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# Design of feedback control for unstable processes with time delay

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**Abstract**: In this paper, the tuning method, based on characteristic areas and Magnitude Optimum (MO) criterion for some unstable processes is presented. The proposed approach is to use inner compensator, of the first or the second order, to stabilise the process. The stabilised process is controlled by 2-DOF PI controller, tuned by using MOMI or DRMO tuning method (depending on desired tracking or disturbance-rejection performance). The proposed method was tested on five linear process models. The responses were relatively fast and without oscillations, all according to the MO criterion.

*Keywords*: unstable processes, internal feedback, PID control

## 1. INTRODUCTION

Most processes in chemical and process control industries are stable and can be controlled by various types of controller structures and relatively wide range of controller parameters. However, some types of processes, like continuous stirred reactors, bioreactors or polymerisation reactors are inherently unstable. Those processes require closer attention, since, to stabilise them, controller structure and parameters should be carefully chosen (Lee et al., 2010).

Several tuning rules for different types of unstable processes have been proposed so far. Some of the methods are dedicated to PI(D) controller design for unstable processes. Jacob and Chidambaram (1996) provided tuning formulas for the first-order unstable process with delay (FODUP) for PI controllers by using model reference method, synthesis method and internal model control (IMC) method. Park et al., (1998) proposed inner proportional feedback loop for stabilising the process and outer loop with PID controller. The proposed approach is equivalent to using 2-degrees-of-freedom (2-DOF) PID controller, which is also used by Prashanti and Chidambaram (2000) to reduce process overshoots. Construction of PID controller with lead/lag filter for integrating and FODUP processes was proposed by Shamsuzzoha and Lee (2008). Additional set-point filter was applied to reduce the overshoots. Panda (2009) designed PID controller for integrating and unstable processes, based on IMC design.

The proposed approach in this paper is to use internal feedback loop to stabilise the system, similar to Park et al. (1998). However, the inner compensator is of the first or the second order. The parameters of the compensator are calculated so as to equalise characteristic areas of the actual and desired closed-loop transfer functions. Then, Magnitude Optimum Multiple Integration (MOMI) or Disturbance Rejection Magnitude Optimum (DRMO) tuning rules (Vrančić et al.; 1999a, 2001, 2004) are applied to calculate PI(D) controller parameters for such stabilised process. The proposed control scheme is given in Figure 1.

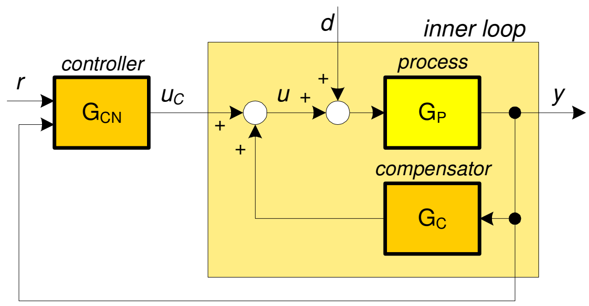


Fig. 1. Block diagram of the proposed closed-loop control.

## 2. DESIGN OF INTERNAL COMPENSATOR

The purpose of the compensator (Fig. 1) is to stabilise the process by forming the inner closed-loop. The compensator parameters depend on desired closed-loop properties. Let us assume that the process transfer function is the following:

$$G_P = K_{PR}\frac{num}{den}e^{-sT_{del}}, \tag{1}$$

where

$$\begin{aligned} num &= 1 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots \\ den &= 1 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots \end{aligned}, \tag{2}$$

and $K_{PR}$ and $T_{del}$ are process gain and time-delay, respectively. Let us choose the following compensator's structure:

$$G_C = K_C\frac{num_C}{den_C}, \tag{3}$$

where

$$\begin{aligned} num_C &= 1 + d_1 s + d_2 s^2 + d_3 s^3 + \cdots \\ den_C &= 1 + c_1 s + c_2 s^2 + c_3 s^3 + \cdots \end{aligned}. \tag{4}$$

