Regionless Explicit MPC of a Distillation Column

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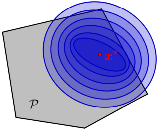
June 30, 2016

Model Predictive Control

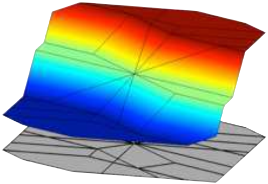
Model Predictive Control

Model Predictive Control

Implicit vs Explicit MPC

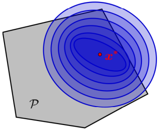


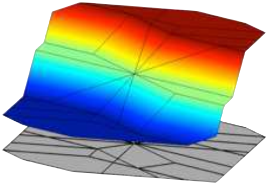
Implicit MPC



Explicit MPC

Implicit vs Explicit MPC



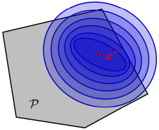


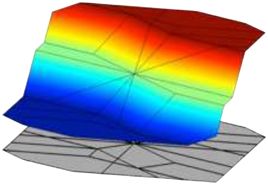
Implicit MPC

Explicit MPC

- Large systems
- Expensive implementation
- Matrix inversions
- Harder to certify

Implicit vs Explicit MPC





Implicit MPC

- Large systems
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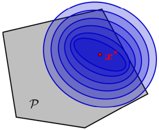
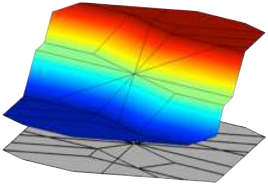
Explicit MPC

- Small systems
- Cheap implementation
- Division-free
- Rigorous analysis

Implicit vs Explicit MPC





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- Large systems
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Explicit MPC

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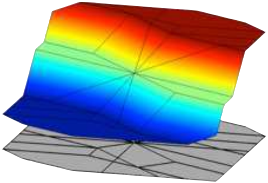
Regionless Explicit MPC

Region-Based Explicit MPC

$$
\begin{aligned}
\min_{U} \; & \frac{1}{2} U^T H U + \theta^T F U \\
\text{s.t. } \; & G U \leq w + S\theta
\end{aligned}
$$

Region-Based Explicit MPC

$$U^\star = V_i\theta + v_i$$
$$\lambda^\star = Q_i\theta + q_i$$
$$\mathcal{R}_i = \{\theta \mid GU^\star \leq w + S\theta, \lambda^\star \geq 0\}$$

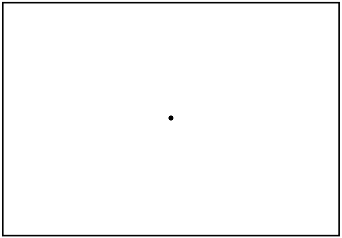


Active Sets – Geometric Method[1]

[1] M. Baotić: Optimal Control of Piecewise Affine Systems – A Multi-parametric Approach, 2005

Active Sets – Geometric Method[1]

Pick an arbitrary point in Ω.



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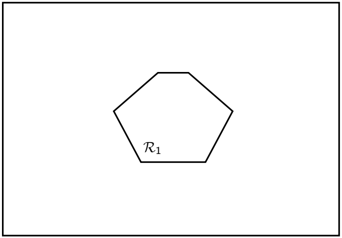
Active Sets – Geometric Method[1]

Construct an initial critical region.



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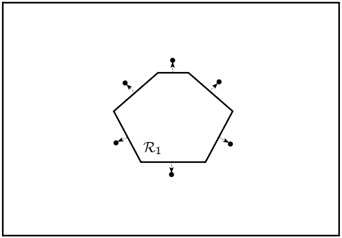
Active Sets – Geometric Method[1]

Pick new points in Ω.



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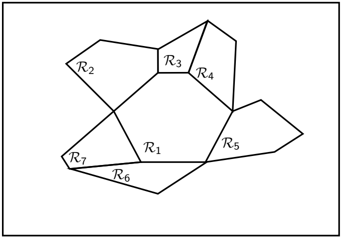
Active Sets – Geometric Method[1]

Construct new critical regions.



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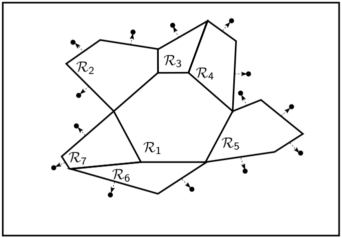
Active Sets – Geometric Method[1]

Repeat recursively until whole Ω is covered.



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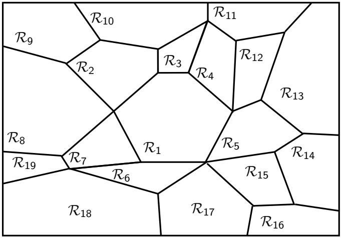
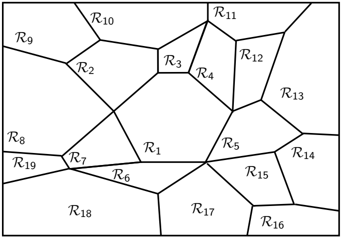


[1] M. Baotić: Optimal Control of Piecewise Affine Systems – A Multi-parametric Approach, 2005

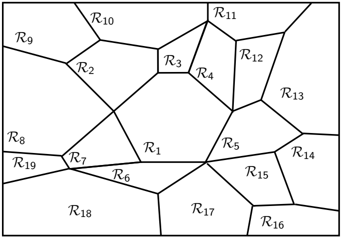
The Idea

Avoid the construction and the storage of regions!



