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Robust Control of a Hydraulic System with Unstructured Uncertainties

M. Karšaiová M. Bakošová A. Vasičkaninová

Department of Information Engineering and Process Control, Faculty of Chemical and Food Technology, Slovak University of Technology, Radlinského 9, 812 37 Bratislava, Slovakia
Tel. + 421 2 59 325 362; e-mail: {maria.karsaiova, monika.bakosova, anna.vasickaninova}@stuba.sk

Abstract: The paper presents simulation results obtained by robust control of a system of three serially connected tanks. The method used for robust controller design is based on the small gain theorem. The robust PID controller is designed that assures the stability of the closed-loop control system for a certain range of unstructured uncertainties.

Key words: unstructured uncertainty, robust controller, robust stability, small gain theorem

1. INTRODUCTION

Uncertainty arises when some aspect of the system model is not completely known at the time of analysis and design. The typical example of a structured uncertainty is the value of a parameter which may vary according to operating conditions. The unstructured uncertainty can be caused by simplified modelling, when it is used to avoid very detailed and complex models. The other reasons for unstructured uncertainties are process non-linearity, changes of operating conditions and external disturbances. Dynamic systems with unstructured uncertainties are widely used to model physical systems.

The small gain theorem (Green and Limebeer, 1994) is a tool for robust controller design for systems with unstructured uncertainty (Karafyllis and Zhong-Ping, 2007). The small gain theorem states that stable systems can be connected to form a stable closed-loop if the loop gain product is less than unity. It is the basis for the general robust stability results.

The paper describes the robust PID controller design for three serially connect tanks. The process is modelled as a system with unstructured additive uncertainty and the robust controller design is based on the small gain theorem. The designed robust controller is tested by simulations.

2. ROBUST STABILITY

Suppose that the transfer function of an uncertain continuous-time system with additive unstructured uncertainty has the form

$$\begin{aligned} G(s) &= G_0(s) + G_{\Delta A}(s) \\ G_{\Delta A}(s) &= W_A(s)\Delta_A(s) \end{aligned} \quad (1)$$

where $G_0(s)$ is the nominal model, $W_A(s)$ is the weight function and $\Delta_A(s)$ is a category of uncertainties that satisfies the condition $\left|\Delta_A(j\omega)\right| \le 1$ for $\forall\omega$.

The task is to find a robust controller for control of the system (1). The design method is based on the small gain theorem (Green and Limebeer, 1994, Veselý and Harsanyi, 2007) and uses the fact that if a feedback loop consists of stable systems and the loop-gain product is less than unity, then the feedback loop is internally stable. The other basis for the design is a fixed point theorem known as the contraction mapping theorem (Khalil, 1996).

According to the small gain theorem, following conditions have to be satisfied: the controller with the transfer function $G_R(s)$ stabilizes the nominal model and for the open-loop transfer function $L(s)$, the condition given in (2) also holds.

$$\begin{aligned} &L(s) = G(s)G_R(s) \\ &\left|L(j\omega)\right| < 1 \end{aligned} \quad (2)$$

The family of the controlled system transfer functions $G(s)$ creates a set, in which $G_0(s)$ is the transfer function of the nominal system and $G_k(s)$ is a transfer function from the set $G(s)$, which differs from $G_0(s)$. Then, the value $l_A(\omega)$ can be calculated as the maximal value of modules as it is shown in (3)

$$\begin{aligned} &l_A(\omega) = max\left|G_k(j\omega) - G_0(j\omega)\right| \\ &\omega \in (0,\infty), \quad k = 1,2..... \end{aligned} \quad (3)$$

The characteristic equation of the closed loop with uncertain controlled system is

$$1 + G_R(s)G(s) = 0 \quad (4)$$

and after the substitution (1) into (4), we obtain

$$\left[1+G_R(s)G_0(s)\right]\left[1+V_0(s)\frac{G_{\Delta A}(s)}{G_0(s)}\right]=0 \tag{5}$$

where $V_0(s)$ is the closed-loop transfer function with the nominal model and has the form

$$V_0(s)=\frac{G_R(s)G_0(s)}{1+G_R(s)G_0(s)} \tag{6}$$

The closed loop must be stable. The small gain theorem requires satisfying also the second condition. It follows from (5) that for the second term in (5) the following condition holds

$$\left[1+V_0(s)\frac{G_{\Delta A}(s)}{G_0(s)}\right]=0 \tag{7}$$

Then after the substitution $s = j\omega$ we obtain

$$\left|V_0(j\omega)\frac{G_{\Delta A}(j\omega)}{G_0(j\omega)}\right|<1 \quad \forall\omega\in(0,\infty) \tag{8}$$

