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MPC-based approximation of dual control by information maximization

Jan Rathouský* Vladimír Havlena**

* *Department of Control Engineering, Czech Technical University in Prague, Czech Republic (e-mail: rathojan@fel.cvut.cz)*

** *Department of Control Engineering, Czech Technical University in Prague, Czech Republic (e-mail: havlena@control.felk.cvut.cz)*

Abstract: This paper proposes multiple-step active control algorithms based on MPC approach that approximate persistent system excitation in terms of the increase of the lowest eigenvalue of the parameter estimate information matrix. It is shown how the persistent excitation condition is connected with a proposed concept of stability of a system with uncertain parameters. Unlike similar methods, the proposed algorithms predict the information matrix for more than one step of control. The problem is formulated as an MPC problem with an additional constraint on the information matrix. This constraint makes the problem non-convex, thus only locally optimal solutions are guaranteed.

Keywords: Adaptive systems and control; Input and excitation design; Stochastic control; Dual control approximation; Model predictive control.

1. INTRODUCTION

Controller design is usually based on some performance specifications that should be satisfied for a system model. Thus the controller design is primarily based on some model that describes the system up to a certain precision. Various algorithms exist that take into account the model uncertainty, based on uncertainty model both in time and frequency domain. Usual algorithms, however, do not take into account the possibility that the control process itself may bring some information about the controlled system and thus improve the model.

The simultaneous optimal control and identification problem is referred to as a dual control problem Feldbaum (1960–61) which is known to be analytically solvable for only very special systems as in Sternby (1976); Åström and Helmersson (1986) as it requires solving the Bellman equation (Bertsekas (2005)). Numerical solution faces the curse of dimensionality problem. There exist approximations of the optimal solution based on suboptimal solution of the original problem, (Lee and Lee (2009); Lindoff et al. (1999, 1998); Wittenmark (1995, 2002); Chen and Loparo (1991)), or on problem reformulation (Filatov et al. (1996)). An overview of the state-of-the-art methods is given in Filatov and Unbehauen (2004), where an algorithm with dual properties is defined as one that actively gathers information during the control process while satisfying given control performance.

In this paper we propose three algorithms based on the idea of the persistent system excitation (Goodwin and Sin (1984)). The persistent excitation condition requires

that the information about the system in the sense of its parameter information matrix is increased linearly, i.e.

$$P_{t+M}^{-1} - P_t^{-1} \geq \gamma I \quad (1)$$

for all t and some given M , where P_k^{-1} denotes the information matrix (i.e. the inverse of the variance matrix) after k steps of control, γ is a given real constant and I denotes the identity matrix of corresponding order. The inequality symbol $>$ is used in the positive definiteness meaning, i.e. for two matrices A and B , $A > B$ means that $A - B$ is a positive definite matrix.

The proposed algorithms are based on a constrained MPC control design that is adjusted such that the persistent excitation condition is satisfied. This control problem is formulated and analysed in section 3. In section 2 we show a motivation example for such design and propose a concept of stability of a system with uncertain parameters. This concept is based on a requirement that the mean value of a given quadratic criterion is finite over infinite control horizon. It is shown on the motivation example that the persistent excitation condition (1) is also sufficient for stability in this sense.

The proposed algorithms predict the information matrix over more than one step of control. This prediction is one of the two major problems of the methods, as the only practically computable prediction based on certainty equivalence assumption is used. The second major problem is the inherent non-convexity, the reason why only local solution is guaranteed to be found when using numerical methods for problem solution.

Section 4 contains derivations and descriptions of individual algorithms. All algorithms are designed for autoregressive systems with external input (ARX), although their formulation allows for future generalization for ARMAX systems with known MA part (Havlena (1993); Peterka

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(1986)). Finally, section 5 shows simulations of the proposed methods and we conclude in section 6.

2. PERSISTENT EXCITATION AND STABILITY

In this section we will introduce a concept of stability of a closed loop system with fixed but unknown parameters and show how this concept is connected with the persistent excitation conditions.