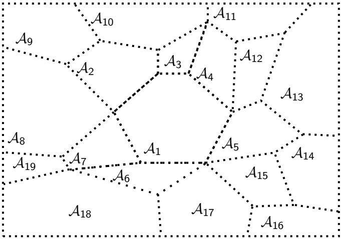
The Idea

$$U^\star = V_i\theta + v_i$$
$$\lambda^\star = Q_i\theta + q_i$$
$$\mathcal{R}_i = \{\theta \mid GU^\star \leq w + S\theta, \lambda^\star \geq 0\}$$



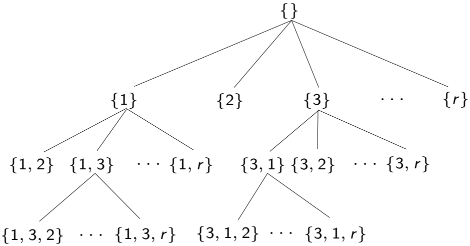
The Idea[2]

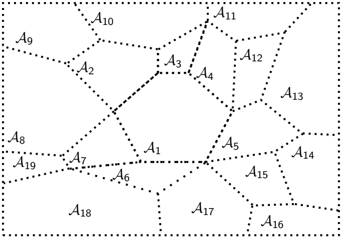
$$U^\star = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$$
$$\lambda^\star = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$$
$$\mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_R\}$$



[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

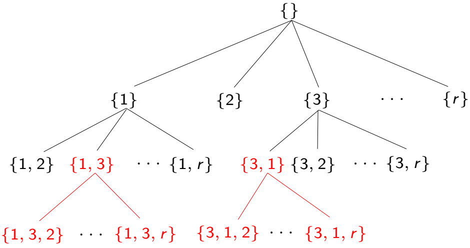
Active Sets – Extensive Enumeration[3]

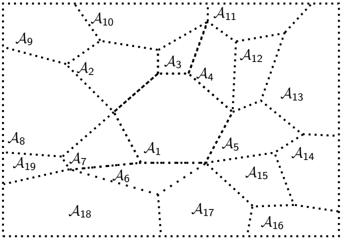




[3] A. Gupta et. al.: A novel approach to multiparametric quadratic programming, Automatica 2011

Active Sets – Extensive Enumeration[3] – Tree Pruning





[3] A. Gupta et. al.: A novel approach to multiparametric quadratic programming, Automatica 2011

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

On-line phase

Region-Based Approach

Off-line phase

1. Active Sets

On-line phase

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

1. Active Sets
   - Geometric method

On-line phase

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

1. Active Sets
    - Geometric method
    - Extensive enumeration

On-line phase

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

1. Active Sets
   - Geometric method
   - Extensive enumeration
2. $U^{\star}, \lambda^{\star}$ from KKT conditions

On-line phase

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

1. Active Sets
   - Geometric method
   - Extensive enumeration
2. $U^\star, \lambda^\star$ from KKT conditions
3. Construction of regions

On-line phase

Explicit MPC Solution Recap

Region-Based Approach

Off-line phase

1. Active Sets
    - Geometric method
    - Extensive enumeration
2. $U^\star, \lambda^\star$ from KKT conditions
3. Construction of regions

On-line phase

4. Function evaluation

Explicit MPC Solution Recap

Regionless Approach

Off-line phase

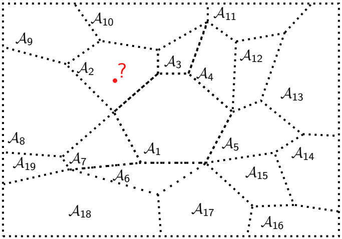
1. Active Sets
   - ~~Geometric method~~
   - Extensive enumeration
2. $U^\star, \lambda^\star$ from KKT conditions
3. ~~Construction of regions~~

On-line phase

4. Point location

Point Location Problem[2]

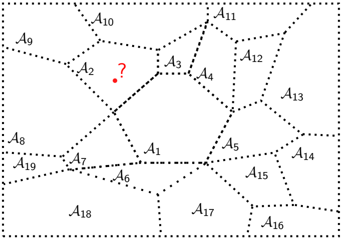
How to find in which region we are, without storing any regions?