The conditions $\left|\Delta_A(j\omega)\right|=1$ and $\left|W_A(j\omega)\right|=l_A(\omega)$ represent the worst cases and so, it is possible to rewritten (8) to the form

$$\left|V_0(j\omega)\right|<\frac{\left|G_0(j\omega)\right|}{l_A(\omega)} \tag{9}$$

Robust controller design is then based on finding parameters of the transfer function $V_0(s)$, the choice of the structure of the robust controller and calculation of the controller parameters.

3. ROBUST PID CONTROLLER DESIGN

A robust PID controller was designed for the process represented by three serially connected tanks. The controlled variable is the liquid level h_3 in the 3rd tank and the manipulated variable is the flow rate of the inlet stream of water q. The inputs, outputs and parameters of the nominal model in a steady state are represented by following values: inlet flow rate q^s =1m^3 min^{-1}, steady-state value of level h_3^s = 0.444 m, valve constants k_{11} = 1 m$^{2.5}$ min^{-1}, k_{22} = 1.5 m$^{2.5}$ min^{-1}, k_{33} = 1.5 m$^{2.5}$ min^{-1} and cross-section areas of tanks F_1 = 0.5 m^2, F_2 = 0.1 m^2, F_3 = 0.1 m^2. Unstructured uncertainties result from simplification of the mathematical model using linearization and changes of the valve constant k_{33}. The family of the transfer functions in three operating points is following

$$G_0(s)=\frac{G_{0n}(s)}{G_{0d}(s)}=\frac{0.9}{0.008s^3+0.186s^2+1.18s+1} \tag{10}$$

$$G_1(s)=\frac{G_{1n}(s)}{G_{1d}(s)}=\frac{1.18}{0.01s^3+0.21s^2+1.2s+1} \tag{11}$$

$$G_2(s)=\frac{G_{2n}(s)}{G_{2d}(s)}=\frac{0.69}{0.006^3+0.16s^2+1.16s+1} \tag{12}$$

where $G_0(s)$ is the nominal model, $G_1(s)$ is the model obtained with k_{33} = 1.3 m$^{2.5}$ min^{-1} and $G_2(s)$ is the model obtained for k_{33}= 1.7 m$^{2.5}$ min^{-1}. Figure 1 shows the function $l_A(\omega)$ which was determined using (3).

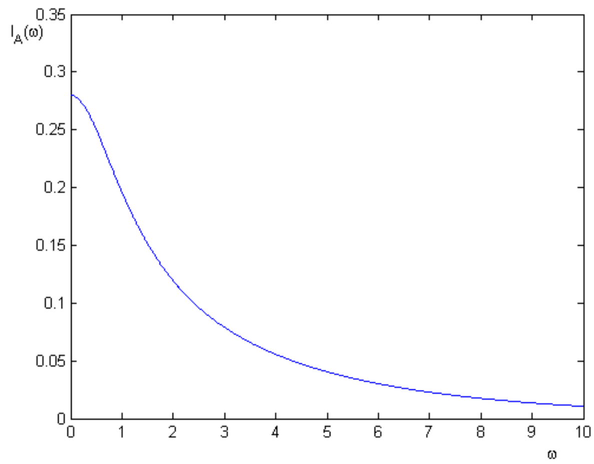

Fig. 1. Dependence of maximal values of modules on the frequency ω

The structure of robust PID controller was selected in the form:

$$G_R(s)=\frac{G_{Rn}(s)}{G_{Rd}(s)}=\frac{1}{K}\frac{C(s)}{s}=\frac{1}{K}\frac{c_1s^2+c_0s+c_{-1}}{s} \tag{13}$$