2.1 The concept of stability

Let us consider a general ARX system. Such system has a form

$$y_k = \sum_{i=1}^n a_i y_{k-i} + \sum_{i=0}^m b_i u_{k-i} + e_k, \quad (2)$$

where u_i and y_i are system inputs and outputs, respectively and e_i is a discrete-time white noise. Let us assume that the system parameters a_i and b_i are fixed but unknown constants. Also let us assume that at the time $k = 0$ we have some estimate of the parameter values, that we will denote $\hat{a}_{i,0}$ and $\hat{b}_{i,0}$. These parameter estimates can be used for controller design and they are expected to get more precise during the future control process. If the estimate is unbiased, the estimate error at time k , $\tilde{a}_{i,k} = a_i - \hat{a}_{i,k}$ and $\tilde{b}_{i,k} = b_i - \hat{b}_{i,k}$ is a random vector with zero mean and variance matrix P_k .

Because the future estimates of parameters are not available at the initial time $k = 0$, it is convenient to model such situation by a stochastic process, the parameters of which are random variables $a_{i,k}$ and $b_{i,k}$ with mean $\hat{a}_i = \hat{a}_{i,0}$ and $\hat{b}_i = \hat{b}_{i,0}$ and variance matrix P_k . The advantage is that properties of the estimate errors remain unchanged. It will also be supposed for simplicity of computations that $a_{i,k}$ and $b_{i,k}$ are independent with respect to time k .

Let us now consider a linear quadratic (LQ) controller for this system. The controller minimizes the following criterion

$$J_N = \sum_{k=1}^N \{ru_k^2 + y_k^2\}, \quad (3)$$

minimization of which leads to a feedback control law. One way to cope with unknown system parameters in controller design is to use the certainty equivalence (CE) approach, i.e. substitute these parameters with their mean value. In the previous model, it means to use \hat{a}_i and \hat{b}_i instead of $a_{i,k}$ and $b_{i,k}$, respectively.

The question now is, whether such control will be stable. If the real parameters are far from their mean values, the LQ control based on CE becomes unstable. If the set of parameters for which the closed loop system becomes unstable has a constant nonzero probability, then the criterion mean $\mathbf{E} J_N$ will go to infinity as $N \rightarrow \infty$. This is the case, when the parameter estimate is not updated during the control process and its variance remains unchanged. The only way to make the criterion mean $\mathbf{E} J_N$ converge to a finite value is to make the probability of the unstable set of parameters decrease sufficiently fast to zero. Based on the previous analysis, we can define the stability of a closed loop system in the following way: A closed loop system is stable, if $\mathbf{E} J^\infty = \lim_{N \rightarrow \infty} \mathbf{E} J_N < \infty$.

We will now show on a simple example that if the variance of the parameters decreases as N^{-1} , or equivalently, if its inverse (or information) increases linearly, than the stability is guaranteed for a CE feedback LQ controller. The condition of a linear information growth is also called a condition of persistent excitation in identification theory and guarantees that the parameter estimates converge fast to the real values.

2.2 Derivation of stability condition

As stated before, we will now show that a linear growth of information is sufficient to guarantee stability in the previously defined sense. We will not show a formal proof but rather use a simple example to demonstrate the idea.

Let us consider the following simple discrete integrator system

$$y_k = y_{k-1} + b_k u_k + e_k, \quad (4)$$

with only one unknown parameter b_k , that is modeled as a random variable in compliance with the previous subsection. For $r = 0$ in (3), the CE feedback control law is

$$u_k = -\frac{1}{\hat{b}} y_{k-1}, \quad (5)$$

where \hat{b} is the parameter mean value. The control law is defined for all nonzero \hat{b} , which is exactly the condition for the system to be controllable. The system output is then

$$y_k = y_{k-1} - \frac{b_k}{\hat{b}} y_{k-1} + e_k = \frac{\tilde{b}_k}{\hat{b}} y_{k-1} + e_k. \quad (6)$$

The noise e_k will be further omitted for simplicity, as it does not change the result. From (6) it follows that

$$y_k = \frac{\prod_{i=1}^k \tilde{b}_i}{\hat{b}^k} y_0, \quad (7)$$

and

$$J_N = \sum_{k=1}^N \{y_k^2\} = \sum_{k=1}^N \left\{ \frac{\prod_{i=1}^k \tilde{b}_i^2}{\hat{b}^{2k}} y_0^2 \right\}. \quad (8)$$

The mean of the criterion is then

$$\mathbf{E} J_N = \mathbf{E} \sum_{k=1}^N \{y_k^2\} = \sum_{k=1}^N \mathbf{E} \left\{ \frac{\prod_{i=1}^k \tilde{b}_i^2}{\hat{b}^{2k}} y_0^2 \right\}. \quad (9)$$