[2]F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Point Location Problem[2]

$$U^\star = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$$
$$\lambda^\star = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$$
$$\mathcal{R}(\mathcal{A}_i) = \{\theta \mid GU^\star \leq w + S\theta, \lambda^\star \geq 0\}$$



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Point Location Problem[2]

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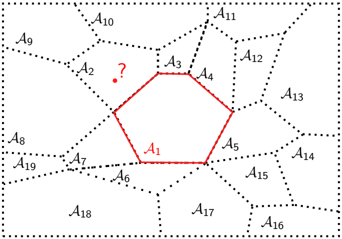

[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Point Location Problem[2]

$$U^\star = V(\mathcal{A}_2)\theta + v(\mathcal{A}_2)$$
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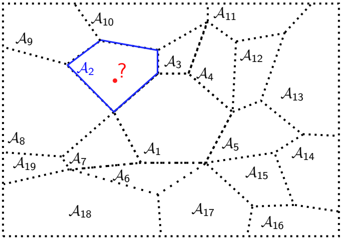

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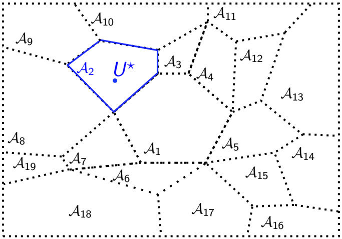

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Sequential search[2]

procedure $\textsc{RegionLess}(\theta)$
    for $i \in \{1, \ldots, R\}$ **do**
        Compute $\lambda = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$
        if $\lambda \geq 0$ **then**
            Compute $U = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$
            if $GU \leq w + S\theta$ **then**
                $U^\star \leftarrow U$
                return $U^\star$
            end if
        end if
    end for
end procedure

[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Sequential search[2]

procedure $\textsc{Regionless}(\theta)$ ▷ given initial condition θ
  for $i \in \{1, \ldots, R\}$ **do**
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Sequential search[2]

procedure $\text{REGIONLESS}(\theta)$ ▷ given initial condition θ

  for $i \in \{1, \ldots, R\}$ **do**

    Compute $\lambda = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$

    if $\lambda \geq 0$ **then** ▷ dual feasibility check

      Compute $U = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$

      if $GU \leq w + S\theta$ **then**

        $U^\star \leftarrow U$

        return $U^\star$

      end if

    end if

  end for

end procedure

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  for $i \in \{1, \ldots, R\}$ **do**

    Compute $\lambda = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$

    if $\lambda \geq 0$ **then** ▷ dual feasibility check

      Compute $U = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$

      if $GU \leq w + S\theta$ **then** ▷ primal feasibility check

        $U^\star \leftarrow U$

        return $U^\star$

      end if

    end if

  end for

end procedure

[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Sequential search[2]

```
procedure REGIONLESS(θ)                              ▷ given initial condition θ
    for i ∈ {1, ..., R} do
        Compute λ = Q(A_i)θ + q(A_i)
        if λ ≥ 0 then                                ▷ dual feasibility check
            Compute U = V(A_i)θ + v(A_i)
            if GU ≤ w + Sθ then                      ▷ primal feasibility check
                U* ← U
                return U*                            ▷ obtain globally optimal U*
            end if
        end if
    end for
end procedure
```

[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Sequential search[2]

procedure $\textsc{Regionless}(\theta)$ ▷ given initial condition θ
  for $i \in \{1, \ldots, R\}$ **do**
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    if $\lambda \geq 0$ **then** ▷ dual feasibility check
      Compute $U = V(\mathcal{A}_i)\theta + v(\mathcal{A}_i)$
      if $GU \leq w + S\theta$ **then** ▷ primal feasibility check
        $U^\star \leftarrow U$
        return $U^\star$ ▷ obtain globally optimal $U^\star$
      end if
    end if
  end for
end procedure

$Q(\mathcal{A}_i), q(\mathcal{A}_i), V(\mathcal{A}_i), v(\mathcal{A}_i), G, w, S$ are precomputed and stored off-line.

[2] F. Borrelli and M. Baotić: On the computation of linear model predictive control laws, Automatica 2008

Sequential search[4]

procedure REGIONLESS(θ) ▷ given initial condition θ
  for $i \in \{1, \ldots, R\}$ **do**
    Compute $\lambda = Q(\mathcal{A}_i)\theta + q(\mathcal{A}_i)$
    if $\lambda \geq 0$ **then** ▷ dual feasibility check
      Compute $U = -H^{-1}(F^T\theta + G_{\mathcal{A}_i}^T\lambda)$ ▷ U parametrized by λ
      if $GU \leq w + S\theta$ **then** ▷ primal feasibility check
        $U^\star \leftarrow U$
        return $U^\star$ ▷ obtain globally optimal $U^\star$
      end if
    end if
  end for
end procedure

[4] M. Kvasnica, et. al.: On Region-Free Explicit Model Predictive Control, Conference on Decision and Control 2015

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            if $GU \leq w + S\theta$ **then** ▷ primal feasibility check
                $U^\star \leftarrow U$
                return $U^\star$ ▷ obtain globally optimal $U^\star$
            end if
        end if
    end for
end procedure

$Q(\mathcal{A}_i), q(\mathcal{A}_i), \mathcal{A}, H^{-1}, F, G, w, S$ are precomputed and stored off-line.

