

# Impact of the controller model complexity on MPC performance evaluation for building climate control

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**KU LEUVEN**

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**Problem: EU spends 400 billion EUR/year on energy.**

\* International Energy Agency, 'Energy efficiency requirements in building codes, energy efficiency policies for new buildings'

2013 OECD/IEA.

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**Solution: Thermal comfort control**

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## Pros:

- Satisfy thermal comfort constraints
- Minimize energy consumption
- Obey technological restrictions

## Cons:

- Implementation in early stages
- Need for a good controller model

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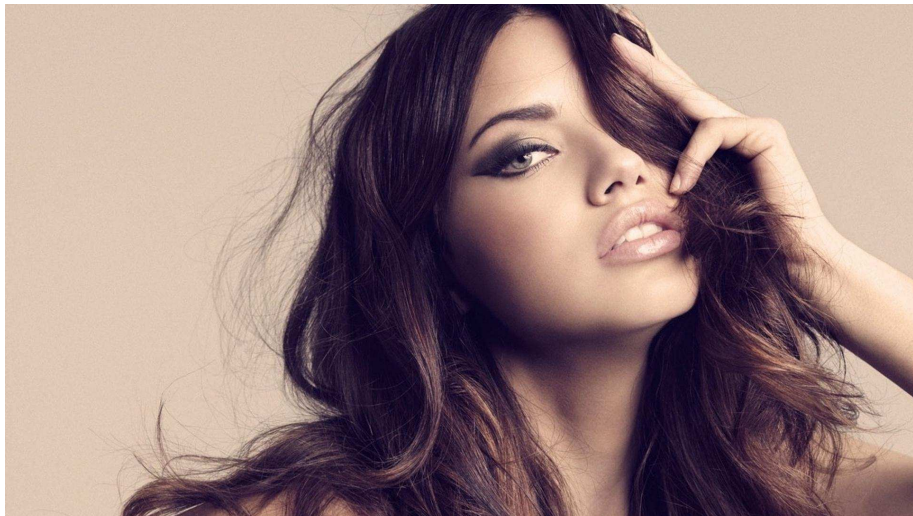
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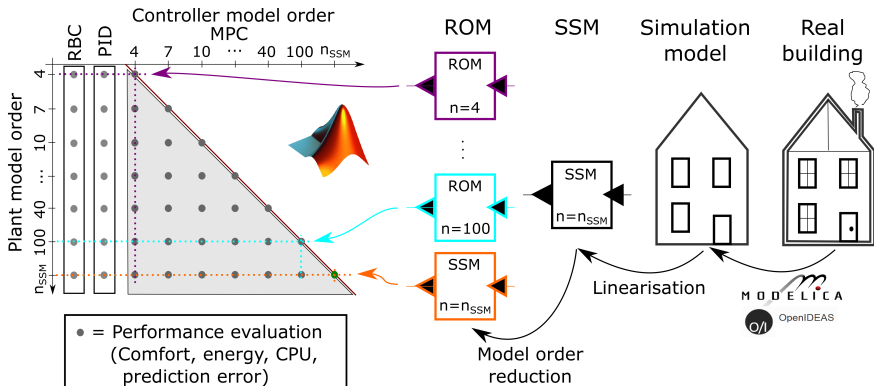
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# What is the Best Model?



COMPLEXITY

simplicity



# Building Description

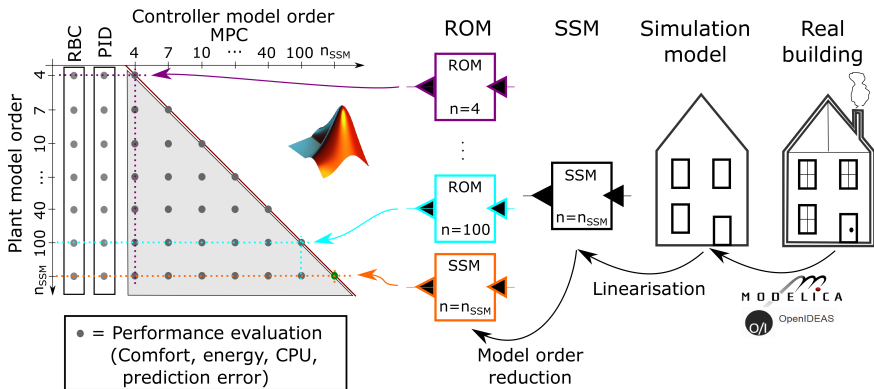


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Floor area	[m <sup>2</sup> ]	48.3
Conditioned volume	[m <sup>3</sup> ]	130.6
Total exterior surface area	[m <sup>2</sup> ]	195
Windows	[-]	5
Walls	[-]	22
Roof and floor surfaces	[-]	12
Thermal zones	[-]	6