The criterion will only converge to a finite value, if the elements of the series converge to zero fast enough. But, using the independence assumption,

$$\mathbf{E} y_k^2 = \mathbf{E} \left\{ \frac{\prod_{i=1}^k \tilde{b}_i^2}{\hat{b}^{2k}} y_0^2 \right\} = \frac{\prod_{i=1}^k \sigma_i^2}{\hat{b}^{2k}} y_0^2, \quad (10)$$

where σ_i^2 is the variance of b_i . Now, if the linear growth of information is guaranteed, i.e. $\sigma_i^2 = \frac{\sigma_i^2}{i}$, it holds

$$\mathbf{E} y_k^2 = \left(\frac{\sigma_i}{\hat{b}} \right)^{2k} \frac{1}{k!} y_0^2, \quad (11)$$

so the series is convergent for any σ_1 and any nonzero \hat{b} . This idea can be even generalized for varying $\hat{b} = \hat{b}_k$, if it is guaranteed that $|\hat{b}_k| > \epsilon$ for all k and some $\epsilon > 0$.

3. PROBLEM FORMULATION AND ANALYSIS

This paper deals primarily with ARX systems that are usually given in a form of the following equation

$$y_k = z_k^T \theta + e_k = x_k^T \theta_x + u_k b_0 + e_k, \quad (12)$$

where $\theta = [b_0, a_1, b_1, \dots, a_n, b_n]^T = [b_0, \theta_x^T]^T$ is a vector of parameters and $z_k = [u_k, y_{k-1}, u_{k-1}, \dots, y_{k-n}, u_{k-n}]^T = [u_k, x_k^T]^T$.

The presented algorithms, however, are derived using state-space descriptions of a linear stochastic discrete-time system (Aström (1970)), in a usual form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ee_k \\ y_k &= Cx_k + Du_k + e_k, \end{aligned} \quad (13)$$

with the usual meaning of symbols, i.e. A, B, C, D and E are system matrices of proper dimensions, u_k, y_k and x_k are the system input, output and state, respectively and e_k is a gaussian white noise sequence with zero mean and constant finite variance.

Therefore the following nonminimal state-space representation of an ARX system (12) will be used

$$\begin{aligned} A &= \begin{bmatrix} a_1 & b_1 & \dots & b_{n-1} & a_n & b_n \\ 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} b_0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} E = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ C &= [a_1 \ b_1 \ \dots \ b_{n-1} \ a_n \ b_n] \quad D = [b_0] \end{aligned} \quad (14)$$

The state in this representation is x_k defined above and is directly measurable, as it is formed by previous inputs and outputs. The symbol θ has the meaning of the current estimate $\hat{\theta}_0$ from section 2 and will be used for simplicity of notation.

3.1 Problem formulation

A standard MPC problem is formulated as a minimization problem

$$U^* = \arg \min_U J_N = \arg \min_U \sum_{k=1}^N \{ru_k^2 + y_k^2\}, \quad (15)$$

$$\begin{aligned} \text{s. t.} \quad x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \\ |u_k| &\leq u_{max}, \quad |y_k| \leq y_{max} \end{aligned}$$

where N is the control horizon, r is a positive real tuning parameter and u_{max}, y_{max} are hard constraints on inputs and outputs, respectively. To ensure persistent system excitation, the criterion must also take into account the improvement of information gained after some amount of control inputs, i.e. the persistent excitation condition (1). In the case of an ARX system (12) it takes the form (Anderson and Moore (2005))

$$P_{t+M}^{-1} - P_t^{-1} = \sum_{k=t+1}^{t+M} \{z_k z_k^T\} \geq \gamma I, \quad (16)$$

where z_k is the system regressor at time k defined in (12). We will consider $t = 0$ for simplicity of notation, the case of general t is straightforward.