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Distillation Column – Process Unit

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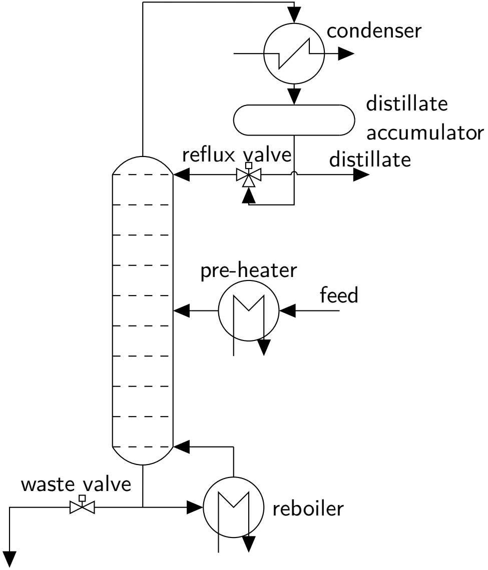

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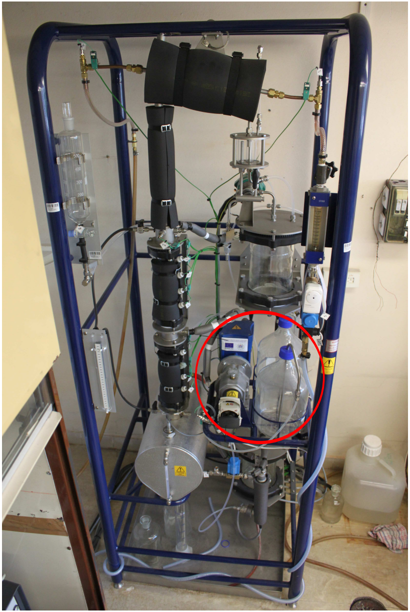

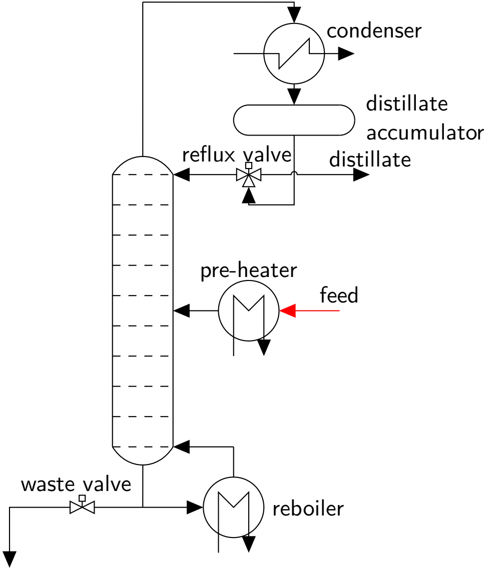

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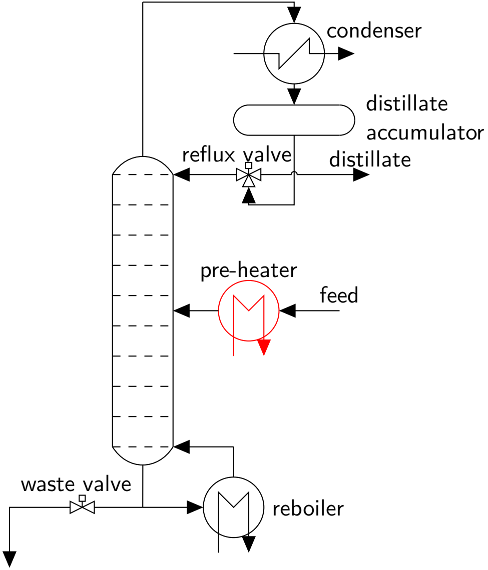

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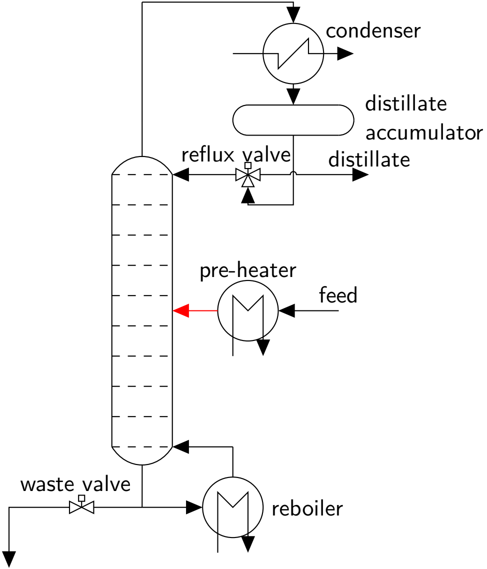