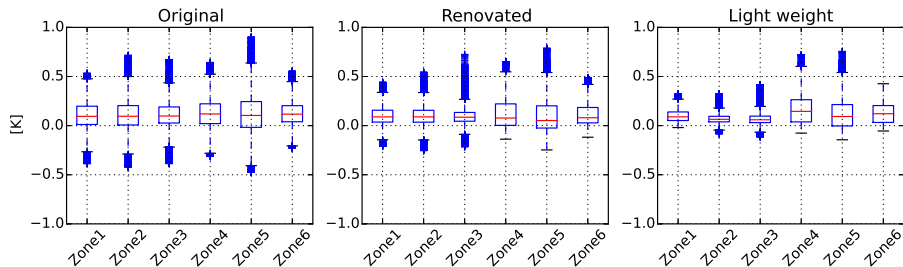
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# Linearisation<sup>1</sup>



<sup>1</sup>Picard, D., Jorissen, F., and Helsens, L. 2015. Methodology for Obtaining Linear State Space Building Energy Simulation Models. In 11th International Modelica Conference, pages 51–58, Paris

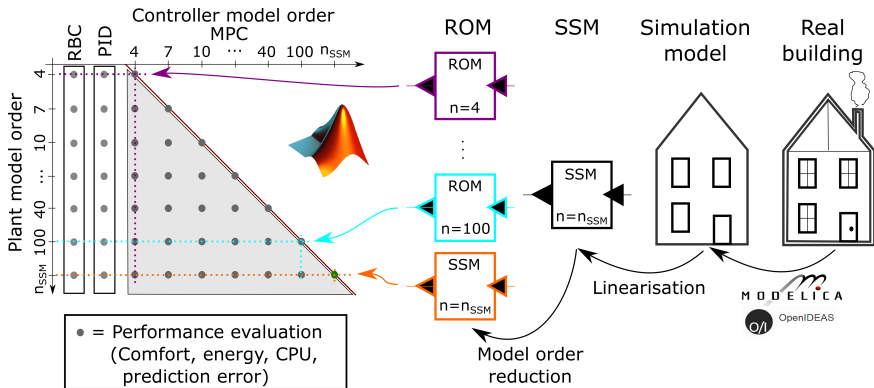
# Linearisation Error



Full year open-loop simulation linearization error below 1 K.



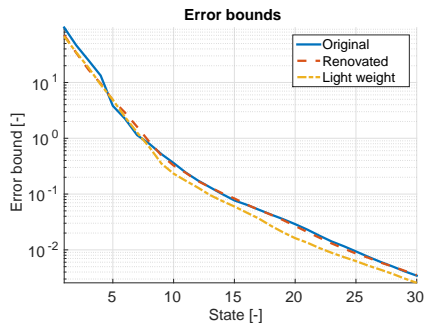
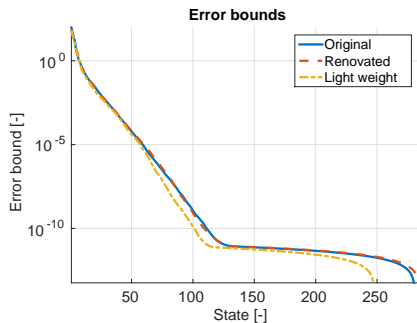
# Model Order Reduction<sup>2</sup>



Square root balanced truncation algorithm, based on Hankel singular values.