Let us now introduce some notation. It holds that $\sum_{k=1}^M z_k z_k^T = Z_M Z_M^T$, where $Z_M = [z_1, \dots, z_M]$. The

regressors are columns of the matrix Z_M and can be expressed as a linear function of the initial condition of the system $x_0 = [y_0, u_0, y_{-1}, u_{-1}, \dots, y_{-n+1}, u_{-n+1}]^T$ and the input vector $U = [u_1, \dots, u_N]^T$ as

$$z_k = F_k \begin{bmatrix} x_0 \\ U \end{bmatrix}, \quad k = 1, \dots, M, \quad (17)$$

where F_k is a matrix of appropriate dimensions. Similarly, the rows of Z_M are formed by shifted inputs and outputs, particularly $[u_1, \dots, u_M]$ to $[u_{-n+1}, \dots, u_{M-n}]$ and $[y_0, \dots, y_{M-1}]$ to $[y_{-n+1}, \dots, y_{M-n}]$. Let us denote the k -th row of Z_M as $w_k, k = 1, \dots, 2n + 1$. Also w_k can be expressed by

$$w_k^T = G_k \begin{bmatrix} x_0 \\ U \end{bmatrix}, \quad k = 1, \dots, 2n + 1, \quad (18)$$

where G_k is a matrix of appropriate dimensions. The vector $Y = [y_1, \dots, y_N]$ can be expressed as

$$Y = H \begin{bmatrix} x_0 \\ U \end{bmatrix}, \quad (19)$$

where H is a matrix of corresponding dimensions. Also let us call M the excitation horizon. Putting together (15) and (16) and using the introduced notation (17), (18) and (19) the problem has the form

$$U^* = \arg \min_U \left\{ rU^T U + [x_0^T \ U^T] H^T H \begin{bmatrix} x_0 \\ U \end{bmatrix} \right\} \quad (20)$$

$$\begin{aligned} \text{s. t.} \quad |u_k| &< u_{max}, \quad |y_k| < y_{max} \\ \sum_{k=1}^M \{z_k z_k^T\} &\geq \gamma I \end{aligned}$$

Because the suitable γ is hard to be stated apriori, it can be seen as a tuning parameter for the algorithm. We can see that there is a tradeoff between the criterion value J_N and the minimum eigenvalue γ . In some cases, it is more natural to reverse the problem – define the maximum criterion value and maximize γ within these constraints. Let us denote the optimal criterion value of the MPC problem (15) as J_N^* . Then the alternative formulation of the problem is

$$\gamma^* = \arg \max_U \gamma, \quad (21)$$

$$\begin{aligned} \text{s. t.} \quad \left\{ rU^T U + [x_0^T \ U^T] H^T H \begin{bmatrix} x_0 \\ U \end{bmatrix} \right\} &\leq J_N^* + \Delta J \\ |u_k| &\leq u_{max}, \quad |y_k| \leq y_{max} \\ \sum_{k=1}^M \{z_k z_k^T\} &\geq \gamma I \end{aligned}$$

for a given maximum criterion change ΔJ .

3.2 Problem analysis

The problem (20) or its alternative (21) differ from the original MPC (15) only in the last condition. As the MPC problem is convex and standard algorithms exist for its solution, the presented algorithms in fact differ only in how they cope with the last condition (16).

Because the information matrix (16) consists of quadratic and bilinear terms, both problems are non-convex in control inputs, as demonstrated in Figure 1, which shows

the lowest eigenvalue of the information matrix of a second order ARX system after two steps of control as a function of the two inputs u_1 and u_2 . This is a difference from simple one-step approaches where the solution always lies on the constraints (Filatov and Unbehauen (2004)) and is a reason for using numerical methods.

The second problem caused by the extra condition (16) is that the sum cannot be actually computed precisely, because it contains future outputs that do not depend only on future inputs, but also on the parameter values and input noise. A lower bound of the mean of the variance matrix could be achieved by computing $\mathbb{E} z_k \mathbb{E} z_k^T \leq \mathbb{E}(z_k z_k^T)$ from Jensen's inequality. However, even computing $\mathbb{E} z_k$ is complicated, as the computation needs higher moments of the parameter joint distribution. Therefore, the conditional mean $\mathbb{E}_{\theta_0} z_k = \mathbb{E}(z_k | \theta_k = \theta_0, \forall k = 1, \dots, M)$ is used instead of the mean $\mathbb{E} z_k$.

Also note that because the information matrix increment in (16) is a sum of M dyads, its rank is less or equal to M . Therefore it is necessary that $M \geq 2n+1$ (i.e. the length of the regressor) to be able to achieve that all its eigenvalues are positive. On the other hand, N should be significantly greater than M so that the control criterion can take into account the future impact of identification procedure on the control quality. The last observation is that as the criterion minimization and information maximization are in contradiction, the persistent excitation condition may not be possible to satisfy, i.e. the problem may easily be infeasible for some choice of ΔJ and γ .

