Impact of the controller model complexity on MPC performance evaluation for building climate control

Damien Picard\textsuperscript{a}, Ján Drgoňa\textsuperscript{b}, Lieve Helsen\textsuperscript{a,c}, and Michal Kvasnica\textsuperscript{b}

\textsuperscript{a}KU Leuven, Department of Mechanical Engineering, Leuven, Belgium
\textsuperscript{b}Slovak University of Technology in Bratislava, Slovakia
\textsuperscript{c}EnergyVille, Thor Park, Waterschei, Belgium

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Problem: EU spends 400 billion EUR/year on energy.

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40% goes into thermal comfort in buildings.*

* International Energy Agency, ‘Energy efficiency requirements in building codes, energy efficiency policies for new buildings’
2013 OECD/IEA.
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Goal: Reduce the energy consumption

Building Control Motivation

Problem: EU spends 400 billion EUR/year on energy. 40% goes into thermal comfort in buildings.*

Goal: Reduce the energy consumption

Solution: Thermal comfort control

Model Predictive Control

Pros:

- Satisfy thermal comfort constraints
- Minimize energy consumption
- Obey technological restrictions

Cons:

- Implementation in early stages
- Need for a good controller model
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What is the Best Model?
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complexity

simplicity
Methodology

- Controller model order
- Plant model order
- RBC
- PID
- MPC
- ROM
- SSM
- Simulation model
- Real building
- Linearisation
- Model order reduction

- Performance evaluation (Comfort, energy, CPU, prediction error)
## Building Description

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor area [m²]</td>
<td>48.3</td>
</tr>
<tr>
<td>Conditioned volume [m³]</td>
<td>130.6</td>
</tr>
<tr>
<td>Total exterior surface area [m²]</td>
<td>195</td>
</tr>
<tr>
<td>Windows [-]</td>
<td>5</td>
</tr>
<tr>
<td>Walls [-]</td>
<td>22</td>
</tr>
<tr>
<td>Roof and floor surfaces [-]</td>
<td>12</td>
</tr>
<tr>
<td>Thermal zones [-]</td>
<td>6</td>
</tr>
</tbody>
</table>
Linearisation

Controller model order
MPC

Plant model order

RBC
PID

ROM
n=4

SSM
n=n_{SSM}

Simulation model
Real building

Linearisation
Model order reduction

= Performance evaluation
(Comfort, energy, CPU, prediction error)

---

Full year open-loop simulation linearization error below 1 K.
Square root balanced truncation algorithm, based on Hankel singular values.

Guarantees of an error bounds and preserves most of the system characteristics in terms of stability, frequency, and time responses.
Single week open loop simulation with realistic control inputs and disturbances.
Reduced Order Models – Prediction Errors

The central line is the median, the box gives the 1st and 3rd quartiles, the whiskers contain 99.5% of the data, the crosses are the outliers.
Control Setup

Controller model order

Plant model order

RBC

PID

MPC

ROM

SSM

Simulation model

Real building

Linearisation

Model order reduction

n = Performance evaluation (Comfort, energy, CPU, prediction error)
Control Scheme

- **MPC**
- **Building**
- **Estimator**

Inputs:
- \( r \)

Outputs:
- \( u \)
- \( y \)

Intermediate Variables:
- \( \hat{x}, \hat{p} \)
- \( d \)
Estimator and Augmented Model

