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ROBUST TUNING OF PI CONTROLLER FOR IPDT PLANT

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Abstract: By considering robust tuning of the PI controller for uncertain Integral Plus Dead Time plant (IPDT) this paper demonstrates possibilities of the new Matlab/Simulink tool based on the performance portrait method. For plants with parameters defined over uncertainty intervals it enables to guarantee transient responses with specified deviations from ideal shapes at the plant output and input and to fulfill additional optimality specification, defined e.g. in terms of the minimal IAE values for the setpoint and disturbance steps, in terms of the maximal integral gain, etc. In difference to the robust tuning methods of the 1st generation considering typically controller parameters calculated from plant parameters specified by a single entry, in this new method uncertain plant parameters are specified by two entries characterizing their extreme values. As the ideal step responses at the plant output monotonic transients are considered, whereas at the plant input one-pulse step responses consisting of two monotonic intervals are required.

Keywords: Proportional control, optimal control, robust control, dead time.

1. INTRODUCTION

Tuning of the PI controller for the IPDT plant

$$F(s) = \frac{K_s}{s} e^{-T_d s}$$
$$K_s \in \langle K_{smin}, K_{smax} \rangle ; T_d \in \langle T_{dmin}, T_{dmax} \rangle \quad (1)$$

is frequently treated both in the process control and in the motion control areas. In connection with appropriate model reduction techniques it enables to approximate broad range of processes Åström and Hägglund (2005), Skogestad (2003). Consequently, high number of different "optimal" tuning rules based on this model may be found in the literature O'Dwyer (2006).

From the early beginning of PID control, for the controller tuning both the analytical (see e.g. Oldenbourg and Sartorius (1944,1951)) as well as experimental methods Ziegler and Nichols (1942) were used.

When considering tuning rules appropriate for education & practice, it is to agree with Skogestad (2003) that they should be 1. well motivated, 2. preferably model-based, 3. analytically derived, 4. simple and easy to memorize and 5. work well on a wide range of processes. When continuing with requirements of Skogestad (2006), controller tuning should enable achieving trade-off between: fast speed of response, good disturbance rejection, stability & robustness, less input usage and less sensitivity to measurement noise.

But, on the other hand, also the experimental controller tuning played always an important role, what may e.g. be demonstrated by the high popularity of the early tuning by Ziegler and Nichols (1942) that still gives inspiration for many new approaches based on the accumulated knowledge and broad simulation possibilities Åström and Hägglund (2004), Hägglund and Aström (2002). Of course, except of the analytical design the main requirements on such tuning remain mostly the same as above.

In this paper we are going to show that the requirements of robust control may be combined with high requirements on control performance, when the proposed tuning will guarantee specified performance not only for the nominal working point, but for any loop parameters of the uncertainty intervals in (1). Similarly as Åström and Hägglund (2004),Hägglund and Aström (2002), or Ziegler and Nichols (1942) the new method is based on carrying out series of simulation experiments on some sample of representative processes under requirement of chosen shape-related performance measures. Such an approach can today be easily performed by using tremendous power of computers for organizing and evaluating experiments, as well as for processing, visualizing, storing and recalling the achieved results for large number of control loops typical in practice. Thereby, one can easily extend spectrum of different qualitative & quantitative properties that will be evaluated and stored in computer database, to be chosen "on demand" and in different combinations by engineer carrying out design requiring particular specifications.

The paper is structured as follows. To characterize basic properties of the first generation of robust controller tuning methods and to enable their systematic comparison with the new proposed method, in Chapter 2 several tuning methods are discussed. In Chapter 3, basic requirements on robust controller tuning are summarized and in Chapter 4 performance measures for robust controller tuning in the time domain are introduced. In Chapter 5 the performance portrait for plant (1) is described and then used in Chapter 6 for controller tuning based on minimization of IAE values of setpoint step responses, or maximization of the integral gain values subject to shape related constraints for the plant input and output. The achieved results are compared with those corresponding to the first-generation robust tuning methods. Basic conclusions are summarized in Chapter 7.

2. FIRST GENERATION OF ROBUST CONTROLLER TUNING METHODS

Next we will briefly introduce several robust tuning methods that may be used for the IPDT plant and are interesting with respect to the paper aims.

2.1 Analytical controller design - TRDP

Based on generalization of the double real dominant closed loop pole Oldenbourg and Sartorius (1944,1951) to the triple real dominant pole (TRDP), whereby the PI controller is extended by the setpoint weighting according to

$$U(s) = K_c\,[bW(s) - Y(s)] + \frac{K_c}{sT_i}\,[W(s) - Y(s)] \qquad (2)$$

interesting nominal tuning was analytically derived both for regulatory as well as tracking control tasks in Vítečková and Víteček (2008a), Vítečková and Víteček (2008b). The setpoint weighting can be shown to be equivalent to using prefilter

$$F_p(s) = \frac{bT_i s + 1}{T_i s + 1} \qquad (3)$$

with T_i being the integral time constant. The approach is based on solving closed loop characteristic equation for a triple pole s_0 that for

$$\begin{aligned} A(s) &= s^2 T_i e^{T_d s} + K_r K_s (T_i s + 1) \\ \dot{A}(s) &= 2sT_i e^{T_d s} + s^2 T_d T_i e^{T_d s} + K_r K_s T_i \\ \ddot{A}(s) &= 2T_i e^{T_d s} + 4sT_d T_i e^{T_d s} + s^2 T_d^2 T_i e^{T_d s} \end{aligned} \qquad (4)$$

requires to fulfill

$$A(s_0) = 0; \dot{A}(s_0) = 0; \ddot{A}(s_0) = 0 \qquad (5)$$

Solution of the last equation in (5) yields root

$$s_0 = -(2 - \sqrt{2})/T_d \qquad (6)$$

for which from the first two equations in (5) one gets stable tuning with parameters

$$\begin{aligned} K_c &= 2(\sqrt{2} - 1)e^{\sqrt{2}-2}/(K_s T_d) \approx 0.461/(K_s T_d) \\ T_i &= (2\sqrt{2} + 3)T_d \approx 5.828 T_d \end{aligned} \qquad (7)$$

For the root $s_0 = -(2 + \sqrt{2})/T_d$ the resulting values $K_c = -0.1588/(K_s T_d); T_i = 0.17157 T_d$ with negative loop gain do not guarantee the closed loop stability.

Zero of the closed loop transfer function

$$F_{wy} = \frac{K_s K_c (T_i s + 1)}{s^2 e^{T_d s} + K_s K_c (T_i s + 1)} \qquad (8)$$

can be cancelled by the prefilter denominator in (3) that removes overshooting typical for one-degree-of-freedom PI controllers. Simultaneously, by cancelling one of the triple pole (6) by the prefilter numerator (3) that further accelerates the transient responses, one gets the setpoint weighting coefficient

$$b = \frac{1/|s_0|}{T_i} = \frac{2 - \sqrt{2}}{2} \approx 0.293 \qquad (9)$$

The corresponding maximal sensitivity and the complementary sensitivity peaks are $M_s = 1.70; M_t = 1.44$.

