guaranteed cost.

3.2 Iterative algorithm

Using the Schur complement formula and defining $\boldsymbol{S}$ is equal to $\boldsymbol{P}^{-1}$, the inequality (7) is transformed to the following LMIs

$$\begin{bmatrix} \boldsymbol{S}_k \boldsymbol{A}_i^T + \boldsymbol{A}_i \boldsymbol{S}_k - \boldsymbol{B}_i \boldsymbol{R}^{-1} \boldsymbol{B}_i^T & \boldsymbol{S}_k \sqrt{\boldsymbol{\Psi}_i} \\ \sqrt{\boldsymbol{\Psi}_i}\boldsymbol{S}_k & -\mathbf{I} \end{bmatrix} \le 0$$
$$\gamma \boldsymbol{I} < \boldsymbol{S}_k, \quad i = 1, \ldots, N \tag{16}$$

where

$$\boldsymbol{\Psi}_i = \boldsymbol{C}_i^T \boldsymbol{F}_{k-1}^T \boldsymbol{R} \boldsymbol{F}_{k-1} \boldsymbol{C}_i + \boldsymbol{Q} \quad (17)$$

and $\gamma > 0$ is any positive constant.

Using $\boldsymbol{P} = \boldsymbol{S}^{-1}$, the inequality (11) can be rewritten to the following LMIs

$$\begin{bmatrix} -\boldsymbol{R} & \boldsymbol{B}_i^T \boldsymbol{P} + \boldsymbol{R}\boldsymbol{F}_i \boldsymbol{C}_i & \boldsymbol{0} \\ \boldsymbol{P}_k \boldsymbol{B}_i + \boldsymbol{C}_i^T \boldsymbol{F}_k^T \boldsymbol{R} & \varphi_i & \boldsymbol{C}_i \boldsymbol{F}_k^T \\ \boldsymbol{0} & \boldsymbol{F}_k \boldsymbol{C}_i & \boldsymbol{R} \end{bmatrix} \leq 0$$
$$i = 1, \ldots, N \quad (18)$$

where

$$\varphi_i = \boldsymbol{A}_i^T \boldsymbol{P}_k + \boldsymbol{P}_k \boldsymbol{A}_i^T - \boldsymbol{P}_k \boldsymbol{B}_i^T \boldsymbol{R}^{-1} \boldsymbol{B}_i^T \boldsymbol{P}_k + \boldsymbol{Q} \quad (19)$$

The iterative algorithm for static output simultaneous stabilization of the system (1) with the guaranteed cost (6) is following (Veselý (2002)).

(1) Set parameter γ and required values of the weight matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ due to the cost function (6).
(2) Set $k := 0$ and initial value of matrix $\boldsymbol{F}_0$.
(3) Set $k := k + 1$.
(4) Compute $\boldsymbol{S}_k = \boldsymbol{S}_k^T > 0$ from the inequalities (16). If the solution of (16) is not feasible, the system (18) is not simultaneously stabilizable by static output feedback.
(5) Set $\boldsymbol{P}_k := \boldsymbol{S}_k^{-1}$.
(6) Compute $\boldsymbol{F}_k$ from the inequalities (18). If the solution of (18) is not feasible, the closed-loop system (5) is not quadratically stable with guaranteed cost. Then change $\boldsymbol{Q}$, $\boldsymbol{R}$ or γ in order to find feasible solutions.
(7) If $||\boldsymbol{F}_k - \boldsymbol{F}_{k-1}|| \leq tolerance$ then stop else go to the *Step* 3.
(8) If the solutions of (16), (18) are feasible, then the system (1) is simultaneously stabilizable and the system (5) is quadratically stable with guaranteed cost control algorithm $\boldsymbol{u}^*(t) = \boldsymbol{F}\boldsymbol{y}(t)$ and $J^* = \boldsymbol{x}_0^T \boldsymbol{P} \boldsymbol{x}_0$ is the guaranteed cost.

4. CASES OF ROBUST CONTROLLER DESIGN

Solving presented algorithms represents the feasibility problem of convex optimization. The MATLAB environment enables to solve this problem by Robust Control toolbox and YALMIP toolbox with solver SeDuMi. The Robust Control toolbox uses function `setlmis` to initialize the LMI generating. Function `lmivar` enables to define the properties of optimization variable. Function `getlmis` generates LMI in the form required for processing by the function `feasp`. This function enables to solve the LMI feasibility optimization problem. The YALMIP uses function `sdpvar` to set the properties of optimization variable. The constraints are set simple by using the parentheses in the form `[ expression ]` . Function `solvesdp` enables to solve the optimization problem. To obtain the calculated value, the function `double` can be used.