The transfer function $V_0(s)$ is in the form

$$V_0(s)=\frac{G_{0n}(s)}{G_{0n}(s)+D_{0d}(s)Ks} \tag{14}$$

Parameter K is an optional parameter and the function $V_0(s)$ has to satisfy (9). The polynomial $D_{0d}(s)$ is optional, too, and the following equation has to be satisfied

$$G_{0d}(s) = D_{0d}(s)C(s) \tag{15}$$

Parameters d_1, c_1, c_0, c_{-1} are calculated from the following equation

$$0.008s^3 + 0.18s^2 + 1.18s + 1 = (d_1 s + 1)(c_1 s^2 + c_0 s + c_{-1}) \tag{16}$$

and their values are: $d_1 = 1.1765$, $c_1 = 0.0068$, $c_0 = 0.0035$, $c_{-1} = 1$. The transfer function $V_0(s)$ has the form

$$V_0(s) = \frac{1}{d_1 K s^2 + Ks + 1} \tag{17}$$

and it is affected by the choice of the parameters K. Figure 2 illustrates the dependence of $\frac{|G_0(j\omega)|}{l_A(\omega)}$ and the dependence of $|V_0(j\omega)|$ on ω calculated for various values of K (blue lines). It is clear from Figure 2 that the boundary value of K is $K = 0.11$.

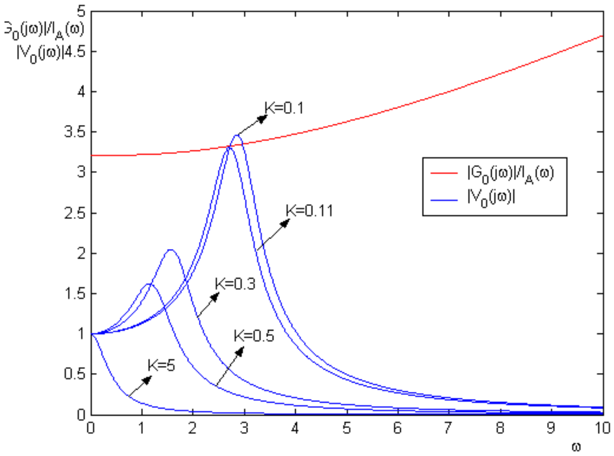

Fig. 2. Amplitude characteristics of V_0 for various K

The system of three serially connected tanks was controlled using three robust PID controllers designed by the small gain theorem for various feasible values of the parameter K. The transfer functions of found controllers are:

$$G_R(s) = 0.0117 + \frac{3.333}{s} + 0.0227s, \quad K = 0.3 \tag{18}$$

$$G_R(s) = 0.007 + \frac{2}{s} + 0.136s, \quad K = 0.5 \tag{19}$$

$$G_R(s) = 0.0007 + \frac{0.2}{s} + 0.0014s, \quad K = 5 \tag{20}$$

Designed controllers were tested by simulation experiments for the nominal model with $k_{33} = 1.5$ (the index 0), the model with $k_{33} = 1.3$ (the index 1) and the model with $k_{33} = 1.7$ (the index 2). The set point was $w = 0.4$ and it changed at the time 40 min to $w = 0.6$. The simulation results obtained using the PID robust controllers (18) – (20) are presented in Figs. 3 – 5.

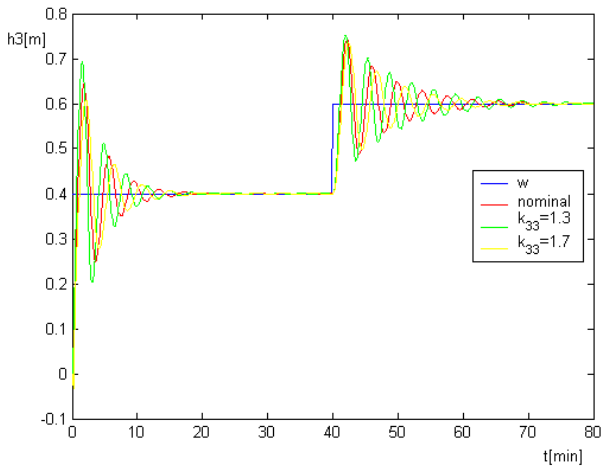

Fig. 3. Control responses of the system of three serially connected tanks with the controller (18)