Examples of achieved transients compared with other tuning approaches are given in Figs. 2-3, 6-7 and 8-9. Basic advantage of the nominal tuning is given by compactness and elegance of its derivation. Though the method gives fast and smooth responses both in regulatory as well as tracking control, its extension to uncertain plants (1) and balancing different requirements on the setpoint and the disturbance response transient shapes at the plant input and output make already problems - the method does not include free tuning parameter enabling dynamics modifications.

2.2 SIMC PI Controller

As the 2nd example illustrating the analytical controller tuning we will mention the popular SIMC PI-rule (abbreviation from Simple/Skogestad Internal Model Control) for fast response with good robustness Skogestad (2003).

Firstly, by considering direct controller synthesis Rivera et al. (1986), Skogestad (2003) leading for a general first order plus dead time (FOPDT) plant

$$F_s = \frac{K_s e^{-T_d s}}{s + 1/T_1} \qquad (10)$$

to a simple first-order setpoint-to-output closed loop transfer function with time constant τ_c

$$F_{wy} = \frac{R(s)F(s)}{1 + R(s)F(s)} = \frac{R(s)K_s}{(s + 1/T_1)e^{T_d s} + R(s)K_s} \qquad (11)$$

$$F_{wy} \stackrel{!}{=} \frac{1}{1 + \tau_c s} e^{-T_d s} \qquad (12)$$

the PI controller

$$R(s) = \frac{s + 1/T_1}{K_s(\tau_c + T_d)s} \qquad (13)$$

is derived, whereby the exponential term may be eliminated by using its first-order Taylor series approximation

$$e^{-T_d s} \approx 1 - T_d s \qquad (14)$$

what requires to use $\tau_c \geq T_d$. For stable 1st order systems it is usually chosen $T_i = T_1$ and $K_c = 1/(K_s T_1(\tau_c + T_d))$. However, for integral systems, when $T_1 \to \infty$, solution (13) is actually approaching the proportional controller, what leads to poor rejection of input (load) disturbances. Of course, it is still possible to choose PI controller and to look for its appropriate tuning by other means, but it is no more the above mentioned direct controller synthesis of the IMC control. Therefore, the first question arises if the abbreviation SIMC is still appropriate for integral plants. In Skogestad (2003) tuning for such systems is derived by analyzing conditions of the critically damped closed loop system with the PI controller and integral delay-free plant ($T_d = 0$), when the double real dominant pole may be achieved by choosing

$$T_i = 4/(K_s K_c) \tag{15}$$

Finally, to consider dead time, the closed loop time constant in (12) was chosen as $\tau_c = T_d$ what yields

$$K_c = 1/(2K_s T_d); T_i = 8T_d \tag{16}$$

Such tuning that might be considered as simplification of the above method (double real dominant pole instead of the triple one) is not only simple, easy to remember but for the lag dominant plants it brings a reasonable improvement of the input-disturbance dynamics in comparing with the traditional IMC tuning rules and also with other tested methods (see Figs. 2-3, 6-7 and 8-9). It yields a reasonably fast response with moderate input usage and good robustness margins both in regulatory as well as tracking control. The analytical controller derivation is no more as compact as in the above case and as it was already mentioned above, it is no more the IMC control. The PI controller was not analytically derived, but chosen. Tuning of the integral part was made for delay-free system what leads to a question, in which range of the dead-time values it will keep the expected performance. But, on the other hand, together with the "half-rule" enabling to deal effectively with more complex plants it shows on necessity to link the controller design to approximative loop modelling and detecting its weakest points by the possibly simplest means.

When comparing integral loops with controller (16) with the IMC control of stable plants, it is also to note that for the integral plant the output setpoint step responses typically have overshooting, whereas in controlling stable 1st order plants (10) the closed loop step responses (12) are monotonic both at the plant input and output. When aiming to monotonic setpoint step responses at the plant output also in controlling integral plants, it is again possible to introduce setpoint weighting (2-3). However, since the method does not give information about the dominant closed loop poles, the calculation based on cancelling one real closed loop pole requiring to choose

$$b = 1/|s_1|\, T_i = 0.702 \tag{17}$$

does no more guarantee purely monotonic output (due to the obviously complex remaining dominant closed loop pole). So, a setpoint weighting guaranteeing purely monotonic output can be determined just experimentally as

$$b = 0.592 \tag{18}$$

What is again to be stressed is that the tuning is typically done just in a nominal point. By using specifications in the frequency domain, it is indeed shown that for integrating processes the suggested settings (16) give the gain margin $GM = 2.96$, the phase margin $PM = 46.9°$, the maximal sensitivity and the complementary sensitivity peaks $M_s = 1.70; M_t = 1.30$, and the maximum allowed time delay error with respect to stability is $1.59T_d$, but the controller tuning does not directly depend on the extreme values of the plant parameters in (1). The method neither includes free parameter enabling to balance dynamics of the setpoint and disturbance responses.

2.3 Non-Convex Optimization Based PI Control

As the 3rd tuning approach to be compared with the newly developed tuning the numerical non-convex optimization method Åström et al. (1998) will be mentioned. Based on the frequency-domain loop specifications by the maximum and complementary sensitivity peaks $M_s = 1.40$ and $M_t = 1.45$ it gives

$$b = 0.66; K_c = 0.282/(K_s T_d)$$
$$K_i = \frac{K_c}{T_i} = \frac{0.0418}{K_s T_d^2} \Rightarrow T_i = 6.746 T_d \tag{19}$$

The optimization problem used for derivation of above results was specified as follows: find controller parameters that maximize the integral gain $K_i = K_c/T_i$ subject to the constraints that the closed-loop system is stable, the Nyquist curve of the loop transfer function satisfies the encirclement condition and that it is outside a circle that has the M_s and M_t circles in its interiors. Although it might seem at the first glance that standard optimization routines yield sufficient tools to solve this problem numerically, it was shown that *"the optimization problem is nontrivial because the constraint, which is infinite dimensional, defines a set in parameter space which is not convex"* Åström et al. (1998) and as a result the found controller parameters do give IAE values (Figs. 2-3, 6-7 and 8-9) that are much larger than those corresponding to other tested approaches. In the nominal case they do not allow achieving monotonic output setpoint step response even when choosing $b = 0$. But, they give relatively good responses for the relatively large deviation from the nominal case. So, they give a nice illustration of the fact that despite apparent simplicity the dynamics of the PID control is still tricky enough to be solved by standard, as well as specialized optimization routines. The dynamics specification in the frequency domain that is usually sufficient in dealing with robust stability problem seems not to be the best alternative for characterizing higher performance requirements in terms of the deviations from the shape related properties defined through the time domain responses.