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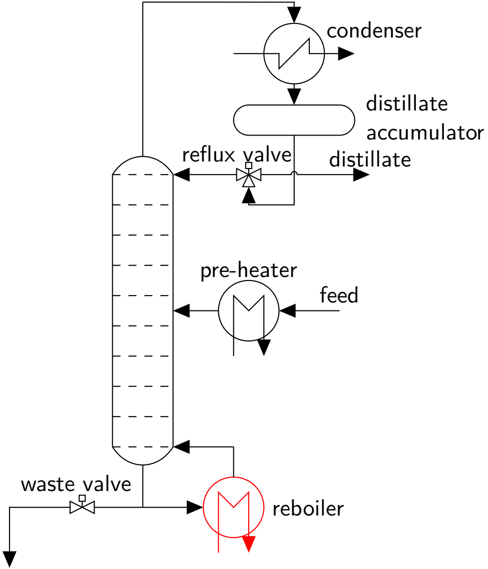

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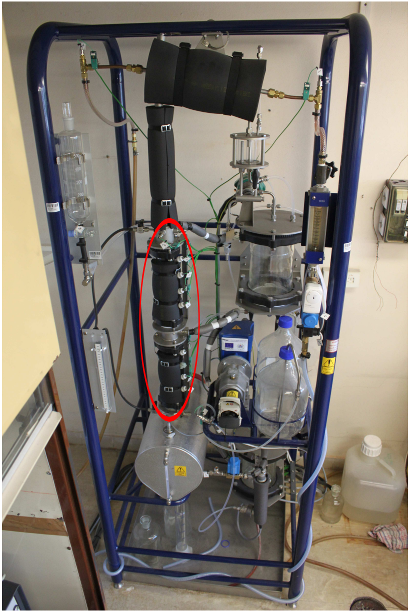

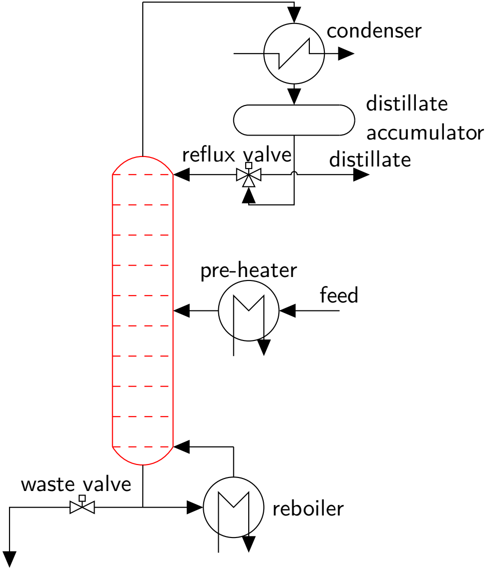

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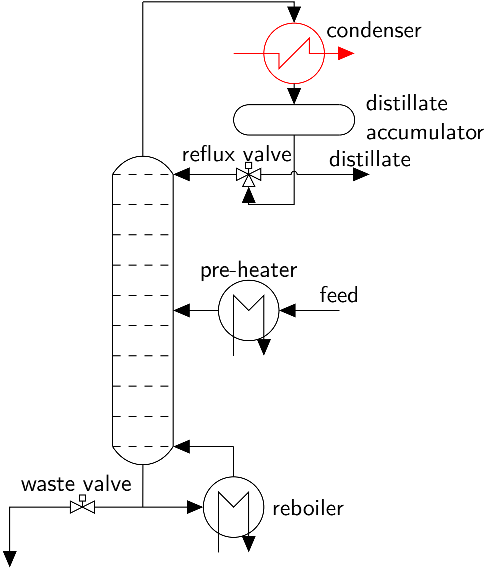

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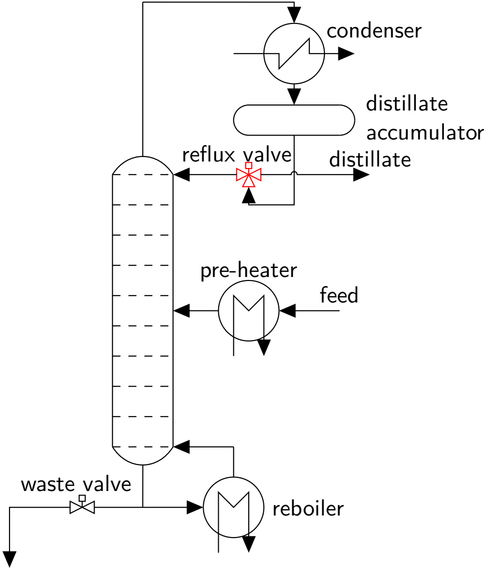

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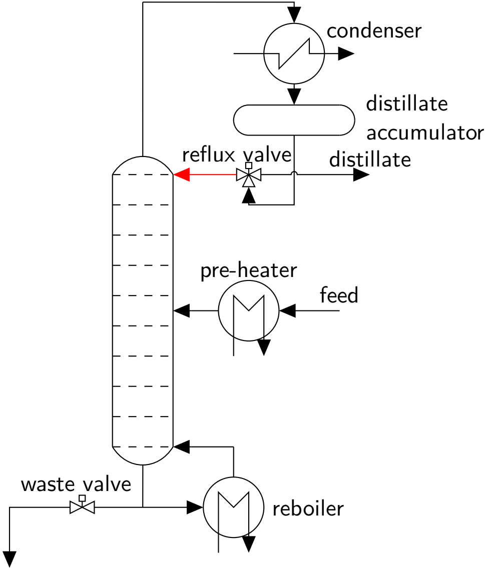