4. PROBLEM SOLUTION

In the previous section, the problem was formulated as a non-convex problem. The non-convexity introduced by (16) can be handled in several ways. This section presents three different methods to solve the problem (20) and (21).

4.1 Rank 1 algorithm

The rank 1 algorithm is based on a convex relaxation of the problem and concentrating all non-convexity into a rank constraint. Using the notation (17), (16) is rewritten as

$$\sum_{k=1}^M F_k \begin{bmatrix} x_0 \\ U \end{bmatrix} [x_0^T \ U^T] F_k^T > \gamma I, \quad (22)$$

or in a simplified form

$$\sum_{k=1}^M F_k U_X F_k^T > \gamma I, \quad (23)$$

using the notation

$$\begin{bmatrix} x_0 \\ U \end{bmatrix} [x_0^T \ U^T] = \begin{bmatrix} x_0 x_0^T & x_0 U^T \\ U x_0^T & U U^T \end{bmatrix} = U_X. \quad (24)$$

The matrix U_X consists of constant terms $x_0 x_0^T$, terms $x_0 U^T$ and $U x_0^T$ linear in U , and the term $U U^T$ quadratic in U . The quadratic term makes the problem (23) unsolvable as an LMI directly, and therefore the following reformulation is used

$$U_{X2} = \begin{bmatrix} x_0 x_0^T & x_0 U^T \\ U x_0^T & U_q \end{bmatrix} \quad (25)$$

$$\text{s. t. } \text{rank}(U_{X2}) = 1, \quad (26)$$

where U_q is now a general symmetric, positive definite matrix, replacing the quadratic term $U U^T$. All non-convexity is now concentrated in the rank constraint (26) and dropping this constraint the task can be solved as a normal LMI problem (Boyd et al. (1994)) in more variables, known also as Schor's relaxation (Vandenberghe and Boyd (1996); Lasserre (2000)).

Expressing the criterion as a Schur complement (Bernstein (2005)) this relaxation makes it possible to solve the original problem as a rank constrained LMI

$$U^* = \arg \min_U \lambda \quad (27)$$

$$\text{s. t. } \begin{bmatrix} \lambda & [x_0^T \ U^T] H^T & U^T \\ H \begin{bmatrix} x_0 \\ U \end{bmatrix} & I & 0 \\ U & 0 & \frac{1}{r} I \end{bmatrix} \geq 0$$

$$|u_k| < u_{max}, \quad |y_k| < y_{max}$$

$$\sum_{k=1}^M F_k U_{X2} F_k^T > \gamma I$$

$$\text{rank } U_{X2} = 1$$

Again, two versions corresponding to formulation (20) or (21) are possible.

4.2 Gershgorin circle algorithm

This algorithm is based on eigenvalue approximation in terms of Gershgorin circles (Bernstein (2005)). For a real matrix A with entries a_{ij} define $R_i = \sum_{j \neq i} |a_{ij}|$, i.e. the sum of absolute values of elements of the i -th row without the diagonal element. Then each eigenvalue lies in at least one of the Gershgorin circles defined as intervals $[a_{ii} - R_i; a_{ii} + R_i]$ for every i . This idea can be used to create constraints on the elements of the information matrix P_M^{-1} . If the diagonal elements a_{ii} are greater than some γ_1 and the nondiagonal sum less than γ_2 , then the lowest eigenvalue must be greater than $\gamma_1 - \gamma_2$.

Let us now formulate the above idea as an optimization problem. The standard MPC part of the algorithm is formed by the first two lines of (20) and the additional constraints are imposed on the elements a_{ij} of the information matrix $P_M^{-1} = Z_M Z_M^T$. Using the fact that $a_{ij} = w_i w_j^T$ and notation (18), it is necessary to ensure that

$$b_{ij} > [x_0^T \ U^T] G_i^T G_j \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i, j = 1, \dots, 2n+1, i < j$$

$$b_{ij} > - [x_0^T \ U^T] G_i^T G_j \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i, j = 1, \dots, 2n+1, i < j$$

$$b_{ij} = b_{ji}$$

$$\gamma_2 > \sum_{j \neq i} b_{ij}, \forall i = 1, \dots, 2n+1$$

$$\gamma_1 < [x_0^T \ U^T] G_i^T G_i \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i = 1, \dots, 2n+1 \quad (28)$$

where b_{ij} are artificial variables that have the meaning of absolute values of a_{ij} . Because the matrix P_M^{-1} is symmetrical, the first two constraints in are only required for $i < j$.

