MPC-Based Reference Governors for Thermostatically Controlled Residential Buildings

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January 3, 2016

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Building Control Motivation

Problem: EU spends 400 billion EUR/year on energy.

* International Energy Agency, ‘Energy efficiency requirements in building codes, energy efficiency policies for new buildings’ 2013 OECD/IEA.

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Solution: Thermal comfort control

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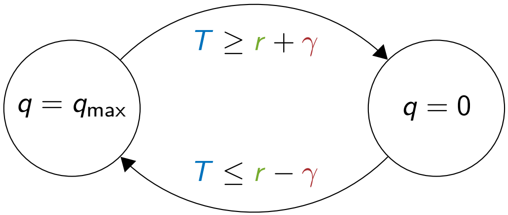
Digital Relay Thermostat

Relay Thermostat Dynamics



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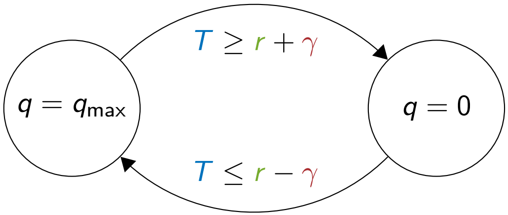

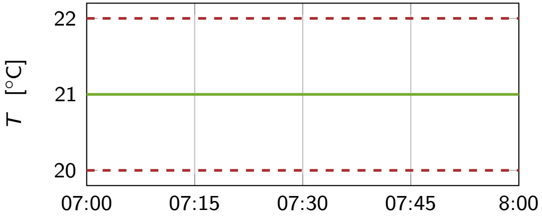

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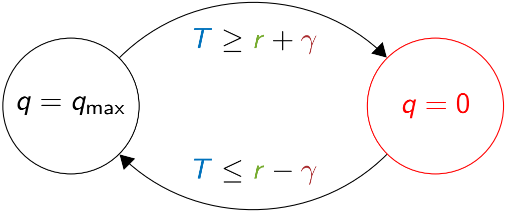

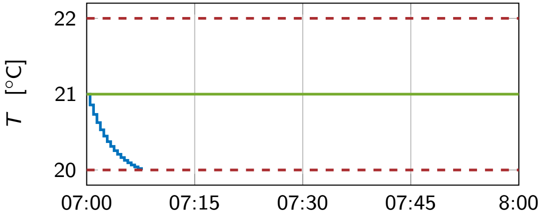

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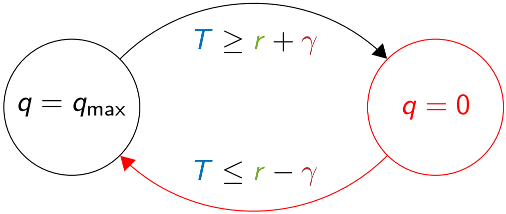

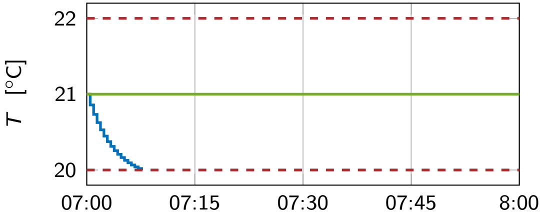

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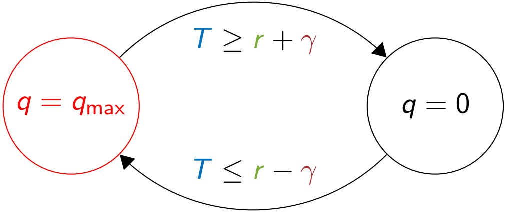