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + L \left( y_{m,k} - \hat{y}_{k|k-1} \right)
\]

\[
\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_{k|k} + Ed_{k|k}
\]

\[
\hat{y}_{k|k} = C\hat{x}_{k|k} + Du_{k|k}
\]
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\[ \hat{y}_{k|k} = C\hat{x}_{k|k} + Du_{k|k} \]

\[
\begin{bmatrix}
\hat{x}_{k+1} \\
\hat{p}_{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
A & 0 \\
0 & I
\end{bmatrix}
\begin{bmatrix}
\hat{x}_k \\
\hat{p}_k
\end{bmatrix}
+ 
\begin{bmatrix}
B \\
0
\end{bmatrix}
\begin{bmatrix}
u_k
\end{bmatrix}
+ 
\begin{bmatrix}
E \\
0
\end{bmatrix}
\begin{bmatrix}d_k
\end{bmatrix}
\]

\[ \hat{y}_k = 
\begin{bmatrix}
C & F
\end{bmatrix}
\begin{bmatrix}
\hat{x}_k \\
\hat{p}_k
\end{bmatrix}
+ 
\begin{bmatrix}
D \\
0
\end{bmatrix}
\begin{bmatrix}u_k
\end{bmatrix} \]
MPC Formulation

\[
\min_{u_0, \ldots, u_{N-1}} \sum_{k=0}^{N-1} \left( \|s_k\|^2_{Q_s} + \|u_k\|^2_{Q_u} \right)
\]

s.t. \(x_{k+1} = Ax_k + Bu_k + Ed_k\)

\(y_k = Cx_k + Du_k\)

\(lb_k - s_k \leq y_k \leq ub_k + s_k\)

\(u \leq u_k \leq \bar{u}\)

\(x_0 = \hat{x}(t)\)

\(\forall k \in \{0, \ldots, N - 1\}\)
State Condensing

\[ x_1 = Ax_0 + Bu_0 + Ed_0 \]
\[ x_2 = A(Ax_0 + Bu_0 + Ed_0) + Bu_1 + Ed_1 \]
\[ \vdots \]
\[ x_{k+1} = A^{k+1}x_0 + \ldots \]
\[ \begin{bmatrix} A^k B \ldots AB & B \end{bmatrix} \begin{bmatrix} u_0^T & \ldots & u_k^T \end{bmatrix}^T + \ldots \]
\[ \begin{bmatrix} A^k E \ldots AE & E \end{bmatrix} \begin{bmatrix} d_0^T & \ldots & d_k^T \end{bmatrix}^T \]
\[ y_k = CA^k x_0 + \ldots \]
\[ C \begin{bmatrix} A^{k-1} B \ldots AB & B \end{bmatrix} \begin{bmatrix} u_0^T & \ldots & u_{k-1}^T \end{bmatrix}^T + \ldots \]
\[ C \begin{bmatrix} A^{k-1} E \ldots AE & E \end{bmatrix} \begin{bmatrix} d_0^T & \ldots & d_{k-1}^T \end{bmatrix}^T + Du_k + Fp_0 \]

Significantly reduces the number of the optimization variables.
Simulation Setup

One year performances of RBC, PID, MPC.
Three types of 6-zone buildings, with 300 states.

\[ T_s = 900, \quad N = 40 \text{ steps (i.e., 10 hours)}, \quad \frac{Q_s}{Q_u} = 10^8. \]
One year performances of RBC, PID, MPC.
Three types of 6-zone buildings, with 300 states.

\[ T_s = 900, \ N = 40 \text{ steps (i.e., 10 hours)}, \ \frac{Q_s}{Q_u} = 10^8. \]
Temperature disturbances

Power disturbances

Power per surface disturbances

52 disturbances
Comfort satisfaction, spanning from 93.9% to 95.2%.
Comfort satisfaction, spanning from 95.6% to 99.6%. Energy savings around 6%.
Comfort satisfaction, close to 100.0%.
Energy savings around 13%.
Performance Evaluation

(a) Total comfort rate

(b) Total heating cost

(c) Total one-step ahead prediction error

(d) CPU time

Original
Renovated
Light weight
Original (OSF)
Renovated (OSF)
Light weight (OSF)
Conclusions

1. Influence of controller model accuracy on controller performance.

2. Minimum of 30 states was necessary for 6-rooms house.

3. When a dense formulation is used a CPU time becomes independent of the number of states of the controller model.

4. Use a controller model which emulates the real building as accurately as possible!

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