2.4 AMIGOs tuning for PI Controller

Obviously being aware of too conservative tuning (19), in Hägglund and Åström (2002) new tuning rules were published based on Approximative M_s-constrained Integral Gain Optimization (AMIGO). These results corresponding to the maximum and complementary sensitivity peaks $M_s = 1.48$ and $M_t = 1.39$ extended by the choice b=0 to achieve monotonic step responseswill be used, when

$$b = 0; K_c = 0.35/(K_s T_d); T_i = 7T_d \tag{20}$$

The corresponding transient responses for setpoint and disturbance steps are in Figs. 2-3, 6-7 and 8-9. The nominal properties are slightly improved and by choosing b=0 the setpoint step responses are nearly monotonic at the plant output.

3. NEEDS FOR ROBUST CONTROLLER DESIGN

The most important feature of all above mentioned design methods is that they give robust tuning based on a single nominal point. The fact that real plants have just exceptionally properties characterized by fixed completely known point, is considered just indirectly, by choosing controller tuning that is sufficiently conservative to be usable also in the case of possible plant-model mismatch. So, possible uncertainty due to finite measurement precision or due to nonlinear character of real processes is paid by conservativeness of the tuning. All methods for controller tuning based on single set of parameters of the nominal plant model must be sufficiently robust against plant model uncertainties to be usable in practice. But, with exception of possible parameter changes allowed with respect to the robust stability, all the up to now mentioned methods do not directly give information specifying, how far the model parameters may deviate from the nominal point to keep the specified plant dynamics. They are just working with a conservativeness degree chosen equally for all possible applications. Some flexibility of the non-convex optimization in Åström et al. (1998) allowed by choice of the maximal sensitivity M_s is far from the originally proclaimed aims *"...to have a design parameter to change the properties of the closed-loop system. Ideally, the parameter should be directly related to the performance of the system, it should not be process oriented. There should be good default values so a user is not forced to select some value... The design parameter should also have a good physical interpretation and natural limits to simplify its adjustment."* All above mentioned methods are working with M_s values from a relatively narrow range 1.4-1.7, but despite to this their robustness and performance reasonably differ.

4. PERFORMANCE MEASURES FOR ROBUST CONTROL

Next, we are going to look for more appropriate tuning parameter(s) and method enabling to fulfil aims of robust control without leading to unnecessarily conservative tuning. From the performance point of view, at the plant output the expected dynamics is frequently specified by the setpoint step responses yielding monotonic transients.

The ideal continuous signal at the plant input giving after integration by the plant dynamics monotonic output will be denoted here as the one-pulse control. It may be characterized as a pulse having one extreme point that is dividing the overall transient into two monotonic control intervals.

Both such shape-related properties were, however, just rarely in focus of contemporary control research. Monotonic control together with a performance index for its evaluation was e.g. mentioned in Åström and Hägglund (2004), Hägglund and Aström (2002). One of recent reviews on PID control Keel et al. (2008) is mentioning just output non-overshooting control, without discussing possible specifications at the plant input and output that may be much harder. This is consequence of the development of last decades, when methods applied were dominated by the mathematical convenience and concentrated mostly on traditional performance criteria like gain margin, phase margin, maximum sensitivity, H_∞ norm, ISE, etc. Because of lacking analytical tools, the controller will be robustly tuned by using numerically derived areas of parameters corresponding to the above mentioned shape-related properties. The aim is to expand such nice dynamics of the nominal case as e.g. given by the tuning (7-9) that simultaneously fulfillis requirements on ideal shapes both at the plant input and output, but corresponding just to a single point with exactly known plant parameters to plant parameters known over uncertainty intervals Huba (2009), Huba et al. (2009), Huba (2010).

4.1 Ideally nonovershooting, monotonic and one-pulse responses

By its nature, definitions of the one-pulse control may be based on definition of the monotonic output control. This represents subset of non-overshooting control that represents subset of stable control.

The output transients $y(t)$ with $y(0) = 0$ corresponding to the setpoint step, $w = const \neq 0$ are classified according to validity of

$$y(t)/w \leq 1, \forall t \in (0, t_{sim}) \tag{21}$$

as *non-overshooting control.*

When fulfilling relations

$$0 \leq y(t_1)/w \leq y(t_2)/w \leq 1; \forall 0 \leq t_1 < t_2 \leq t_{sim} \tag{22}$$

the output response may be denoted as the *monotonic control* and in the case of the output fulfilling (22) and the input fulfilling

$$\begin{aligned} sign(\dot{u}(t_1))sign(u(t_m)) \geq 0, \forall t_1 \in \langle 0, t_m \rangle \cup \\ \cup sign(\dot{u}(t_2))sign(u(t_m)) \leq 0, \forall t_2 \in \langle t_m, t_{sim} \rangle \end{aligned} \tag{23}$$

the dynamics may be denoted as *one-pulse control*. For all that $u(t_m); t_m \geq 0$ corresponds to the maximal control signal amplitude during transient and t_{sim} represents simulation time that should be larger than maximal possible settling time.

Since the settling time used for characterizing speed of output transient strongly depends on the defined measurement precision (given e.g. by ϵ), the much less dependent IAE (Integral of Absolute Error) defined as

$$IAE = \int_0^\infty |e(t)| \, dt \tag{24}$$

will be used as the time-related performance index for quantitative evaluation of responses.

4.2 Amplitude deviations from ideal shapes

In practice, but also in case of computer simulation, it has sense to weaken the above conditions on non-overshooting control by introducing some tolerable overshooting defined by new small positive constant

$$\epsilon > 0 \tag{25}$$

and to find in this way controller parameters corresponding to

$$y(t)/w \leq 1 + \epsilon, \forall t_1 \in (0, t_{sim}) \tag{26}$$

E.g. by choosing $\epsilon = 0.01$, the setpoint step responses with overshooting up to 1% of the setpoint value w will be tolerated and included under denotation as the non-overshooting control. In this paper this approach will only be used for $\epsilon \leq 0.1$, because responses with larger overshooting may also be achieved in other ways (e.g. without using setpoint weighting) and so the design should consider also other alternatives.

A continuous nearly monotonic signal $y(t)$ with the initial value $y_0 = y(0)$ and with the final value $y_\infty = y(\infty)$ will be denoted as ϵ_y -monotonic when it fulfills condition

$$\begin{array}{l} [y(t) - y(y-T)]\, sign(y_\infty - y_0) \geq -\epsilon_y \\ T \leq t < \infty, T \in (0, T_{max}), \forall T_{max} > 0 \end{array} \tag{27}$$

Thereby, in order not to prolong the time required for testing with any positive T_{max}, this has to be chosen to enable capturing sufficient part (e.g. half-period) of the superimposed signal. Number of samples that need to be tested Huba (2010) may be decreased, if all subsequent local extreme points fulfill condition

$$[y_{le,i+1} - y_{le,i}]\, sign(y_\infty - y_0) \geq -\epsilon_y; i = 1, 2, 3, \ldots \tag{28}$$

The amplitude deviations from one-pulse control (23) are based on evaluating amplitude deviations from monotonicity over both monotonic intervals before and after the dominant extreme point $u(t_m); t_m \geq 0$.