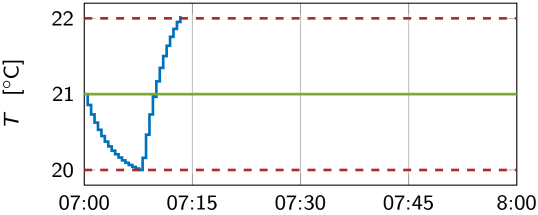

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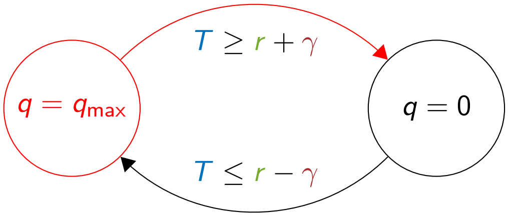

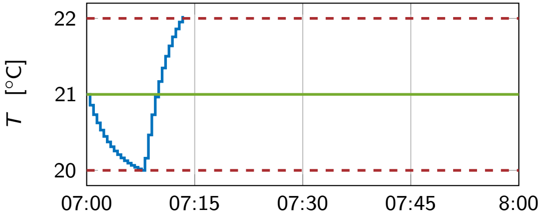

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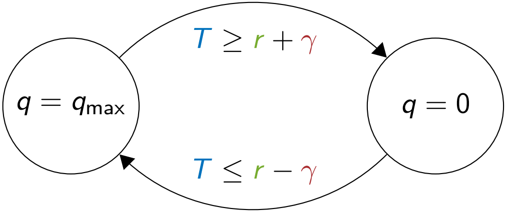

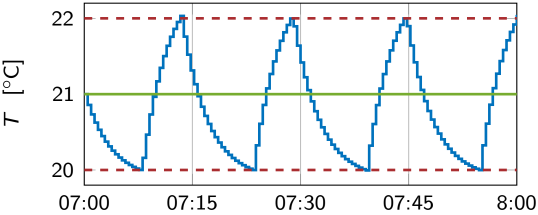

Intelligent Thermostat

Low Cost Intelligent Thermostat

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Model Predictive Control

Pros:

- Satisfy thermal comfort constraints
- Minimize energy consumption
- Obey technological restrictions

Cons:

- Computational Demands

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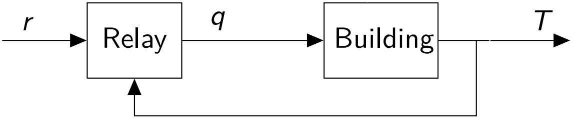
Standard Control Architecture



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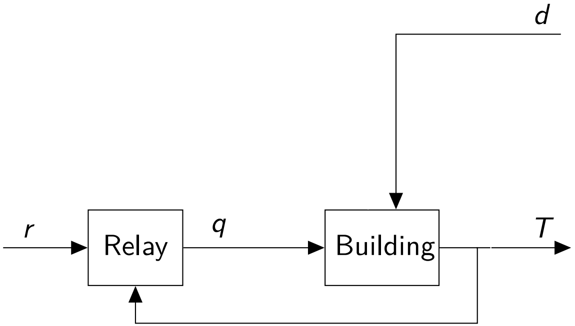
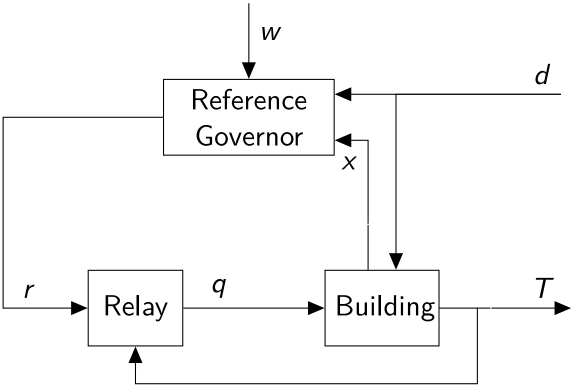