Then, the closed-loop transfer function of the inner feedback loop (between signals $u_C$ and $y$ in Fig. 1) is:

$$G_{CL} = K_{PR}\frac{num \cdot den_C \cdot e^{-sT_{del}}}{den \cdot den_C - K_1 num \cdot num_C \cdot e^{-sT_{del}}}. \tag{5}$$

Let us define a desired closed-loop transfer function of the inner loop to have the same steady-state gain, numerator and pure time-delay as in (5):

$$G_{CLD} = \frac{K_{PR}}{1-K_1}\frac{num \cdot e^{-sT_{del}}}{den_R}, \tag{6}$$

where

$$K_1 = K_{PR}K_C, \tag{7}$$

and $den_R$ is a desired closed-loop denominator:

$$den_R = 1 + e_1 s + e_2 s^2 + e_3 s^3 + \cdots. \tag{8}$$

In order to make $G_{CL}$ (5) and $G_{CLD}$ (6) equal, the following sub-functions should become equal:

$$\begin{aligned} G_1 &= \frac{den_C}{\frac{den \cdot den_C}{1-K_1} - \frac{K_1}{1-K_1} num \cdot num_C \cdot e^{-sT_{del}}} \\ G_2 &= \frac{1}{den_R} \end{aligned}. \tag{9}$$

However, exact matching of $G_1$ and $G_2$ is not possible, since $G_1$ contains numerator and pure time-delay in denominator, which cannot be developed into finite number of terms. One possibility to make $G_1$ as close as possible to $G_2$ is to make them equal in lower frequency region by equating terms of their "characteristic areas" (Rake, 1987; Vrančić et al, 1999a). Namely, for the following transfer function:

$$G = \frac{K_P\left(1 + \beta_1 s + \beta_2 s^2 + \beta_2 s^3 + \cdots\right)}{\left(1 + \alpha_1 s + \alpha_2 s^2 + \alpha_2 s^3 + \cdots\right)}e^{-sT_d}, \tag{10}$$

the characteristic areas can be calculated as (Vrančić et al., 1999a,b):

$$\begin{aligned} A_0 &= K_P \\ A_1 &= K_P\left(\alpha_1 - \beta_1 + T_d\right) \\ A_2 &= K_P\left(\beta_2 - \alpha_2 - \beta_1 T_d + 0.5 \cdot T_d^2\right) + A_1\alpha_1 \\ A_3 &= K_P\left(\alpha_3 - \beta_3 + \beta_2 T_d - 0.5\beta_1 T_d^2 + \frac{T_d^3}{6}\right) + A_2\alpha_1 - A_1\alpha_2 \\ &\vdots \end{aligned}. \tag{11}$$

Function $G_1$ (9) can be expressed in terms of parameters $\alpha_i$ and $\beta_i$ (10) by applying Taylor's expansion of time-delay term in denominator:

$$e^{-sT_{del}} \approx 1 - sT_{del} + \frac{s^2T_{del}^2}{2!} - \frac{s^3T_{del}^3}{3!} + \cdots, \tag{12}$$

as follows:

$$\begin{aligned} \alpha_1 &= \frac{K_1\left(b_1 + d_1 - T_{del}\right) - a_1 - c_1}{K_1 - 1} \\ \alpha_2 &= \frac{K_1\left(-T_{del}d_1 - T_{del}b_1 + d_2 + b_1d_1 + b_2 + 0.5T_{del}^2\right) - c_2 - a_1c_1 - a_2}{\left(K_1 - 1\right)} \\ \alpha_3 &= \frac{K_1\left(d_3 + b_1d_2 + b_2d_1 + b_3 - \frac{T_{del}^3}{6} + 0.5T_{del}^2\left(b_1 + d_1\right) - T_{del}\left(d_2 + b_2 + b_1d_1\right)\right) - c_3 - a_1c_2 - a_2c_1 - a_3}{\left(K_1 - 1\right)} \\ &\vdots \\ \beta_1 &= c_1 \\ \beta_2 &= c_2 \\ \beta_3 &= c_3 \\ &\vdots \end{aligned}. \tag{13}$$

Function $G_2$ (9) can be simply expressed in terms of parameters $\alpha_i$ and $\beta_i$ (10) as follows:

$$\begin{aligned} \alpha_1 = e_1,\ \alpha_2 = e_2,\ \alpha_3 = e_3, \cdots \\ \beta_1 = 0,\ \beta_2 = 0,\ \beta_3 = 0, \cdots \end{aligned}. \tag{14}$$

In order to simplify derivations, denominator $den_C$ (3) will be chosen a-priori. Its main task is to filter out the process output noise signal, so it should be of the same or higher order ($n$) than the numerator:

$$den_C = \left(1 + sT_F\right)^n, \tag{15}$$

where $T_F$ can be chosen as several times smaller than absolute values of the process time constants.

