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New Mathematical Tools for Analysis and Control of Platoons of Cars in Future Automated Highway Systems

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Abstract. This paper introduces new mathematical tools for stabilization and asymptotic following in infinite platoons of vehicles in future automated highway systems. The platoon description, behavior analysis and control is approached in 2-D polynomial framework, that is, the dynamics of the problem are described using a fraction of two bivariate polynomials. In contrast to some previous works, the platoon here assumes a leader (and an infinite number of followers), therefore the often used bilateral z-transform should not be used here since it was developed for doubly infinite vehicular strings. The unilateral z-transform seems better suited. However, it brings about the need to take the boundary conditions into consideration; among other, the leader vehicle comes into the scene. The necessary formalism is introduced in the paper and used to provide elegant alternative proofs of some well-known facts about the platooning problem.

Keywords: Automated highway systems, automated guided vehicles, multidimensional systems, multivariable polynomials, polynomial equations, polynomial methods.

In current studies automated highway systems [2], platooning is conceived as a way of expanding the envelope of capacity and safety that can be achieved by road vehicles. When the vehicles are organized in platoons, they can operate much closer together than is possible under manual driving conditions. Each highway lane can therefore carry several times as much traffic as it can today, which should make it possible to greatly reduce highway congestion. Also, at close spacing aerodynamic drag is significantly reduced, which can lead to major reductions in fuel consumption and exhaust emissions. The high-performance vehicle control system also increases the safety of highway travel, reduces driving stress and tedium, and provides a very smooth ride.

In the microscopic description of the highway, vehicles are individually modeled. The vehicles headway is defined as the time taken for the vehicle to traverse

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the inter-vehicle spacing ahead of it. There are various control policies for the platoon characterized by different speed-spacing relationships that the vehicle control system aims to guarantee. A popular control policy is that of constant time headway which makes the desired ranges proportional to vehicle speeds.

When vehicles follow each other automatically in a lane, they behave like a coupled system, the behavior of which depends on the control actions of the individual vehicles. A phenomenon known as "slinky effect" [2] where a small tracking error or disturbance in the response of a lead vehicle gets amplified as it propagates along the platoon or string of vehicles is commonly observed in today's driving. The longitudinal control system of each system has to be designed so to guarantee platoon or string stability, which in turn implies the absence of slinky effects.

As the highway lane capacity increases with the length of the platoon [2], very long platoons are desirable. This fact is reflected in theoretical literature differently: Either platoons with a finite number of cars are studied and then the limit infinite case is taken. Or doubly infinite strings of vehicles are considered allowing to apply bilateral transforms that neglect any boundary conditions.

Instead, we advocate the use semi-infinite platoons with a leader where the leading vehicle is labeled by 0 and the follow-up cars are numbered by $1, 2, \ldots$. Positions, distances and velocities in the platoon are described by spatial sequences of time functions corresponding to the equally indexed vehicles $\{f(t, k)\} = f(t, 0), f(t, 1), f(t, 2), \ldots, \ t \in [0, \infty)$. To handle such sequences, we introduced an original joint unilateral Laplace and (shifted) unilateral z-transform denoted $\mathcal{LZ}_1$ which is defined by

$$\mathcal{LZ}_1 \{f(t, k)\} = \int_{0^-}^{\infty} \left(\sum_{k=1}^{\infty} f(t, k) z^{-k} \right) e^{-st} \, dt.$$

Application of the $\mathcal{LZ}_1$ transform opens the door to rich world 2-D polynomial systems theory [5]. In addition, it reveals that the leading vehicle movement is actually boundary condition for the coupled platoon system.

The transform has been employed to solve various problems of analysis and control for platoons with leader [1, 6].

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