Non-overshooting specifications (not distinguishing between non-overshooting and monotonic control) exist also in the frequency domain (see e.g. Keel et al., 2008) but their application is extremely complicated, especially when speaking about dead time systems.

4.3 Integral deviations from ideal shapes

Specific integral measure for deviations from monotonicity was introduced by Åström and Hägglund (2004), Hägglund and Aström (2002). Here, we will prefer new measures for deviations from monotonic and one-pulse shapes that may be easily tested numerically, by evaluating simulated or experimentally measured transients corresponding to the setpoint and disturbance step responses and are also appropriate for constrained control.

To evaluate control effort required to achieve the required output behavior, Total Variance (TV) criterion was proposed Skogestad (2003), Skogestad and Postlethwaite (2007) defined as

$$TV = \int_0^\infty \left| \frac{du}{dt} \right| dt \approx \sum_i |u_{i+1} - u_i| \tag{29}$$

Under non-perfect control it is not easy to be evaluated analytically. So, typically, its values are computed by simulation after appropriate discretization with sampling period as small as possible. According to Skogestad and Postlethwaite (2007) in Matlab it may be simple computed by the command `sum(abs(diff(u))`.

Very simple integral measure for evaluating deviations from strict monotonicity defined for the plant output $y(t)$ with the initial value $y(0)$ and the final value $y(\infty)$ by modification of the TV criterion will be denoted here as the TV_0 criterion Huba (2010)

$$TV_0 = \sum_i |y_{i+1} - y_i| - |y(\infty) - y(0)| \tag{30}$$

$TV_0 = 0$ just for strictly monotonic response, else $TV_0 > 0$.

In controlling unstable and integral plants the number of significant control pulses cannot decrease below the number of unstable poles Huba (2009), Huba (2010). To stress contribution of the superimposed oscillation in systems with 1P dominant control it is then appropriate to work with the TV_1 criterion defined as

$$TV_1 = \sum_i |u_{i+1} - u_i| - |2u_m - u(\infty) - u(0))| \geq 0 \tag{31}$$

This gives zero values just for strictly 1P control signal and may be applied also to constrained control signal. For control signals with superimposed higher harmonics it takes positive values.

Graphically represented in the plane of loop parameters, together with quantitative measures, such properties will be giving *performance portrait* of particular control loop. In this way, the new approach continues in developing trends recommended e.g. by Ackermann (2002)

5. CLOSED LOOP PERFORMANCE PORTRAIT (PP) AND ROBUST CONTROLLER DESIGN

The closed loop PP represents information about the loop performance corresponding to the setpoint and the disturbance step responses expressed over a grid of (possibly normalized) loop parameters including all possible working points. By containing information about required loop properties for different loop paramterers, the PP may be used both for optimally choosing the nominal controller tuning for a comletely known plant, or for the robust controller tuning of a plant with interval parameters.

For a loop represented by a parameter vector

$$P = \{p_1, p_2, \ldots p_S, p_{S+1}, p_{S+I}\} \tag{32}$$

with the dimension

$$D = S + I \tag{33}$$

each entry of the first subset of parameters $p_i; i = 1, \ldots, S$ is given as a signle value that has to be fixed during the controller tuning.

There may also exist some uncertain (plant) parameters

$$p_i \in \langle p_{imin}, p_{imax} \rangle ; \qquad i = S+1, \ldots, S+I \tag{34}$$

that vary over some (known) intervals. Next, we define such limits also for the first subset of parameters (e.g. by some preliminary robust stability analysis method), so that all parameters may be expressed in the above form.

In computation of the PP all parameters p_i take just discrete $n_i + 1$ levels

$$\begin{array}{l} p_{i,j} = p_{imin} + (p_{imax} - p_{imin}) j / n_i; \\ j = 1, 2, \ldots, n_i; n_i > 1; i = S+1, \ldots, S+I \end{array} \tag{35}$$

Both the nominal as well as the robust control design may now be carried out in two ways: as determination of an optimal controller parameter set, or as a determination of an optimal working point of a controller expressed by means of the plant parameters vectors. When the number of the controller parameters exceeds number of the plant parameters, combination of both approaches is possible.

When e.g. working with the uncertain plant model (1), the controller (2- 3) is specified by three parameters b, K_c, T_i. In addition to the plant parameters K_s and T_d, specification of the setpoint response (12, 13) additionally requires determination of at least one time constant τ_c. It means that in total there are 6 parameters that determine the resulting dynamics. If two of them, $K_s \in (K_{smin}, K_{smax}); T_d \in (T_{dmin}, T_{dmax})$ are uncertain, the task of the control design may be formulated as:

a) to find directly the controller parameters b, K_c, T_i, or

b) for the controller parameter defined by formulas introduced ich Chapter 2 to find an appropriate location of the operating point K_{s0}, T_{d0} and the free design parameters b.

Both has to be done in such a way that over all grid points corresponding to chosen tuning and to all possible values of the uncertain interval parameters the required shape-related performance measures will be achieved. The necessary amout of computation and the achieved precision will obviously depend on the level of quantization and on the choice of the limits introduced for the free parameters that have to be determined.

PP required for such a design may be generated by simulation, or by real time experiments. When it is based on normalized parameters, it may then be repeatadly used for different tasks with different values of particular loop parameters. Although such PP generation may be connected with numerical problems, especially those related to the nature of grid computations, when one has to balance precision of achieved results (quantization level in considered grid) with the total number of evaluated points and the corresponding computation time, it gives very promising results especially when dealing with dead time systems.

The first attempt to analyze optimal robust tuning of the IPDT plant by the performance method was done by Huba et al. (2009) in 2D space of normalized parameters $K = K_c K_s T_d$ and $\Omega_f = T_d/T_i$. The setpoint weighting b has been chosen as minimum of the optimal nominal values calculated from the two above parameters over the uncertainty set, what was leading to slightly conservative tuning. Now, the setpoint weighting will be considered as the third independent coordinate and the performance portrait will be generated over grid of points in 3D with the uncertainty subspace given by the coordinates vector $K = K_c K_s T_d$; $\tau_i = T_i/T_d$; and parameter b representing the third coordinate. The integer variable describing particular levels of this parameter will be displayed in the following figures as k.

6. COMPARATIVE ANALYSIS OF PI TUNINGS

6.1 Nominal Tuning for min IAE

The simplest strategy for designing robust controller tuning seems to be to find such controller parameters b, K_c, T_i that will guarantee for all possible plant parameters (1) minimal mean IAE values subjected to amplitude or integral deviations on the plant input and output.