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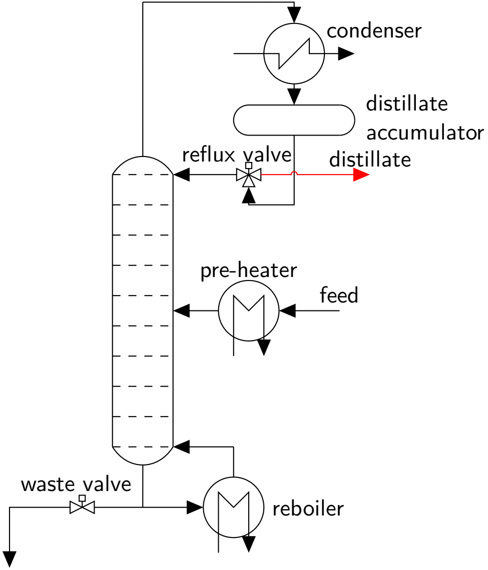
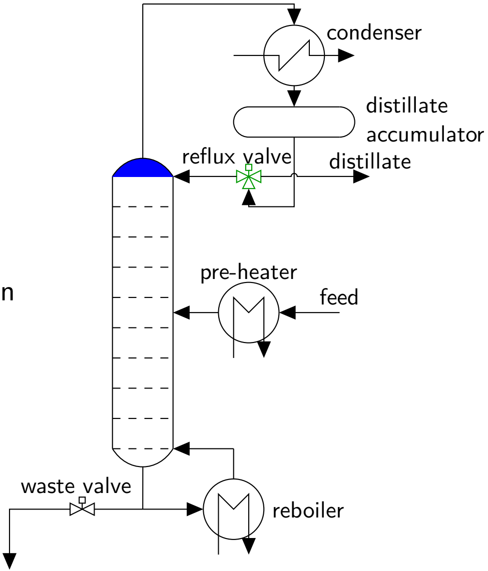


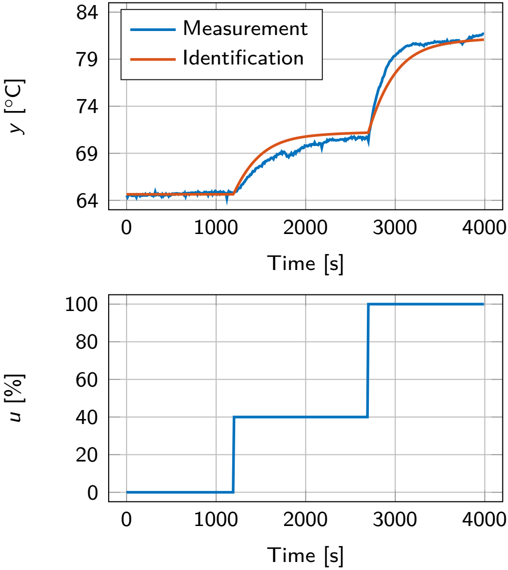
Distillation Column – Variables



Process variable: y
Temperature at top of the column

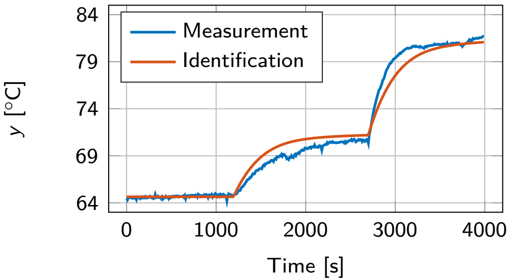
Manipulated variable: u
Reflux ratio

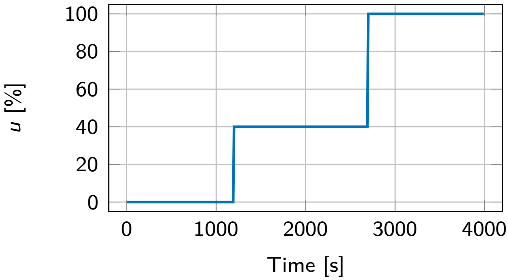
$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$



Model Identification

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$
$$n_x = 10$$





Disturbance Modelling

Design model:

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k$$

Disturbance Modelling

Augmented design model:

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k \\
d_{k+1} &= d_k \\
y_k &= Cx_k + Ed_k
\end{aligned}
$$

Disturbance Modelling

Augmented design model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ d_{k+1} &= d_k \\ y_k &= Cx_k + Ed_k \end{aligned}$$

Luenberger observer:

$$\begin{aligned} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k+1} &= \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + L\left(y_{\mathrm{m},k} - \hat{y}_k\right) \\ \hat{y}_k &= \begin{bmatrix} C & E \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k} \end{aligned}$$