Reference Governor Architecture



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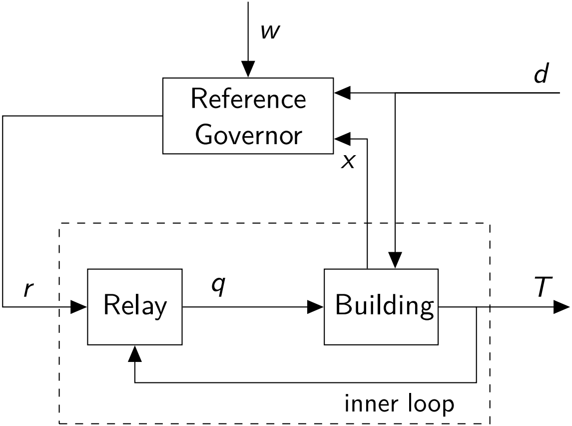


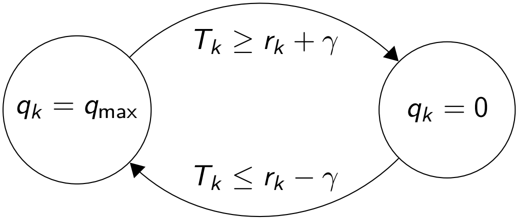
Reference Governor MPC Formulation

$$\min_{r_0,\dots,r_{N-1}} \sum_{k=0}^{N-1} (q_k + \alpha s_k)$$

$$\text{s.t. } x_{k+1} = Ax_k + Bq_k + Ed_k$$

$$T_k = Cx_k$$

$$w - \theta - s_k \leq T_k \leq w + \theta + s_k$$



Reference Governor MPC Formulation

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\begin{aligned}
\min_{r_0,\ldots,r_{N-1}} \quad & \sum_{k=0}^{N-1} (q_k + \alpha s_k) \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bq_k + Ed_k \\
& T_k = Cx_k \\
& w - \theta - s_k \leq T_k \leq w + \theta + s_k \\
& q_k = \begin{cases} q_{\max} & \text{if } z_k = 1 \\ 0 & \text{otherwise} \end{cases} \\
& z_{k+1} = f(z_k, T_k, r_k)
\end{aligned}
$$

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\min_{r_0,\ldots,r_{N-1}} \quad & \sum_{k=0}^{N-1} (q_k + \alpha s_k) \\
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& z_{k+1} = \begin{cases} 1 & \text{if} \quad (z_k = 1 \ \wedge \ \sim (T_k \geq r_k + \gamma)) \vee \\ & \qquad (z_k = 0 \ \wedge \ \ (T_k \leq r_k - \gamma)) \\ 0 & \text{otherwise} \end{cases}
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& q_k = \begin{cases} q_{\text{max}} & \text{if } z_k = 1 \\ 0 & \text{otherwise} \end{cases} \\
& z_{k+1} = \begin{cases} 1 & \text{if } (z \wedge \sim \delta_{\text{a}}) \vee (\sim z \wedge \delta_{\text{b}}) \\ 0 & \text{otherwise} \end{cases} \\
& [\delta_{\text{a}} = 1] \Leftrightarrow [T \geq r + \gamma], \\
& [\delta_{\text{b}} = 1] \Leftrightarrow [T \leq r - \gamma].
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\min_{r_0,\ldots,r_{N-1}} \quad & \sum_{k=0}^{N-1} (q_k + \alpha s_k) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B q_k + E d_k \\
& T_k = C x_k \\
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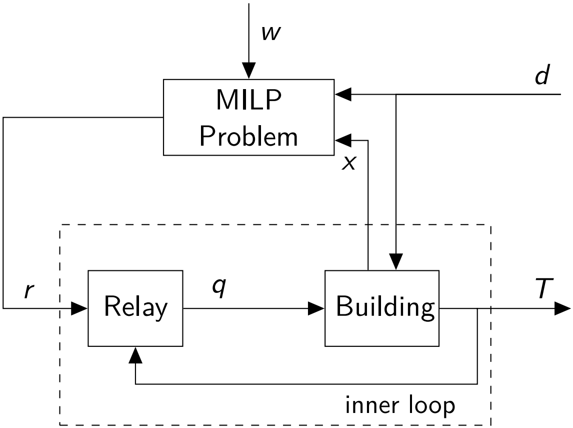
& q_k = \begin{cases} q_{\mathrm{max}} & \text{if } z_k = 1 \\ 0 & \text{otherwise} \end{cases} \\
& z_{k+1} = \begin{cases} 1 & \text{if } (z \wedge \sim \delta_{\mathrm{a}}) \vee (\sim z \wedge \delta_{\mathrm{b}}) \\ 0 & \text{otherwise} \end{cases} \\
& m_{\mathrm{a}} \delta_{\mathrm{a}} \le r + \gamma - T \le M_{\mathrm{a}}(1 - \delta_{\mathrm{a}}), \\
& m_{\mathrm{b}} \delta_{\mathrm{b}} \le T - r + \gamma \le M_{\mathrm{b}}(1 - \delta_{\mathrm{b}}).
\end{aligned}
$$