Fig. 1 shows several windows of one layer (with $k = 16$) of the 3D performance portrait calculated for the setpoint step responses over 27x27x21 points for $K \in \langle 0.1, 1.4\rangle$; $\tau_i \in \langle 3.5, 15.5\rangle$; $b \in \langle 0, 1\rangle$. The position of the optimal operating point gives minimal IAE value for the tolerated output amplitude deviation from monotonicity and input amplitude deviation from one-pulse control $\epsilon_y = \epsilon_u = 10^{-3}$.

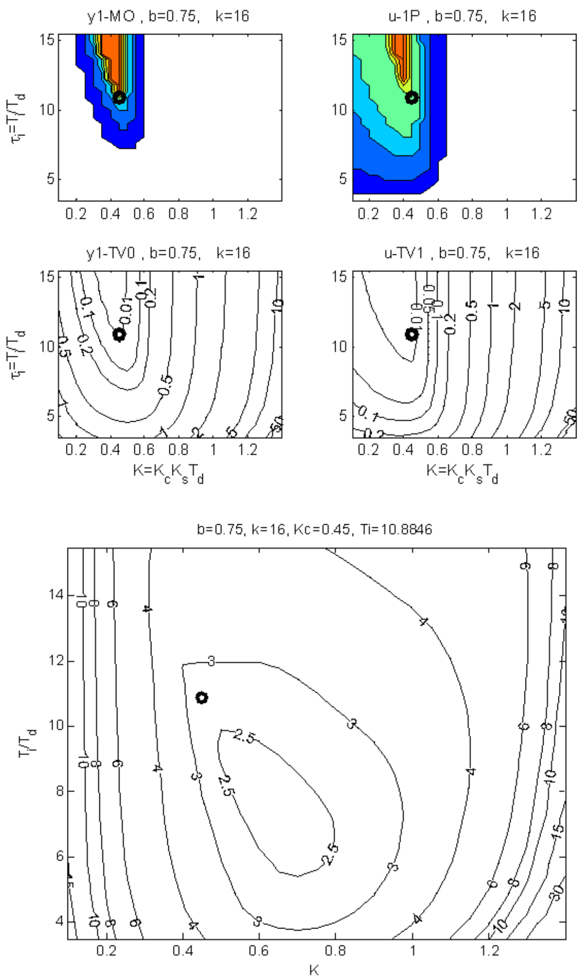

Fig. 1. One layer of the PP ($k = 16$) calculated for the setpoint step responses over 27x27x21 points and containing the optimal nominal tuning corresponding to min IAE. Note the similarities between the amplitude and integral measures for the plant output and input

By comparing the amplitude and integral deviations from ideal shapes it is possible to conclude that usually it would be enough to work with one set of such measures, whereby, due to their simplicity, the integral measures could be preferred and the amplitude deviations could be estimated as

$$\epsilon_y \leq TV_0(y)/2; \epsilon_u \leq TV_1(u)/2 \quad (36)$$

The identity holds just then when the analyzed transition has exactly one additional pulse with the amplitude given by the particular value of ϵ.

The setpoint and disturbance responses in Fig. 2 and Fig. 3 corresponding to the found optimal parameters

$$K_c = 0.45; T_i = 10.88; b = 0.75 \quad (37)$$

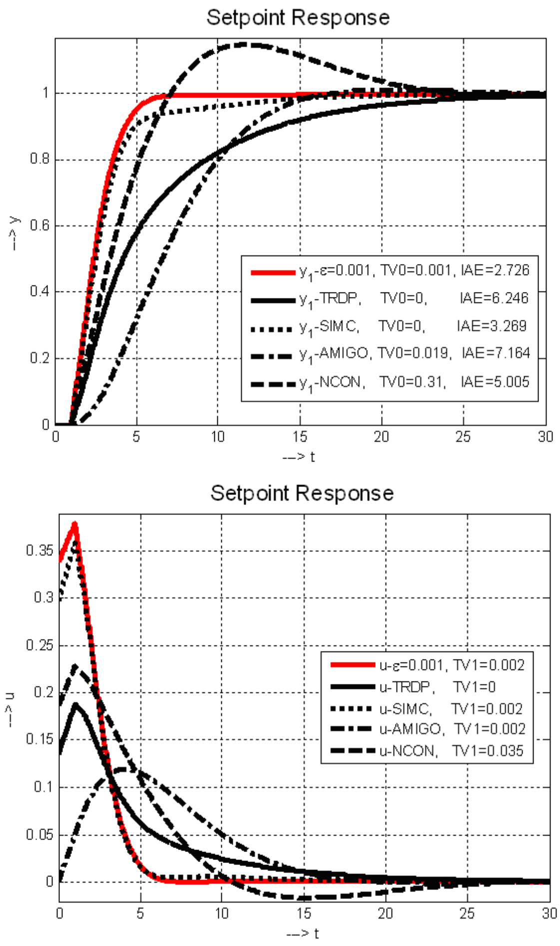

Fig. 2. Setpoint step responses at the plant output and input corresponding to the optimal tuning according to Fig. 1 (red) compared with TRDP (7-9), SIMC (16) and non-convex optimization (19).

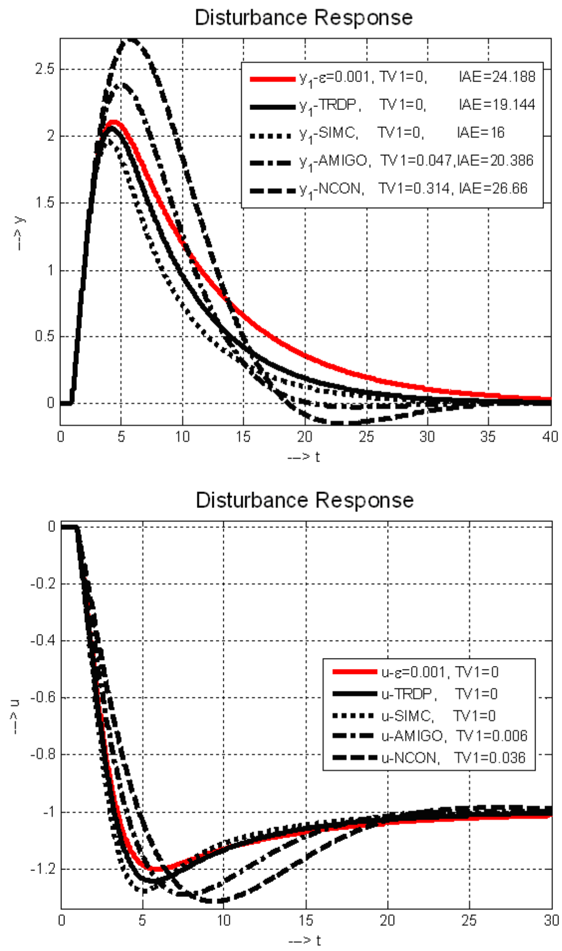

Fig. 3. Load-disturbance step responses at the plant output and input corresponding to the optimal tuning according to Fig. 1 (red) compared with TRDP (7-9), SIMC (16) and non-convex optimization (19).

do not represent an absolute optimum. By broadening the PP to larger integral time constants and by increasing number of grid points (decreasing the quantization step), the identified optimal solutions tend to those corresponding to pure P control and $T_i \rightarrow \infty$. This trivial handicap (with respect to the disturbance response) can be avoided by optimizing weighted sum of the setpoint and disturbance responses. But already without such modification, the achieved results show that the new method enables to optimize the setpoint responses by keeping acceptable disturbance response.