Now, the internal compensator's parameters can be calculated by equating characteristic areas (11) of $G_1$ and $G_2$ (9). In order to simplify practical realisation of the compensator, the first- and the second-order numerator ($num_C$) will be derived (note that it does not limit us to calculate higher-order compensators). When choosing the first-order compensator's numerator, the first two areas (11) of sub-processes $G_1$ (13) and $G_2$ (14) should be equal. The following parameters are obtained:

$$K_1 = \frac{T_{del}(e_1 - a_1) + a_1 b_1 - b_1 e_1 + e_2 + e_1 c_1 - a_1 c_1 - a_2}{b_1^2 + e_2 - b_2 - b_1 e_1 + e_1 c_1 - b_1 c_1 + c_2 + T_{del}(e_1 + c_1 - b_1) + 0.5 T_{del}^2}$$
$$d_1 = T_{del} - b_1 + c_1 + e_1 + \frac{a_1 - e_1}{K_1} \qquad (16)$$

By equating the first three areas, we get:

$$K_1 = \frac{\begin{bmatrix} e_3 - a_3 + b_1 a_2 + c_2 e_1 - b_1 e_2 + c_1 e_2 - a_1 c_2 + a_1 b_2 - a_2 c_1 - \\ - b_2 e_1 - b_1 c_1 e_1 + a_1 b_1 c_1 - b_1^2 a_1 + b_1^2 e_1 + 0.5 T_{del}^2 (e_1 - a_1) + \\ + T_{del}(c_1 e_1 + a_1 b_1 - a_1 c_1 - b_1 e_1 + e_2 - a_2) \end{bmatrix}}{\begin{bmatrix} c_3 + e_3 - b_3 + c_2 e_1 + c_1 e_2 + 2 b_2 b_1 - b_1 e_2 - c_1 b_2 - c_2 b_1 - \\ - b_2 e_1 + c_1 b_1^2 + b_1^2 e_1 - b_1 c_1 e_1 - b_1^3 + \\ + T_{del}(c_1 e_1 - b_1 e_1 + b_1^2 - b_2 + e_2 + c_2 - c_1 b_1) + \\ + 0.5 T_{del}^2 (c_1 + e_1 - b_1) + \frac{T_{del}^3}{6} \end{bmatrix}}$$
$$d_1 = T_{del} - b_1 + c_1 + e_1 + \frac{a_1 - e_1}{K_1} \qquad (17)$$
$$d_2 = c_1 e_1 + c_2 - b_2 + e_2 + (T_{del} - b_1)(e_1 + c_1 - b_1) + 0.5 T_{del}^2 + \frac{a_1 c_1 + a_2 + T_{del} a_1 - T_{del} e_1 - b_1 a_1 + b_1 e_1 - e_2 - e_1 c_1}{K_1}$$

Compensator gain $K_C$ can be calculated from (7) as:

$$K_C = \frac{K_1}{K_{PR}} . \qquad (18)$$

Note that the areas (11) can also be calculated in time-domain by integrating the process input and output signals after changing the process (10) set-point (Vrančić et al., 1999b).