For robust controller design the above presented non-iterative algorithm has been applied. The values of matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ in the cost function (6) and parameter γ used for controller tuning are shown in Table 5.

Table 5. Parameters for controller design

Cost function	Q	R	γ
1	$\begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{pmatrix}$	$\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$	0.001
2	$\begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{pmatrix}$	$\begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$	0.001

Due to the various weight matrices in the cost function (6) and using different MATLAB toolboxes, the four different cases of controller tuning have been obtained (Table 6). The following initial conditions of system (1) have been considered (Bakošová and Oravec (2010))

$$\boldsymbol{x}_0 = (\Delta c_A, \Delta c_B, \Delta T, \Delta T_C)^T \quad (20)$$

$$\begin{aligned} \boldsymbol{x}_0 &= (2.1210, 0.8644, 335.4726, 325.7271)^T - \\ &-(1.8614, 1.0113, 338.4080, 328.0599)^T = \\ &= (0.2596, -0.1469, -2.9354, -2.3328)^T \end{aligned} \quad (21)$$

Table 6. Cases of the robust controller design

Case	Method	Cost function	Used toolbox
1	non-iterative	1	*Robust Control*
2	non-iterative	1	YALMIP
3	non-iterative	2	*Robust Control*
4	non-iterative	2	YALMIP

5. RESULTS AND DISCUSSION

Four different controllers (F_1–F_4) for considered system (1) have been designed for various cases of the robust stabilization controller tuning (Table 6). The designed controllers are shown in Table 7. In this table are shown the maximal evaluated values of cost function J in comparison to guaranteed values of the cost function J^* for each designed controller. These values have been evaluated for considered initial values (21). The maximal eigenvalue of all uncertain systems (EV_{max}) have been calculated. The time measured in seconds needed for each LMI solving procedure is shown in column t_{CPU}. These data have been evaluated by computer with 3.20 GHz CPU and memory 4 GB RAM.

Table 7. Properties of the designed robust static output feedback controllers

Case	Controller F_c	J^*	J	EV_{max}	t_{CPU}
1	$\begin{bmatrix} 0.0456 & 0.0294 \\ 0.0569 & 0.0496 \end{bmatrix}$	0.2765	0.1286	−0.016	$\begin{bmatrix} 0.08 \\ 0.05 \end{bmatrix}$
2	$\begin{bmatrix} 0.0026 & -0.0026 \\ 0.0179 & 0.0160 \end{bmatrix}$	0.3221	0.2151	−0.036	$\begin{bmatrix} 0.58 \\ 0.25 \end{bmatrix}$
3	$\begin{bmatrix} 0.0333 & 0.0170 \\ 0.0261 & 0.0189 \end{bmatrix}$	0.7340	0.2638	−0.020	$\begin{bmatrix} 0.08 \\ 0.05 \end{bmatrix}$
4	$\begin{bmatrix} 0.0028 & -0.0029 \\ 0.0020 & 0.0005 \end{bmatrix}$	0.5546	0.4158	−0.033	$\begin{bmatrix} 0.51 \\ 0.47 \end{bmatrix}$

All the designed controllers assure the lower value of cost function J than the guaranteed value J^* as the theory predicts. All designed controllers guarantee the negative value of maximal eigenvalue for all uncertain closed–loop systems. The lower values of cost function J have been obtained for the controllers tuned by Robust Control toolbox.

In general, YALMIP toolbox needs more CPU time for solving the optimization problem than Robust Control toolbox. On the other hand, the YALMIP toolbox offers more comfortable environment. For system (1) and all the controllers F_1–F_4, closed–loop behaviours have been generated with corresponding control inputs using MATLAB–Simulink environment. In the Figures 3 – 12 are shown performances of the closed–loop system controlled using controllers designed by non-iterative algorithm.