6.2 Nominal Tuning for max K_i

Next we are going to compare the new Performance Portrait method with the optimization based approaches. By their numerical procedures both approaches are very close each other. Similarly as in the PI controller tuning by the non-convex optimization Åström et al. (1998), or by its later modification Hägglund and Aström (2002), also the PP method will be used to find the maximal integral gain, but instead of the previously considered constraints on the maximum sensitivity, the search will now be subject to the shape related constraints puts on the plant input and output step responses.

Fig. 4 shows several windows of one layer of the 3D performance portrait from the above example corresponding to the location of the optimal nominal point yielding

$$K_c = 0.65; T_i = 4.42; b = 0 \tag{38}$$

The corresponding setpoint and disturbance responses in Fig. 6 and Fig. 7 show that this approach does not give the absolutely best setpoint response (this was not required), but the achieved disturbance response is already the absolutely best one. Again, the look up of the optimal tuning was fully based on the performance portrait corresponding just to the setpoint step response. Although the TV_1 values of the disturbance response are relatively close to the absolute minimum, when necessary, this parameter may be further improved by considering also the PP of the disturbance response.

Transients in Fig. 6 and Fig. 7 show that the best nominal setpoint step responses are achieved by the SIMC tuning that also gives relatively good disturbance responses.

Surprisingly, the disturbance responses corresponding to the non-convex optimization NCON Åström et al. (1998), or the AMIGOs tuning Hägglund and Aström (2002) derived by optimization for the optimal disturbance response (maximal K_i gain) give the worst IAE results.

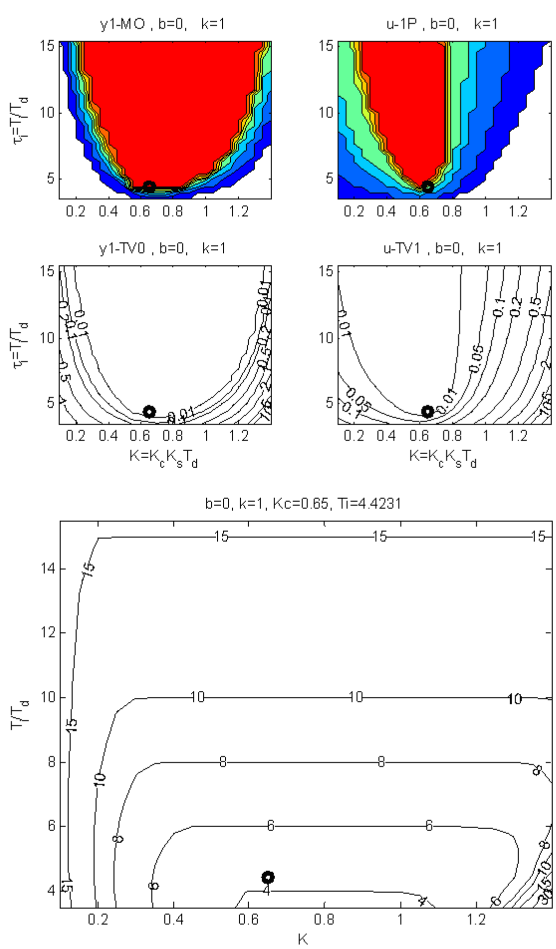

Fig. 4. The first layer ($k = 1$) of the PP calculated for the setpoint step responses over 27x27x21 points for $K \in \langle 0.1, 1.4\rangle$; $\tau_i \in \langle 3.5, 15.5\rangle$; $b \in \langle 0, 1\rangle$ and indicating the optimal nominal tuning corresponding to max K_i corresponding to the tolerated output amplitude deviation from monotonicity and input amplitude deviation from one-pulse control $\epsilon_y = \epsilon_u = 10^{-3}$. Note the similarities and differences between the amplitude and integral measures for the plant output and input.

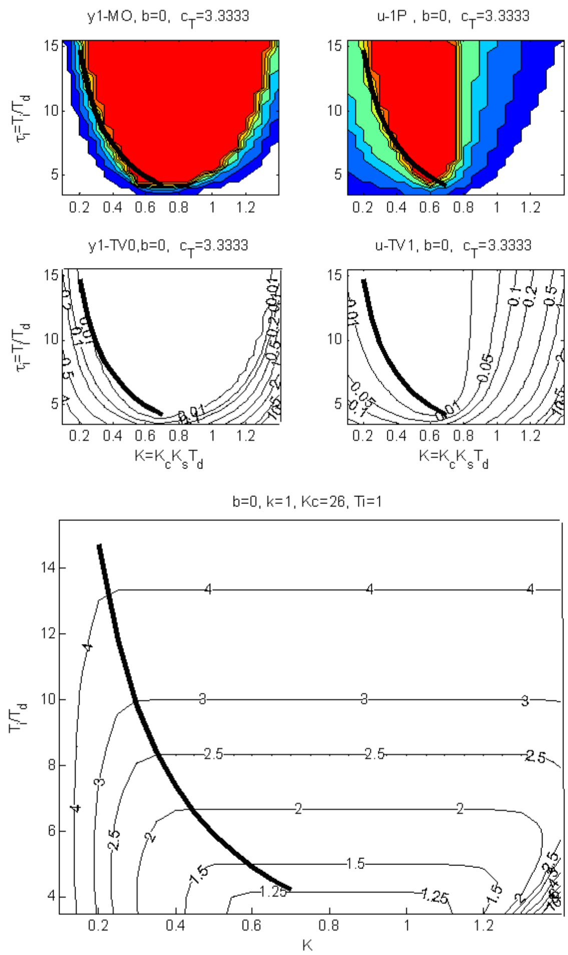

Fig. 5. The first layer ($k = 1$) of the PP calculated for the setpoint step responses over 27x27x21 points for $K \in \langle 0.1, 1.4\rangle$; $\tau_i \in \langle 3.5, 15.5\rangle$; $b \in \langle 0, 1\rangle$ and displaying the amplitude and integral deviations from monotonicity at the plant output and from the 1P at the plant input (above) and from the output IAE values (below); The Uncertainty Curve Segment corresponding to $T_d \in \langle 0.3, 1.0\rangle$ and $\epsilon_y = \epsilon_u = 10^{-3}$ satisfies to the requirement $K_i = K_c/T_i \overset{!}{=} max$.