Model Predictive Control Formulation

$$
\begin{aligned}
\min_{u_0,\ldots,u_{N-1}} \quad & \sum_{k=0}^{N_c-1} \|\Delta u_k\|_{Q_u}^2 + \sum_{k=0}^{N} \|y_k - y_{\text{ref}}\|_{Q_y}^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\
& y_k = Cx_k + Ed_0 \\
& u_k = u_{N_c-1} \\
& \Delta u_k = u_k - u_{k-1} \\
& u_{\min} \leq u_k \leq u_{\max} \\
& \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \\
& x_0 = \hat{x}(t), d_0 = \hat{d}(t), u_{-1} = u(t-1)
\end{aligned}
$$

Model Predictive Control Formulation

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N_c-1} \|\Delta u_k\|_{Q_u}^2 + \sum_{k=0}^{N} \|y_k - y_{\text{ref}}\|_{Q_y}^2$$

reference tracking

$$\begin{aligned}
\text{s.t. } & x_{k+1} = Ax_k + Bu_k \\
& y_k = Cx_k + Ed_0 \\
& u_k = u_{N_c-1} \\
& \Delta u_k = u_k - u_{k-1} \\
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& \Delta u_{\min} \le \Delta u_k \le \Delta u_{\max} \\
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\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\
& y_k = Cx_k + Ed_0 \\
& u_k = u_{N_\text{c}-1} && \text{move blocking} \\
& \Delta u_k = u_k - u_{k-1} \\
& u_\text{min} \leq u_k \leq u_\text{max} \\
& \Delta u_\text{min} \leq \Delta u_k \leq \Delta u_\text{max} \\
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$$

Model Predictive Control Formulation

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\Delta u_{\min} &\leq \Delta u_k \leq \Delta u_{\max} \\
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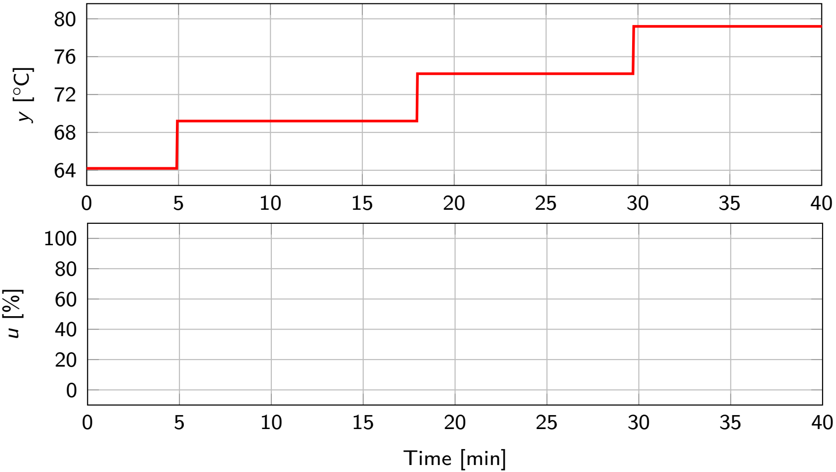
\end{aligned}$$

move blocking

17 parameters!

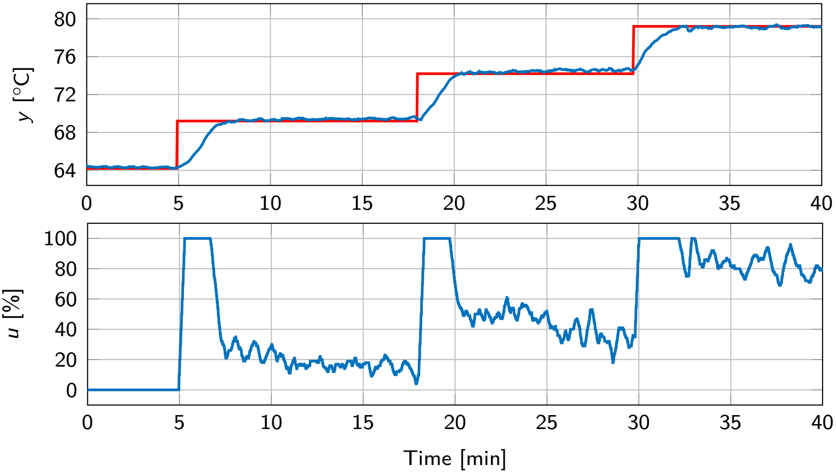
Experimental Results

$$T_s = 1\,\mathrm{s},\ N = 40,\ N_c = 5,\ Q_y = 400,\ Q_u = 1$$



Experimental Results

$$T_s = 1\,\mathrm{s},\ N = 40,\ N_c = 5,\ Q_y = 400,\ Q_u = 1$$



Controller Synthesis and Complexity

on-line		
method	CPU time	suboptimality

Controller Synthesis and Complexity

on-line

method	CPU time	suboptimality
Gurobi	38.74 ms	0 %

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Fiordos 2	3.15 ms	$\approx$ 4 %

Controller Synthesis and Complexity

on-line

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Regionless	1.95 ms	0 %

Controller Synthesis and Complexity

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Execution time improvement by the factor of 6.

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Execution time improvement by the factor of 6.

off-line (17 params., 1095 active sets)

	Regionless	Region-based

Controller Synthesis and Complexity

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Execution time improvement by the factor of 6.