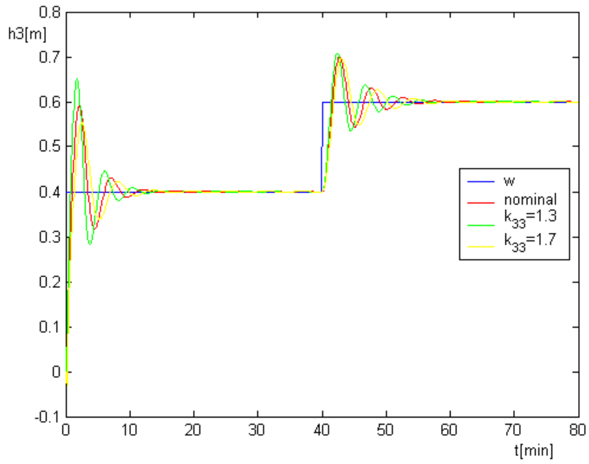

Fig. 4. Control responses of the system of three serially connected tanks with the controller (19)

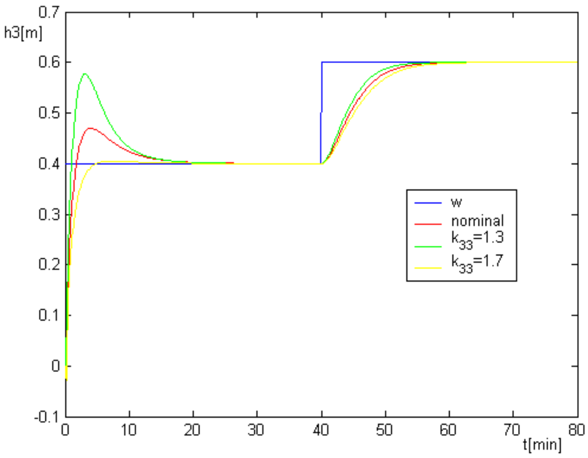

Fig. 5. Control responses of the system of three serially connected tanks with the controller (20)

The control responses obtained using three robust controllers were compared by evaluation of the IAE performance indexes. Their values are summarizes in the Table 1.

Table 1 Comparison of robust controllers using IAE performance indexes

	IAE_0	IAE_1	IAE_2
K=0.3	1.817	2.0541	1.7609
K=0.5	1.26	1.308	1.294
K=5	1.8218	2.1527	1.6828

One PID controller was designed for K higher then 0.11 to show that such choice leads to unstable control responses. The choice was K = 0.1 and the inequality (9) is not satisfied in this case. The transfer function of the found controller is

$$G_R(s) = 0.035 + \frac{10}{s} + 0.068s, \quad K = 0.1 \tag{21}$$

The simulation results obtained using the PID controller (21) are presented in Fig. 6.

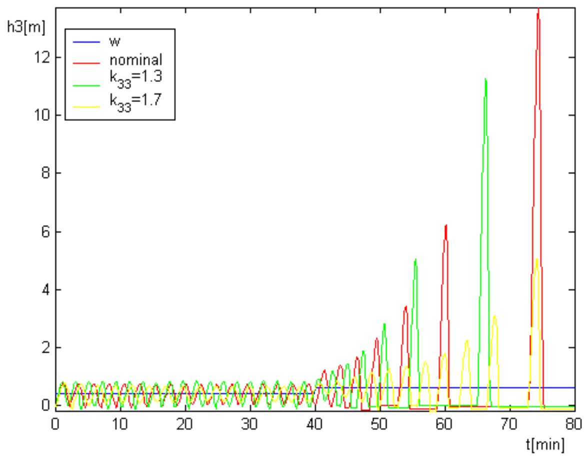

Fig. 6. Control responses of the system of three serially connected tanks with the controller (20)

4. CONCLUSION

Obtained simulation results confirmed, that it is possible to assure good control responses of controlled processes with unstructured uncertainties using robust PID controllers. The optional parameter K used in the controller synthesis depends on the controlled system and on the amplitude of unstructured uncertainties. The stability and quality of the control response depends on the value of optional parameter K and it is important to find its boundary value.

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