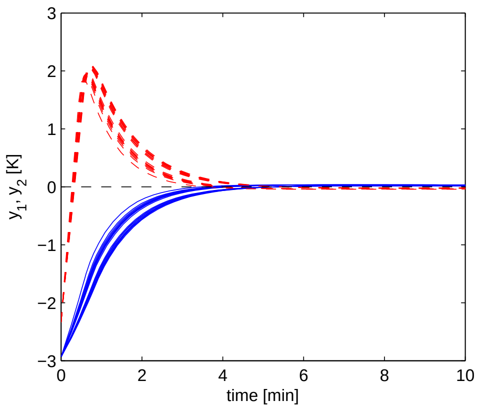

Fig. 3. Closed-loop behaviour of CSTR using the robust controller F_1

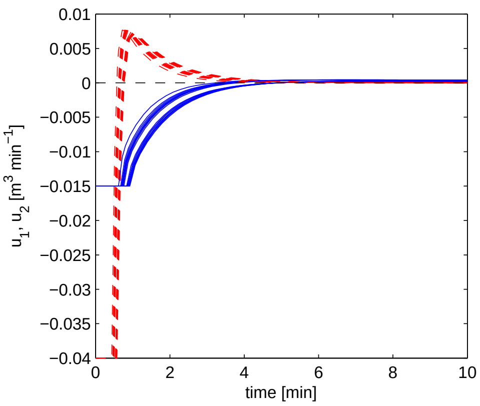

Fig. 4. Control inputs generated by the controller F_1

The controllers F_1, F_2 have been designed for the *cost function 1* and the controllers F_3, F_4 have been tuned for the *cost function 2* (Table 6). As can be seen in the Figure 3 and Figure 5 the control performances of the CSTR assured by the controllers F_1, F_2 obtain similar *overshoot* (Bakošová et al. (2003)). The Figure 3 show that using the controller F_1 tuned by Robust Control toolbox leads to the lower value of *settling time* (Bakošová et al. (2003)) in comparison to the control performance assured by the controller F_2 tuned by YALMIP (Figure 5). The control performance assured by the controller F_4 tuned by YALMIP toolbox obtain lower value of *overshoot* (Figure 9) than is assured by using the controller F_3 (Figure 7). On the other hand, the *setling time* obtained using the controller F_4 is much longer (Figures 11, 12) in comparison to the performance assured by the controller F_3 (Figure 7, 8) tuned by Robust Control toolbox.

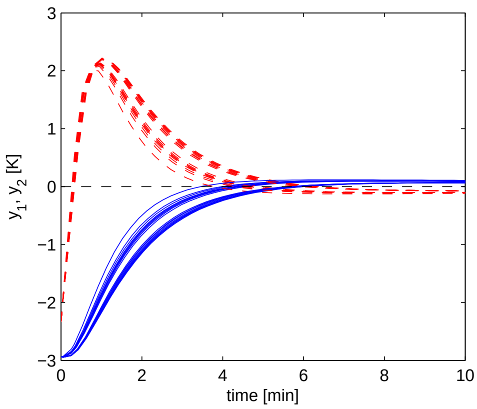

Fig. 5. Closed-loop behaviour of CSTR using the robust controller F_2

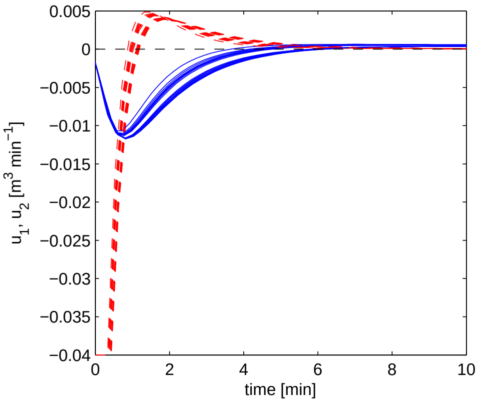

Fig. 6. Control inputs generated by the controller F_2

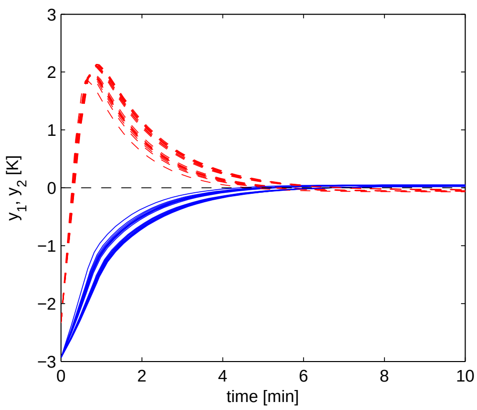

Fig. 7. Closed-loop behaviour of CSTR using the robust controller F_3

6. CONCLUSION

Robust stabilization of the exothermic CSTR with four uncertain parameters using static output feedback controllers was studied. The robust stabilizing multivariable controllers have been designed using the presented simple non-iterative and iterative algorithms, which are based on solving of two sets of LMIs. The problem of their solutions represents the feasibility problem of convex optimization.