Closed-Loop Implementation



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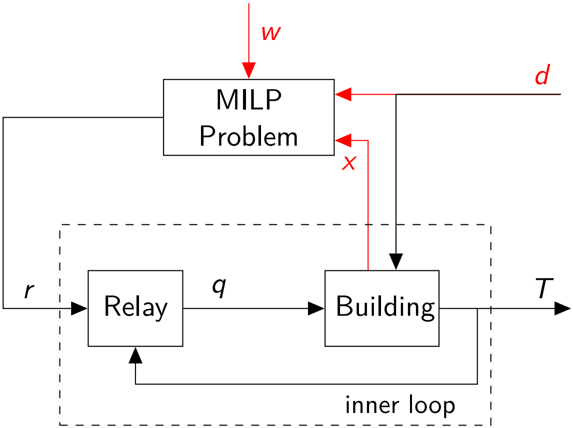

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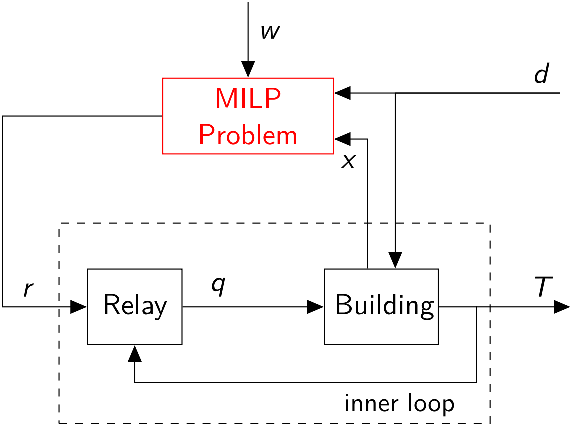

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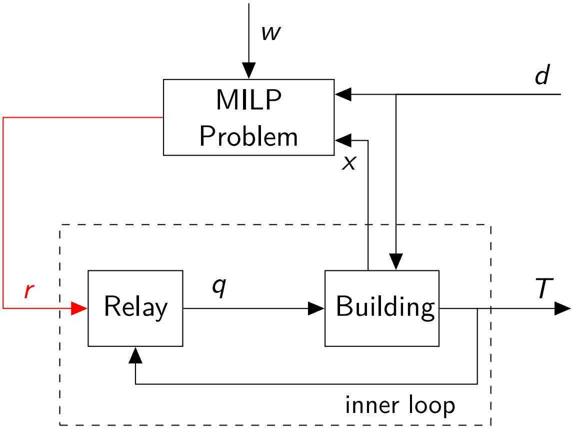

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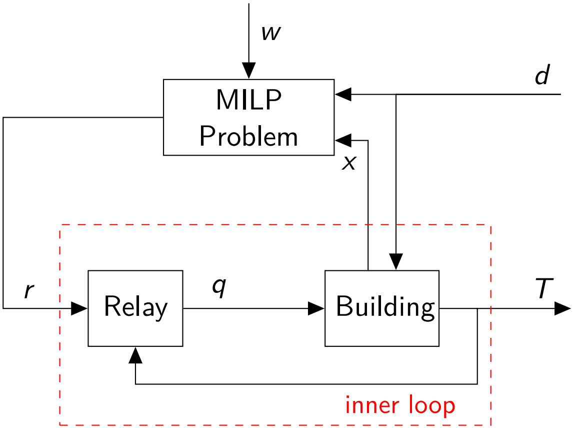