*Illustrative example*

Let us calculate compensator's parameters for the following process transfer function (Panda, 2009; Park et al., 1998):

$$G_P = \frac{e^{-0.5s}}{(1+0.5s)(1-2s)} . \qquad (19)$$

The desired closed-loop denominators (8) are chosen to be of the same order as the process denominator. The first one has been chosen to have the same *absolute* time constants as the process, while the second one has faster response:

$$den_{R1} = (1+0.5s)(1+2s)$$
$$den_{R2} = (1+0.7s)^2 \qquad (20)$$

According to expression (6), the desired closed-loop transfer functions, for both denominators, are:

$$G_{CLD1} = \frac{1}{1-K_1}\frac{e^{-0.5s}}{(1+0.5s)(1+2s)}$$
$$G_{CLD2} = \frac{1}{1-K_1}\frac{e^{-0.5s}}{(1+0.7s)^2} \qquad (21)$$

Note that $K_1$ is not known a-priori. However, it does not have any influence on stability (when $K_1 \neq 1$). The a-priori chosen denominator of the compensator (to filter out high-frequency noise) is:

$$den_C = (1+0.1s)^3 . \qquad (22)$$

Let us now calculate the remaining compensator's parameters by using expressions (17) and (18). The compensators become:

$$G_{C1} = \frac{1.72(1+0.98s+0.29s^2)}{(1+0.1s)^3}$$
$$G_{C2} = \frac{2.32(1+0.95s+0.28s^2)}{(1+0.1s)^3} \qquad (23)$$

Both compensators were tested in the closed-loop configuration, as shown in Fig 1 (without controller gain $G_{CN}$). Response on unity step-change of signal $u_C$ is shown in Fig. 2.

It is clear that the obtained responses (solid lines) are very close to desired responses, defined by function $G_{CLD}$ (6).

## 3. DESIGN OF CONTROLLER

Since the process is already stabilised by the compensator, a controller design is not very critical. Therefore, relatively simple controller structures can be used. In this paper, due to simplicity, the 2-DOF PI controller structure has been chosen:

$$U_C = \left(bK + \frac{K_i}{s}\right)R - \left(K + \frac{K_i}{s}\right)Y , \qquad (24)$$

where $K$, $K_i$ and $b$ are proportional gain, integral gain and proportional weighting factor, respectively. Note that other types of controllers can be applied as well. A Magnitude-Optimum-Multiple-Integration (MOMI) tuning method for PI controllers has been chosen for tracking, since it usually results in a relatively fast closed-loop responses without oscillations for different types of process models (Vrančić et al., 1999a; 2001). If disturbance rejection properties are more important, a DRMO method (modified MOMI method for improving disturbance rejection performance) can be applied (Vrančić et al., 2004).

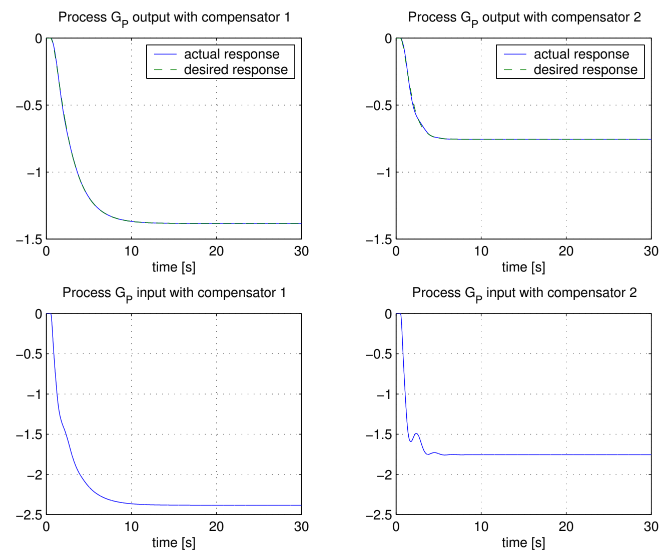


Fig. 2. Response of the inner loop when using both compensators.