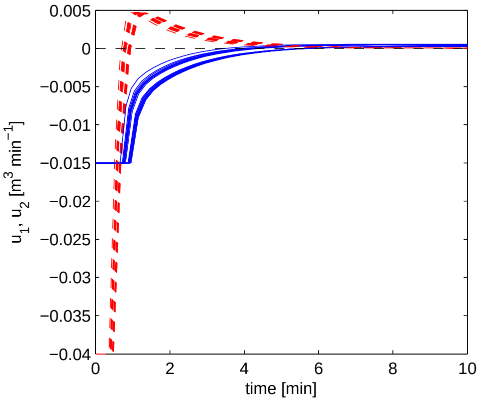

Fig. 8. Control inputs generated by the controller F_3

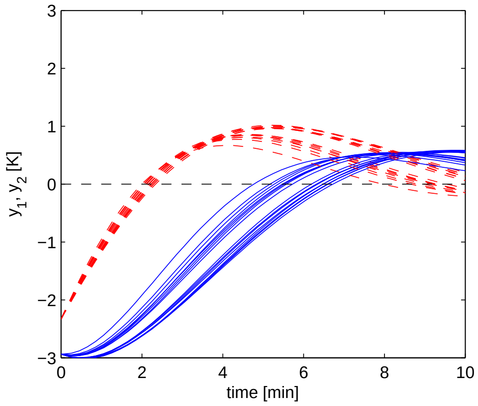

Fig. 9. Closed-loop behaviour of CSTR using the robust controller F_4

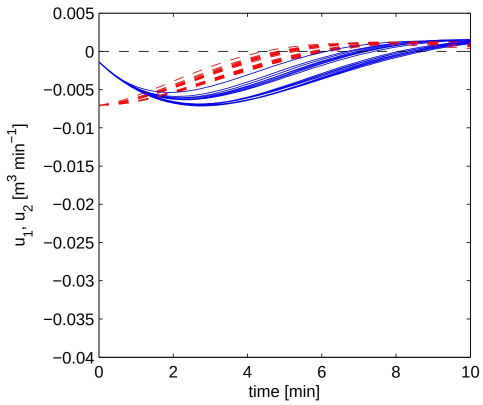

Fig. 10. Control inputs generated by the controller F_4

The Robust Control toolbox and YALMIP toolbox have been used for solving the LMIs and the results have been compared using MATLAB–Simulink environment. Description of optimization problem using YALMIP is more user-friendly. On the other hand, the CPU time decreases using Robust Control toolbox. Despite of simple using of YALMIP, using Robust Control toolbox seems to be more suitable for solving the problem of robust static output feedback stabilization.

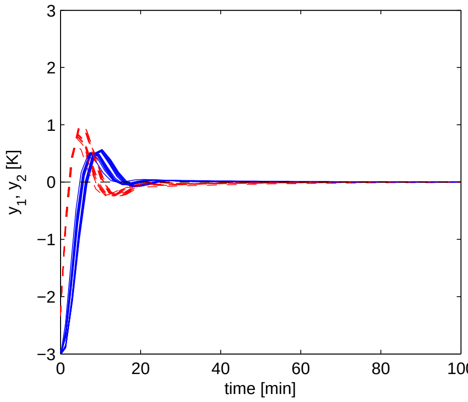

Fig. 11. Closed-loop behaviour of CSTR using the robust controller F_4 using longer evaluation time

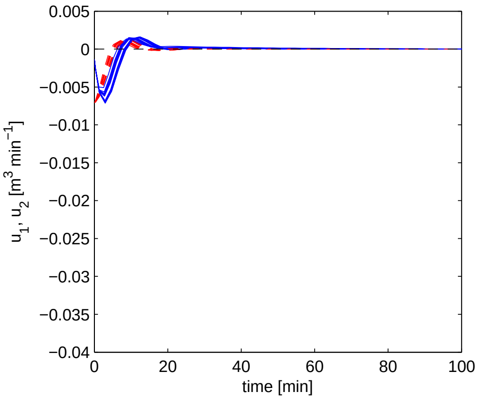

Fig. 12. Control inputs generated by the controller F_4 using longer evaluation time

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