Single Zone Building Model

State (Measured) Variables

x_1 – floor temperature
x_2 – internal facade temperature
x_3 – external facade temperature
x_4 – internal temperature

Controlled Variable

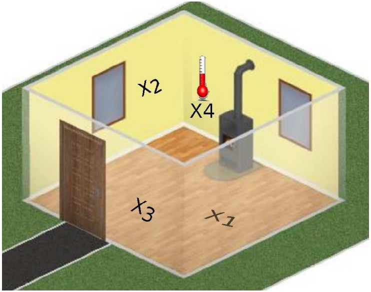
$y = x_4$

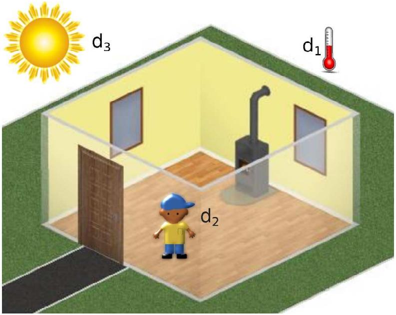
Measured Disturbances

d_1 – external temperature
d_2 – occupancy
d_3 – solar radiation

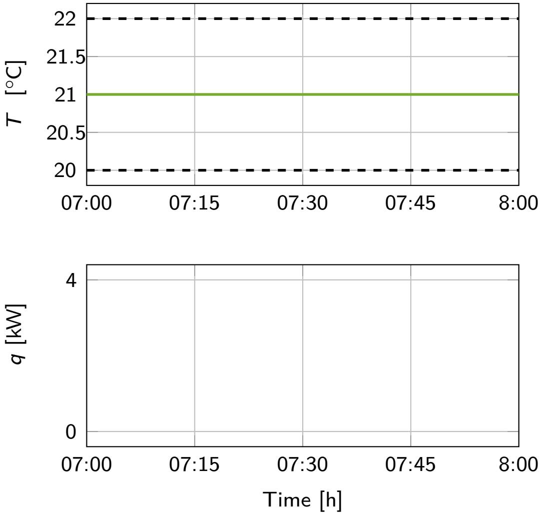
Manipulated Variable

u – heat flow



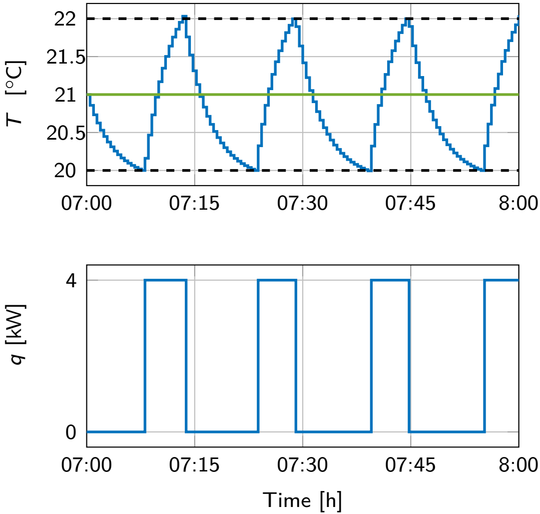


One Hour Window: Relay Thermostat



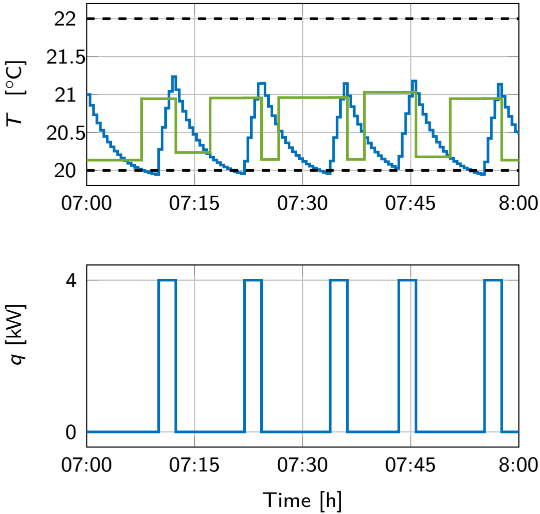