The tuning rule for MOMI method is the following (see Vrančić et al., 1999a; 2001):

$$\begin{aligned} K &= \frac{A_3}{2(A_1A_2 - A_0A_3)} \\ K_i &= \frac{0.5 + A_0K}{A_1} \\ b &= 1 \end{aligned} \quad . \tag{25}$$

The tuning rule for DRMO method is (Vrančić et al., 2004):

$$\begin{aligned} &\xi_1 K^2 + 2\xi_2 K + A_3 = 0 \\ &\xi_1 = A_0^2 A_3 - 2A_0A_1A_2 + A_1^3 \\ &\xi_2 = A_0A_3 - A_1A_2 \\ &K_i = \frac{(1 + A_0K)^2}{2A_1} \\ &b = 0 \end{aligned} \quad , \tag{26}$$

where $K$ can be calculated from the second-order equation in (26). Areas $A_0$ to $A_3$ in (25) and (26) can be calculated from expression (11) if the controlled process is given by expression (10). However, note that the controlled process from controller's viewpoint is the desired closed-loop transfer function (6). Also note that parameters $\alpha_i$ and $\beta_i$ can be expressed by equating expressions (6) and (10):

$$\begin{aligned} &K_P = \frac{K_{PR}}{1 - K_1}, T_d = T_{del} \\ &\alpha_1 = e_1, \alpha_2 = e_2, \alpha_3 = e_3, \cdots, \beta_1 = b_1, \beta_2 = b_2, \beta_3 = b_3, \cdots \end{aligned} \quad . \tag{27}$$

The Matlab toolset, which performs the calculation of the compensator's and PI controller parameters for the chosen and arbitrary linear process models, is available on-line (Vrančić, 2010).

*Illustrative example*

Let us calculate the PI controller parameters for the same process (19) and compensators (23) as in the previous example. The PI controller is actually controlling the closed-loop transfer function (5) which is similar to desired closed-loop transfer function (21). The areas of desired transfer functions can be calculated from expressions (11) and (27) for both compensators:

$$\begin{aligned} &G_{CLD1} : A_0 = -1.38, A_1 = -4.15, A_2 = -9.17, A_3 = -18.79 \\ &G_{CLD2} : A_0 = -0.76, A_1 = -1.43, A_2 = -1.73, A_3 = -1.74 \end{aligned} \quad . \tag{28}$$

The PI controller parameters are calculated by using MOMI (25) or DRMO (26) tuning method for both compensators:

$$\begin{aligned} G_{CLD1}: \quad & K_i = -0.38, K = -0.78, b = 1 \ (MOMI) \\ & K_i = -0.57, K = -0.85, b = 0 \ (DRMO) \\ G_{CLD2}: \quad & K_i = -0.74, K = -0.74, b = 1 \ (MOMI) \\ & K_i = -0.89, K = -0.79, b = 0 \ (DRMO) \end{aligned} \quad . \tag{29}$$

The closed-loop response is given in Fig. 3. It can be seen that responses, when using MOMI method, have faster tracking responses, while DRMO method results in better disturbance rejection performance. Naturally, compensator 2 also gives faster closed-loop responses than compensator 1.

## 4. EXAMPLES

The proposed method will be tested on the following process models:

$$\begin{aligned} G_{P1} &= \frac{2e^{-0.3s}}{(1-3s)(1-s)} \quad & G_{P2} &= \frac{4e^{-2s}}{(1-4s)} \\ G_{P3} &= \frac{e^{-s}}{(1-2s)(1+0.5s)} \quad & G_{P4} &= \frac{e^{-0.4s}}{(1-s)} \end{aligned} \quad , \tag{30}$$

which were tested by some other authors (see Jacob and Chidambaram, 1996; Panda, 2010; Park et al., 1998; Prashanti and Chidambaram, 2000; Shamsuzzoha and Lee, 2008). The desired denominators are:

$$\begin{aligned} den_{R1} &= (1+2s)(1+s) \quad & den_{R2} &= (1+4s) \\ den_{R3} &= (1+s)^2 \quad & den_{R4} &= (1+s) \end{aligned} \quad . \tag{31}$$

The calculated compensators, by using the proposed method, are the following:

$$G_{C1} = \frac{0.16\left(1-18.1s-5.97s^2\right)}{\left(1+0.1s\right)^3}$$
$$G_{C2} = \frac{0.39\left(1+1.105s\right)}{\left(1+0.1s\right)^2}$$
$$G_{C3} = \frac{1.69\left(1+1.13s+0.434s^2\right)}{\left(1+0.1s\right)^3} . \qquad (32)$$
$$G_{C4} = \frac{1.64\left(1+0.228s\right)}{\left(1+0.1s\right)^2}$$