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	Regionless	Region-based
Construction time	6 s	360 s

Controller Synthesis and Complexity

on-line

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Execution time improvement by the factor of 6.

off-line (17 params., 1095 active sets)

	Regionless	Region-based
Construction time	6 s	360 s
Memory demands	224 kB	40 MB

Backup: KKT Conditions

$$
\begin{aligned}
HU^\star + F^T\theta + G_{\mathcal{A}}^T\lambda^\star &= 0 && \text{stationarity} \\
G_{\mathcal{A}}U^\star &= w_{\mathcal{A}} + S_{\mathcal{A}}\theta && \text{primal equality} \\
G_{\mathcal{N}}U^\star &< w_{\mathcal{N}} + S_{\mathcal{N}}\theta && \text{primal inequality} \\
\lambda^\star &\geq 0 && \text{dual inequality} \\
{\lambda^\star}^T(G_{\mathcal{A}}U^\star - w_{\mathcal{A}} - S_{\mathcal{A}}\theta) &= 0 && \text{complementarity slackness}
\end{aligned}
$$

Backup: Extensive Enumeration[3] – Optimality of $\mathcal{A}$

$$
\begin{aligned}
\max_{t,U,\theta,\lambda} \quad & t \\
\text{s.t.} \quad & HU + F^T\theta + G_{\mathcal{A}}^T\lambda = 0 \\
& G_{\mathcal{A}}U = w_{\mathcal{A}} + S_{\mathcal{A}}\theta \\
& t \leq w_{\mathcal{N}} + S_{\mathcal{N}}\theta - G_{\mathcal{N}}U \\
& \lambda \geq t \\
& t \geq 0
\end{aligned}
$$

if LP is feasible with $t^\star > 0$ than $\mathcal{A}$ is optimal set of active constraints

[3] A. Gupta et. al.: A novel approach to multiparametric quadratic programming, Automatica 2011

Backup: Extensive Enumeration[3] – Complexity Bound

$$R_{\mathrm{max}} = \sum_{k=0}^{N_{\mathrm{c}} n_{\mathrm{u}}} \frac{n_{\mathrm{c}}!}{k!(n_{\mathrm{c}} - k)!}$$

$$\begin{aligned} R_{\mathrm{max}} &= \text{maximum number of } \mathcal{A} \\ N_{\mathrm{c}} &= \text{control horizon} \\ n_{\mathrm{u}} &= \text{number of control inputs} \\ n_{\mathrm{c}} &= \text{number of constraints} \\ R &\ll R_{\mathrm{max}} \text{ thanks to pruning} \end{aligned}$$

[3] A. Gupta et. al.: A novel approach to multiparametric quadratic programming, Automatica 2011

Backup: Regionless Explicit MPC Memory Demands

Regionless

$\mathcal{A}$	3918 integers
$Q(\mathcal{A}), q(\mathcal{A})$	54852 floating-point numbers
H, F, G, w, S	584 floating-point numbers
Total memory demands	224 kB

Region-based

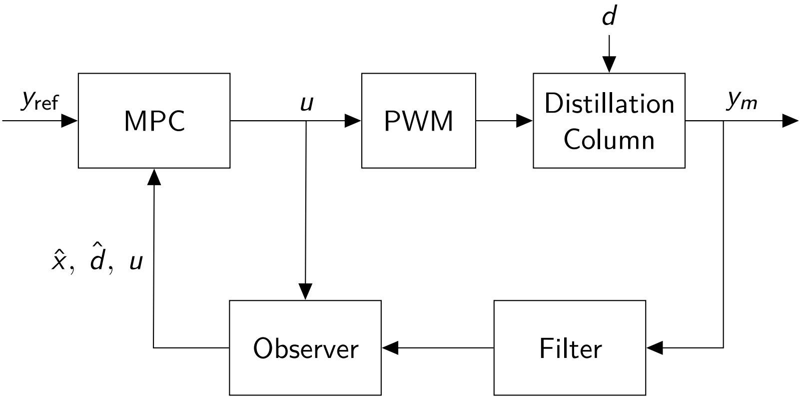
$\mathcal{R}$	10004490 floating-point numbers
Total memory demands	40 MB

integer = 2 bytes, single-precision floating-point number = 4 bytes

Backup: Regionally Optimal Map $\theta \to U^\star$

$$
\begin{aligned}
Q(\mathcal{A}) &= -(G_\mathcal{A} H^{-1} G_\mathcal{A}^T)^{-1}(S_\mathcal{A} + G_\mathcal{A} H^{-1} F^T) \\
q(\mathcal{A}) &= -(G_\mathcal{A} H^{-1} G_\mathcal{A}^T)^{-1} w_\mathcal{A} \\
\lambda^\star &= Q(\mathcal{A})\theta + q(\mathcal{A}) \\
U^\star &= -H^{-1}(F^T\theta + G_\mathcal{A}^T \lambda^\star)
\end{aligned}
$$

Backup: Control Scheme



Backup: Distillation Column – Hardware Solution