$T_s = 2.5$ minutes, $N = 10$ steps, $\theta = \gamma = 1°C$

One Hour Window: Relay Thermostat



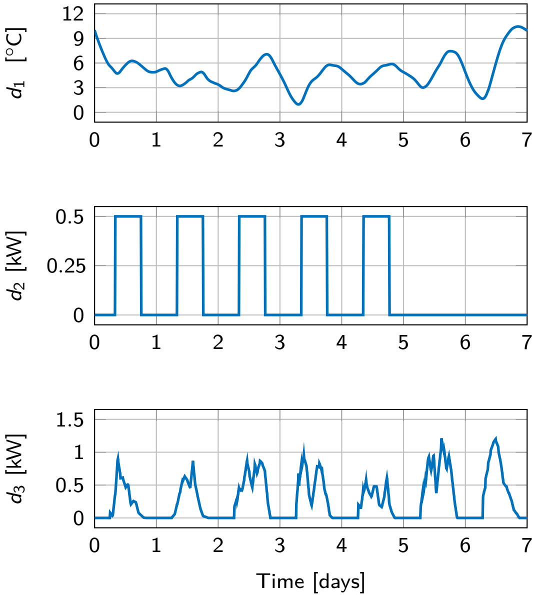
Consumed energy = 1.50 kWh

One Hour Window: MPC Reference Governor

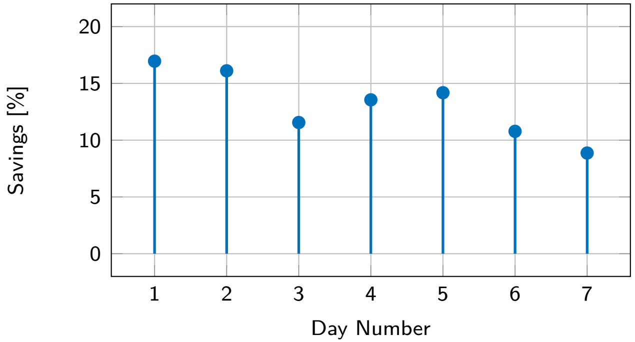


Consumed energy = 0.83 kWh

7 Days Window: Disturbance Profiles

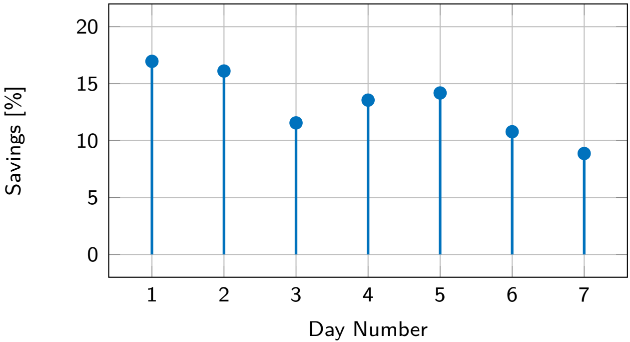


7 Days Window: Energy Savings



13 % savings in average

7 Days Window: Energy Savings



13 % savings in average

Conclusions

1. CPU tractable low cost optimal thermal comfort control
2. Keep existing thermostat in place
3. Case study: 13% energy savings in average