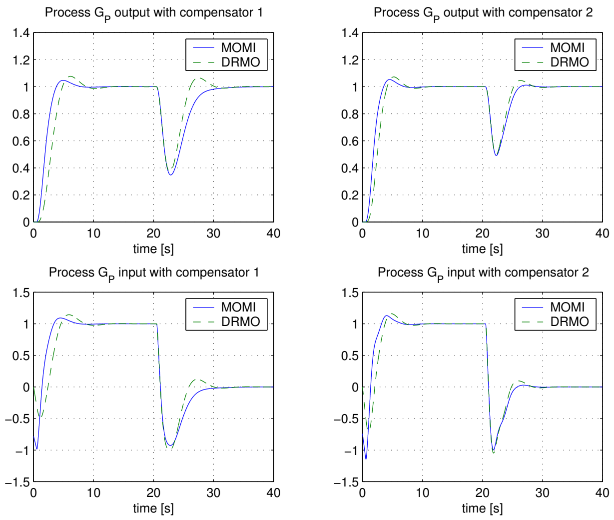

Fig. 3. Closed-loop response when using both compensators when using MOMI and DRMO method.

The calculated controller parameters, for all four process models with compensators, are given in Table 1. Note that Matlab toolset, which performs the calculation of all the parameters for the given process models, is given in Vrančić (2010).

Table 1. PI controller parameters

| | **MOMI** | | | **DRMO** | | |
|---|---|---|---|---|---|---|
| | $K_i$ | $K$ | $b$ | $K_i$ | $K$ | $b$ |
| $G_{P1}$ | 0.15 | 0.325 | 1 | 0.21 | 0.34 | 0 |
| $G_{P2}$ | -0.037 | -0.148 | 1 | -0.056 | -0.166 | 0 |
| $G_{P3}$ | -0.227 | -0.336 | 1 | -0.266 | -0.361 | 0 |
| $G_{P4}$ | -0.81 | -0.816 | 1 | -1.359 | -0.92 | 0 |

The closed-loop responses for all four process models are given in Figs. 4-7. The difference between the desired and the actual inner closed-loop responses are relatively small for all four processes. The closed-loop responses with controller are relatively fast, without oscillations, and with relatively small overshoots, all according to the MO tuning criterion. The tracking performance is better when using MOMI method, while disturbance rejection performance is better with DRMO method.

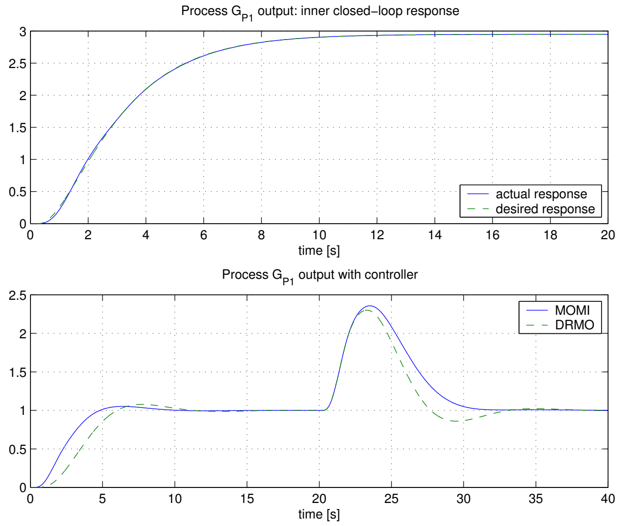

Fig. 4. Closed-loop responses of the process $G_{P1}$ when using MOMI and DRMO method.