4.3 Orthogonal regressors algorithm

This algorithm is based on the idea, that the regressors shape the information ellipsoid, that is the ellipsoid $x^T(P_M^{-1})^{-1}x = x^T P_M x = 1$. The eigenvalues of P_M^{-1} correspond to the ellipsoid radii. Therefore similarly to the previous algorithm, it is necessary to ensure that the regressors' norms $|z_i| > \gamma_1$ and that the regressors are 'as much orthogonal as possible', meaning that for all $i \neq j$, $z_i^T z_j < \gamma_2$. The problem again consists of the first two lines of (20) and the following constraints

$$\begin{aligned} b_{ij} &> [x_0^T \ U^T] F_i^T F_j \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i, j = 1, \dots, M, i < j \\ b_{ij} &> -[x_0^T \ U^T] F_i^T F_j \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i, j = 1, \dots, M, i < j \\ b_{ij} &< \gamma_2, \forall i, j = 1, \dots, M, i < j \\ \gamma_1 &< [x_0^T \ U^T] F_i^T F_i \begin{bmatrix} x_0 \\ U \end{bmatrix}, \forall i = 1, \dots, M \end{aligned} \quad (29)$$

The structure of the problem is similar to the previous one, the difference is in the problem dimension. While the number of constraints is $\frac{(2n+1)(2n)}{2}$ and the dimension of the vectors is M in the Gershgorin algorithm, in this case it is the reverse, i.e. the dimension of regressors is $2n + 1$ and the number of constraints is $\frac{(M)(M-1)}{2}$. From this follows that in this case, M should be equal to $2n + 1$, as the number of regressors should not be higher than their dimension.

4.4 Stability

The stability of the proposed algorithms in the usual (Lyapunov) sense can be guaranteed for the nominal system, i.e. the system for which $a_{i,k} = 0$ and $b_{j,k} = 0$ for all $i = 1 \dots n$, $j = 1 \dots m$ and all $k = 1 \dots N$. This follows from the stability of the MPC controller (Rawlings and Muske (1993)) and from the fact that the criterion is bounded, therefore the difference $\delta u_i = u_i - u_i^*$ is square summable, u_i^* denoting the MPC optimal solution.

5. SIMULATIONS

Simulations of the previously proposed algorithms are shown in this section. The following ARX system was used $y_k = 1.64y_{k-1} - 0.67y_{k-2} + 0.2u_k + 0.22u_{k-1} - 0.12u_{k-2} + e_k$, (30)

which is obtained by discretization of a system $1/(s + 1)^2$ with a sampling period $T_s = 0.2s$ and modified in order to have $b_0 \neq 0$. The system is controlled to zero from the initial state $x_0 = [10, 0, 0, 0]^T$, i.e. the initial output $y_0 = 10$. Note that the nonminimal representation (14) is used, so the system order is 4. The control was designed for $N = 30$, $M = 5$, $r = 1$ and $\Delta J = 0.1J_N^*$. Figures 2 and 3 show the inputs and outputs of a control process for optimal MPC controller and all three designed controllers, respectively. Figure 4 shows the development of the variance matrix in the sense of its greatest eigenvalue.

The Rank 1 algorithm was used in the form of (21) and was solved by YALMIP (Löfberg (2004)) in MATLAB, with

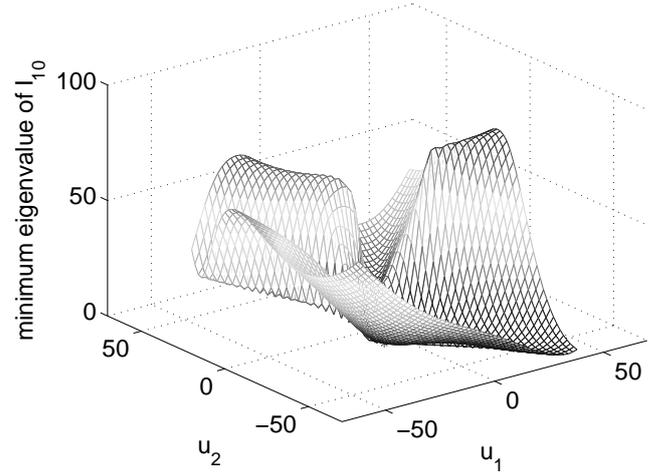


Fig. 1. The lowest eigenvalue of the predicted information matrix after $M = 5$ steps of control as a function of the two first inputs u_1 and u_2 around the optimal MPC solution for the ARX system (30)

help of the LMIRANK solver (Orsi et al. (2006)). As the solver only searches for feasible points, the algorithm was run sequentially with γ varying according to the interval bisection method to find the maximum information. Both the Gershgorin and the regressor algorithm were solved by the MATLAB standard function FMINCON.