6.3 Robust Tuning for max K_i

The seemingly bad results of the nominal tuning based on the nonconvex optimization subject to sensitivity constraints may be explained by considering interval plant parameters. Consider e.g. plant with the dead time uncertainty

$$T_{dmin} = 0.3; T_{dmax} = 1.0 \tag{39}$$

and the corresponding robust controller tuning. Since the uncertain parameter T_d is included both in the PP parameter $K = K_cK_sT_d$, as well as in $\tau_i = T_i/T_d$, all possible operating points given by the optimal controller tuning

$$K_c = 26; T_i = 1; b = 0 \tag{40}$$

sweep in the corresponding layer of PP in Fig. 5 parabolic curve segment. All its points need to satisfy the above given tolerances on the deviations from the output monotonicity and input one-pulse response ($\epsilon_y = \epsilon_u = 10^{-3}$).

For both limit values of T_d show Fig. 8 and Fig. 9 that the new method gives the best disturbance responses by simultaneously keeping the shape related performance measures for the setpoint step responses.

All previously mentioned method were tuned around the symmetrically chosen nominal operating point

$$T_{d0} = (T_{dmin} + T_{dmax})/2 = 0.65 \tag{41}$$

The TRDP method that seems to be slightly conservative in the nominal case gives now good performance over the whole considered uncertainty interval, just for larger difference between extreme dead time values it would already lead to oscillatory behavior.

SIMC method is the best one for $T_d = T_{dmin}$, but for $T_d = T_{dmax}$ it already leads to oscillatory behavior what could be at least partially compensated by non-symmetrical choice of the nominal operating point. The PP method could be used to find new position of the operating point

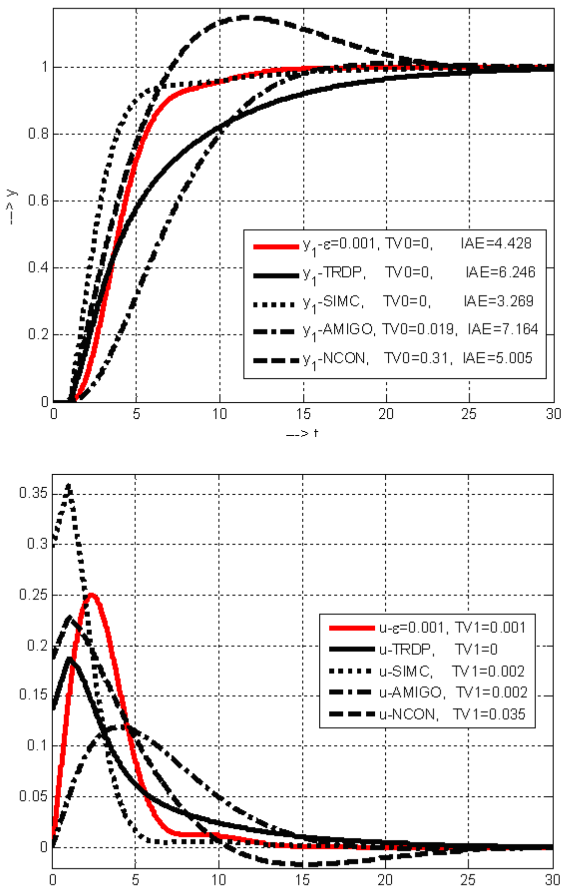

Fig. 6. Setpoint step responses at the plant output and input corresponding to the optimal tuning according to Fig. 4 (red) compared with TRDP (7- 9), SIMC (16) and non-convex optimization (19).

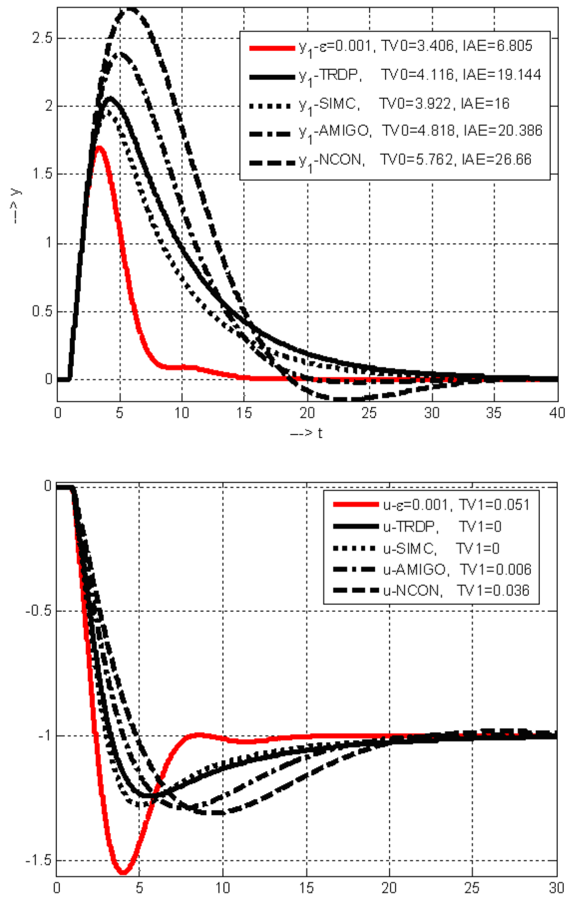

Fig. 7. Load-disturbance step responses at the plant output and input corresponding to the optimal tuning according to Fig. 2 (red) compared with TRDP (7- 9), SIMC (16) and non-convex optimization (19).

symmetrizing deviations corresponding to the limit dead time values.

The NONC and AMIGOs tuning lead for extreme dead time values to surprisingly better performance with lower deviations from ideal schapes than in the nominal case.

7. CONCLUSIONS

New control design method based on amplitude and integral deviations of the transient responses at the plant input and output from their ideal shapes was proposed and illustrated by the frequently treated task of the PI controller tuning in this paper.

The carried out comparative analysis including several first-generation robust tuning approaches for the IPDT uncertain plant has shown their typical features: in some context they may give excellent properties, just to know when, how and which controller tuning and the operating point have to be used. The new approach showed to be much more effective and efficient than the approaches based on the plant characteristics in the frequency domain in all analyzed situations. Whereas the traditional methods are not only typical by a preprogrammed degree of conservativeness and they also do not give information, how the operating point should be chosen with respect to uncertainty intervals of the considered uncertain parameters the new method directly gives solution optimally fitting the specified performance measures without any redundant conservativism for all possible operating points specified by the uncertainty intervals, or indicates that the specified performance may not be achieved by any tuning of the specified controller.

The new method avoids the second step of the traditional approach, when, after deciding, how the controller parameters should be expressed by means of the plant parameters, for plants with parameters taking values from an uncertainty interval it is not clear, how to choose the operating point in order to get the results that are the optimal for all possible values of the uncertain parameter.