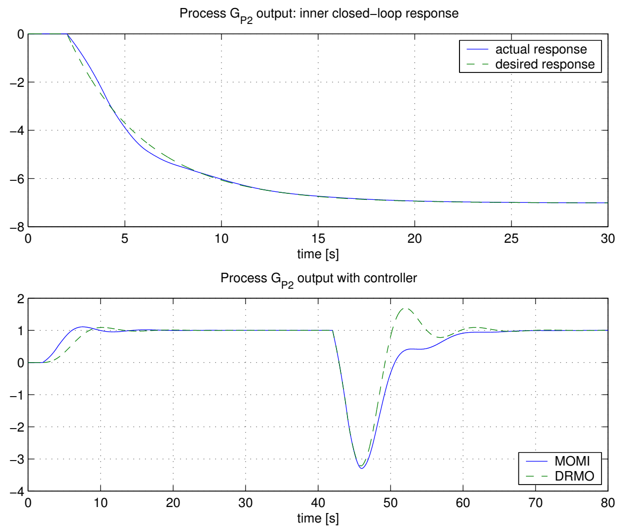

Fig. 5. Closed-loop responses of the process $G_{P2}$ when using MOMI and DRMO method.

## 5. CONCLUSIONS

Controller design is divided into two stages. The first stage is design of inner compensator by means of equating characteristic areas of the actual and desired inner closed-loop transfer function. The comparison of both responses in five examples confirms the efficiency the compensator.

The second stage is design of outer 2-DOF PI controller by applying MOMI or DRMO tuning method. According to all five examples, the proposed approach resulted in a relatively fast responses without oscillations.

The advantages of the proposed method are that it is not limited to the first- or the second-order processes models. Moreover, the method can be extended to higher order compensators or different controller structures (e.g. PID controllers or Smith predictors).

Disadvantage of the proposed method is that it requires, similar to other methods, the a-priori definition of desired closed-loop transfer function. In our case, the desired closed-loop time constants have been chosen to be the same or slightly faster to absolute values of process time constants.

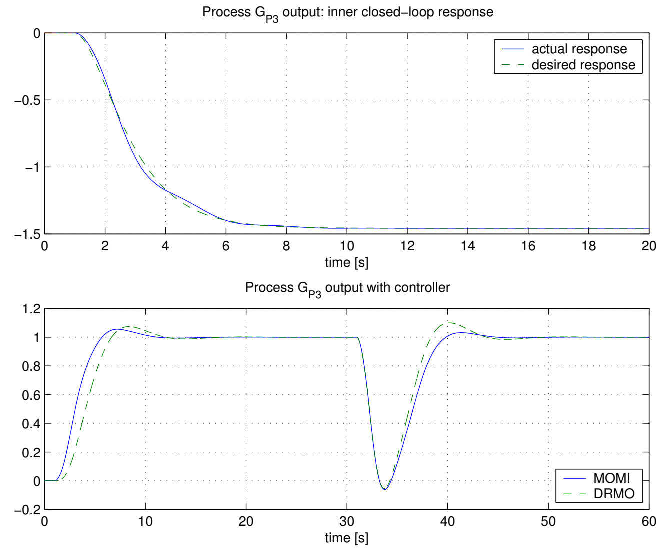


Fig. 6. Closed-loop responses of the process $G_{P3}$ when using MOMI and DRMO method.

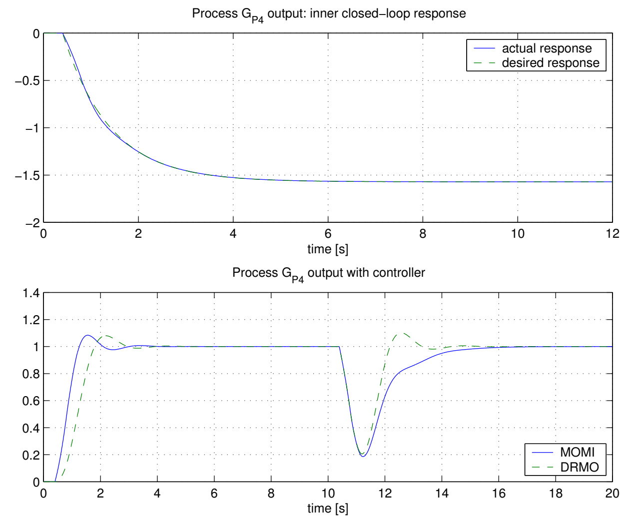


Fig. 7. Closed-loop responses of the process $G_{P4}$ when using MOMI and DRMO method.

In our further work we will investigate robustness of the proposed tuning approach.

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