6. CONCLUSIONS

This paper proposes three different algorithms for simultaneous identification and control, based on a standard MPC approach with a demand on the parameter information matrix in a form of the persistent excitation condition. The paper also shows a motivational example which explains the connection of the persistent excitation with a presented concept of stability.

The proposed algorithms are derived from a general formulation and in some cases it is shown that the persistent excitation may not be satisfied precisely and only approximations are found. However, simulations show that the use of the proposed methods lead to better identification.

The drawback of all three methods is the inherent non-convexity of the problem that causes convergence to local optima only. Therefore the performance is not guaranteed and may vary depending on the algorithms settings such as the starting point or control and excitation horizon.

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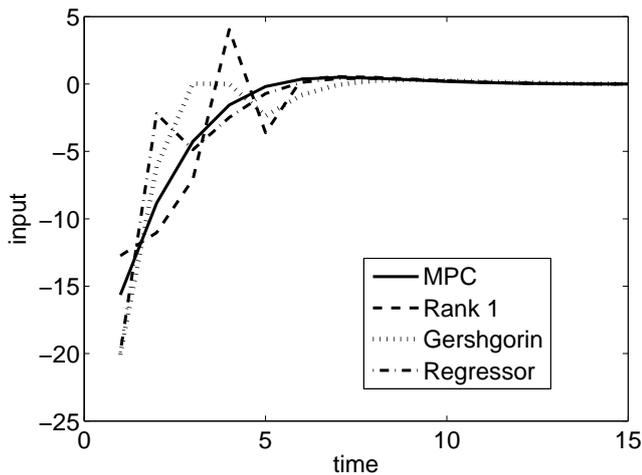


Fig. 2. The control input designed by classical MPC and modifications by all three proposed algorithms for excitation horizon $M = 5$.

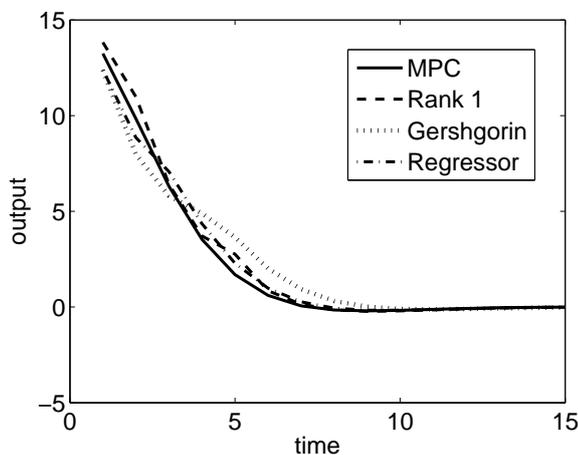


Fig. 3. The output of a system controlled by classical MPC and modifications by all three proposed algorithms for excitation horizon $M = 5$.

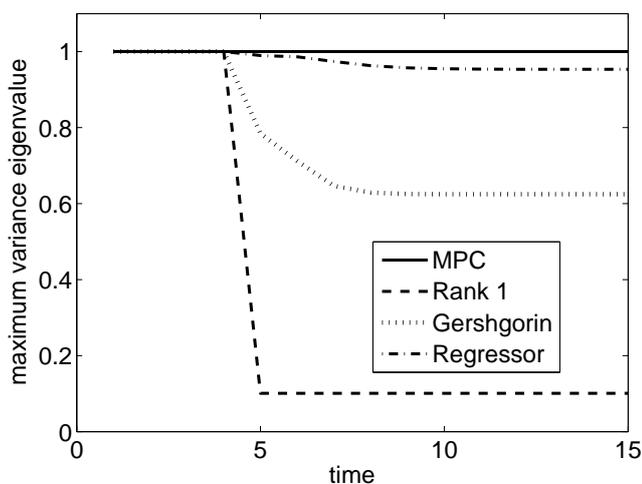


Fig. 4. The maximum eigenvalue of the estimate variance matrix for control designed by classical MPC and modifications by all three proposed algorithms for excitation horizon $M = 5$.

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