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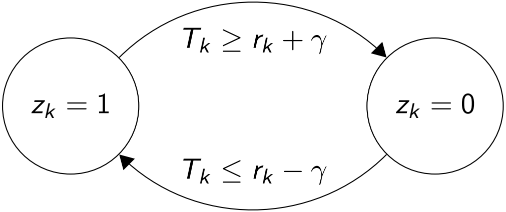
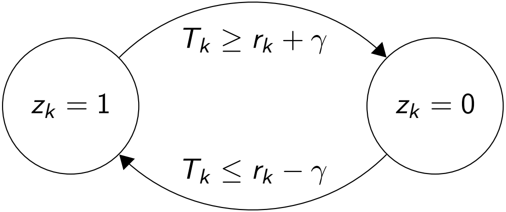
Backup: Hybrid Dynamics of Relay Thermostat



Backup: Hybrid Dynamics of Relay Thermostat

$$u_k = q_{\max} z_k,$$

$$z_{k+1} = \begin{cases} 1 & \text{if} \quad (z_k = 1 \ \wedge \ \sim(T_k \geq r_k + \gamma)) \ \vee \\ & \qquad (z_k = 0 \ \wedge \ \quad (T_k \leq r_k - \gamma)) \\ 0 & \text{otherwise} \end{cases}$$



Backup: Switched Affine System

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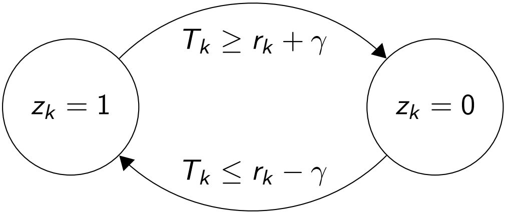

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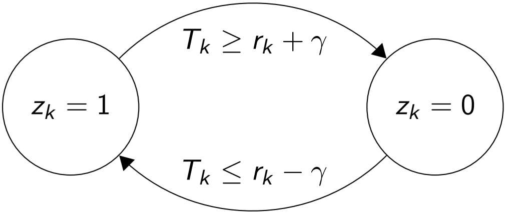

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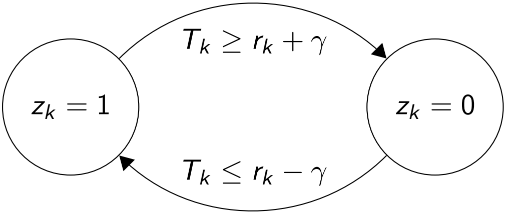

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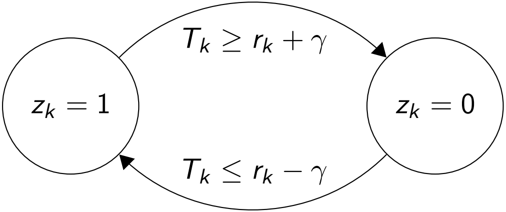

Event Generator:

$$z_{k+1} = \begin{cases} 1 & \text{if } (z \wedge \sim \delta_{\mathrm{a}}) \vee (\sim z \ \wedge \ \delta_{\mathrm{b}}) \\ 0 & \text{otherwise} \end{cases}$$

$$[\delta_{\mathrm{a}} = 1] \Leftrightarrow [T \geq r + \gamma],$$

$$[\delta_{\mathrm{b}} = 1] \Leftrightarrow [T \leq r - \gamma].$$

Backup: Mixed-Integer Modeling

Event Generator:

$$z_{k+1} = \begin{cases} 1 & \text{if } (z \wedge \sim \delta_{\mathrm{a}}) \vee (\sim z \wedge \delta_{\mathrm{b}}) \\ 0 & \text{otherwise} \end{cases}$$

$$m_{\mathrm{a}}\delta_{\mathrm{a}} \leq r + \gamma - T \leq M_{\mathrm{a}}(1 - \delta_{\mathrm{a}}),$$

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Backup: Mixed-Integer Modeling

Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } (z \wedge \sim \delta_\mathsf{a}) \vee (\sim z \wedge \delta_\mathsf{b}) \\ 0 & \text{otherwise} \end{cases}$$