In the comparative analysis it was shown that with respect to symmetry of the deviations from ideal shapes at the plant input and output an intuitive assignment of the operating point of a particular parameter to the centre of its uncertainty interval may not give satisfactory results. It is e.g. important for the SIMC tuning that gives excellent responses around the nominal operating point, but it is non-symmetrically sensitive to the dead-time uncertainty,

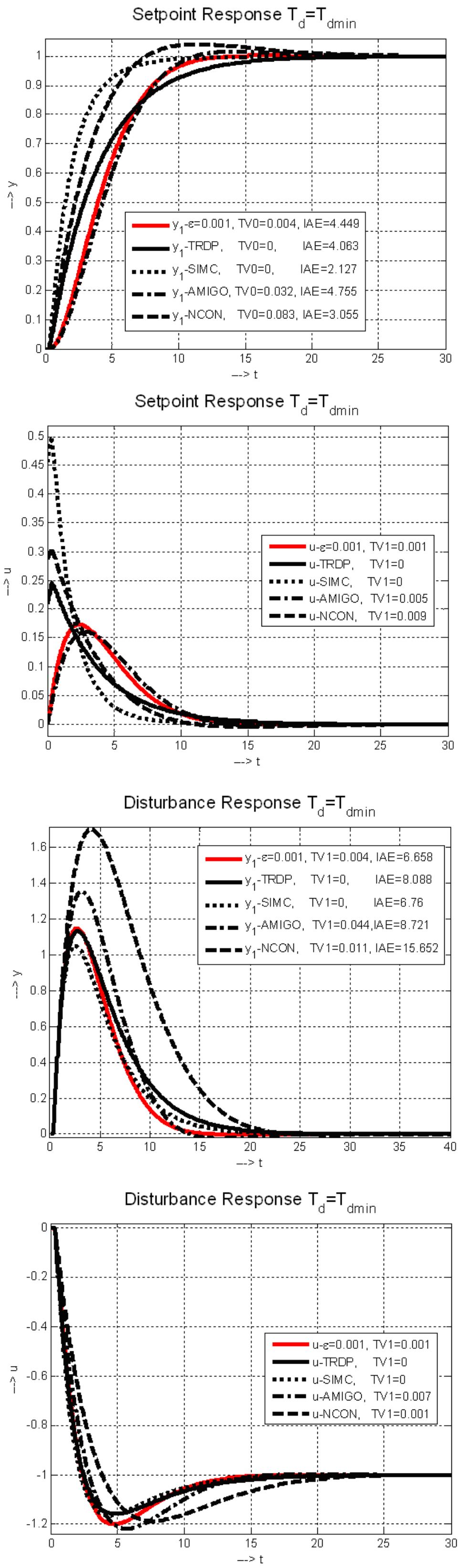

Fig. 8. Setpoint and disturbance steps for $T_d = T_{dmin}$; $\epsilon_y = \epsilon_u = 0.02$.

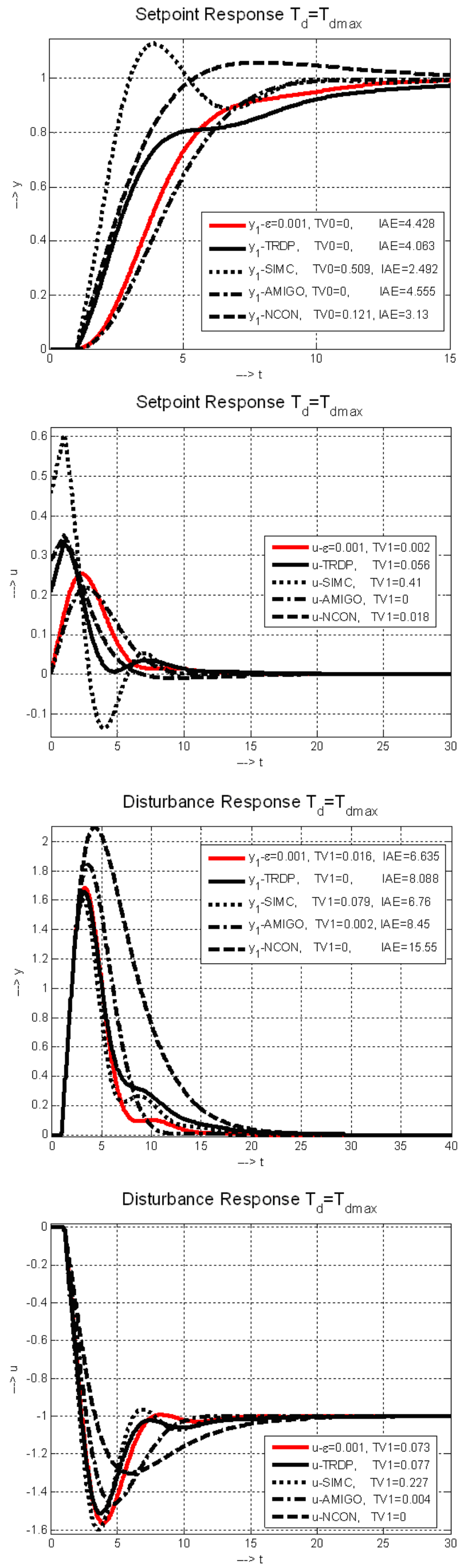

Fig. 9. Setpoint and disturbance steps for $T_d = T_{dmax}$; $\epsilon_y = \epsilon_u = 0.02$.

what consequently requires non-symmetrical choice of the nominal operating point over the uncertainty interval.

On the other hand, the TRDP method is able to guarantee output-monotonic and input-one-pulse transients for a broad neighborhood around the nominal working point, whereby by increasing deviation from the nominal point the conservativeness of the tuning decreases.

The robust tuning based on non-convex optimization (NONC) does not allow monotonic output step responses even in the nominal case, but the shape of responses is rather robust against dead-time perturbation and with increased deviation from the nominal case the performance improves what could explain motivation leading to this design. Its modified version AMIGOs removes the high overshooting of the nominal setpoint step responses and still gives relatively robust responses in the perturbed situations.

Analysis of the new approach to the robust PI controller tuning based on experimental identification of parameter areas corresponding to tolerable deviations from output-monotonic and input-one-pulse control clearly showed that the new method represents new generation of optimal tuning approaches that are able to guarantee believably chosen performance requirements for all considered loop parameters. So it is possible to avoid stiff character of the first-generation tuning formulas that may not only be too conservative, but also too sensitive in some applications.

Since the new method fully relies on a computer support, its use may be very simple and besides of the recommended tuning a lot of additional information characterizing the optimal solution and the overall context of the proposed tuning may be offered. Its drawback is that the designer is no more able to fully rely just on his "pen and paper", but the same happens in many other situations in our life.

For practical use, the above analysis of optimal controller tuning should yet be completed by analysis of the control constraint effects, since the parallel integral control is well known by the integrator windup that can fully destroy the control dynamics.

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