Backup: Mixed-Integer Modeling

Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } \delta_1 \vee \delta_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_1 = \ z \wedge \ \sim \delta_{\mathrm{a}},$$

$$\delta_2 = \sim z \ \wedge \ \delta_{\mathrm{b}},$$

Backup: Mixed-Integer Modeling

Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } \delta_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_1 = \ z \wedge \ \sim \delta_\mathrm{a},$$

$$\delta_2 = \sim z \ \wedge \ \delta_\mathrm{b},$$

$$\delta_3 = \ \delta_1 \vee \delta_2,$$

Backup: Mixed-Integer Modeling

Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } \delta_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_1 \leq z,\ \delta_1 \leq 1 - \delta_\mathrm{a},\ z - \delta_\mathrm{a} \leq \delta_1,$$

$$\delta_2 = \sim z \ \wedge \ \delta_\mathrm{b},$$

$$\delta_3 = \ \delta_1 \vee \delta_2,$$

Backup: Mixed-Integer Modeling

Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } \delta_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_1 \leq z,\ \delta_1 \leq 1 - \delta_\mathsf{a},\ z - \delta_\mathsf{a} \leq \delta_1,$$

$$\delta_2 \leq 1 - z,\ \delta_2 \leq \delta_\mathsf{b},\ -z + \delta_\mathsf{b} \leq \delta_2,$$

$$\delta_3 = \ \delta_1 \vee \delta_2,$$

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Finite State Machine:

$$z_{k+1} = \begin{cases} 1 & \text{if } \delta_3 \\ 0 & \text{otherwise} \end{cases}$$

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$$\delta_3 \geq \delta_1,\ \delta_3 \geq \delta_2,\ \delta_3 \leq \delta_1 + \delta_2.$$

Backup: Mixed-Integer Modeling

Mode Selector:

$$
\begin{gathered}
z_{k+1} = \delta_3 \\
\delta_1 \leq z,\ \delta_1 \leq 1 - \delta_a,\ z - \delta_a \leq \delta_1, \\
\delta_2 \leq 1 - z,\ \delta_2 \leq \delta_b,\ -z + \delta_b \leq \delta_2, \\
\delta_3 \geq \delta_1,\ \delta_3 \geq \delta_2,\ \delta_3 \leq \delta_1 + \delta_2.
\end{gathered}
$$

Backup: MILP Reformulation of MPC Problem

$$
\begin{aligned}
\min_{r_0,\ldots,r_{N-1}} \quad & \sum_{k=0}^{N-1} (\alpha_1 u_k + \alpha_2 s_k) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_k, \\
& T_k = C x_k, \\
& w - \theta - s_k \leq T_k \leq w + \theta + s_k, \\
& u_k = q_{\max} z_k, \\
& m_{\mathrm{a}} \delta_{\mathrm{a},k} \leq r_k + \gamma - T_k \leq M_{\mathrm{a}}(1 - \delta_{\mathrm{a},k}), \\
& m_{\mathrm{b}} \delta_{\mathrm{b},k} \leq T_k - r_k + \gamma \leq M_{\mathrm{b}}(1 - \delta_{\mathrm{b},k}), \\
& \delta_{1,k} \leq z_k, \ \delta_{1,k} \leq 1 - \delta_{\mathrm{a},k}, \ z_k - \delta_{\mathrm{a},k} \leq \delta_{1,k}, \\
& \delta_{2,k} \leq 1 - z_k, \ \delta_{2,k} \leq \delta_{\mathrm{b},k}, \ -z_k + \delta_{\mathrm{b},k} \leq \delta_{2,k}, \\
& \delta_{3,k} \geq \delta_{1,k}, \ \delta_{3,k} \geq \delta_{2,k}, \ \delta_{3,k} \leq \delta_{1,k} + \delta_{2,k}, \\
& z_{k+1} = \delta_{3,k}.
\end{aligned}
$$