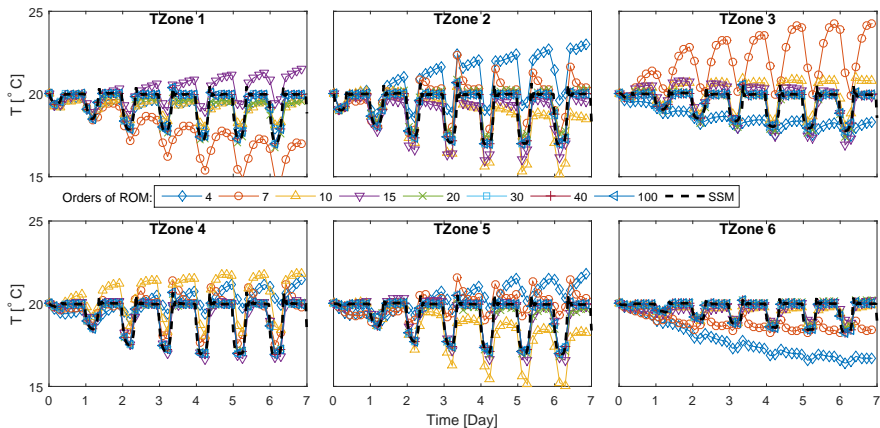
<sup>2</sup> Antoulas, A. C. and Sorensen, D. C. 2001. Approximation of large-scale dynamical systems: An overview. Applied Mathematics and Computer Science.

# Reduced Order Models – Error Bounds



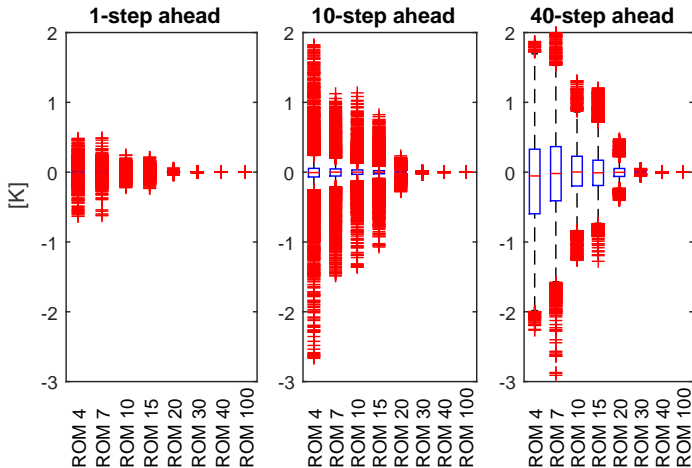
Guarantees of an error bounds and preserves most of the system characteristics in terms of stability, frequency, and time responses.

# Reduced Order Models – Open Loop Simulation



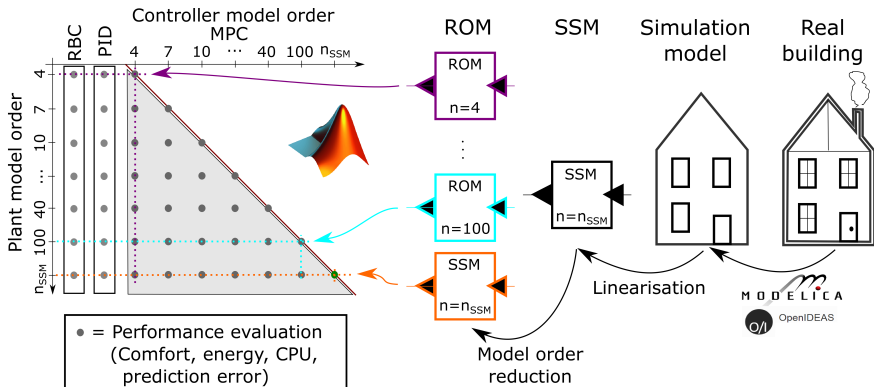
Single week open loop simulation with realistic control inputs and disturbances.

# Reduced Order Models – Prediction Errors

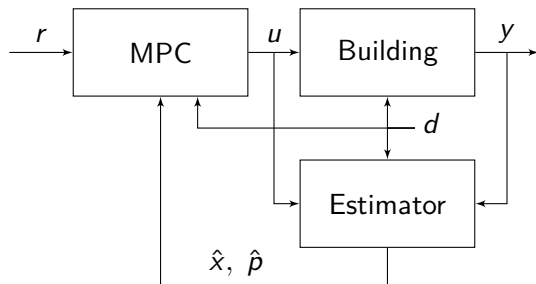


The central line is the median, the box gives the 1st and 3rd quartiles, the whiskers contain 99.5% of the data, the crosses are the outliers.

# Control Setup



# Control Scheme



# Estimator and Augmented Model

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L \left( y_{m,k} - \hat{y}_{k|k-1} \right)$$

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_{k|k} + Ed_{k|k}$$

$$\hat{y}_{k|k} = C\hat{x}_{k|k} + Du_{k|k}$$

$$\underbrace{\begin{bmatrix} \hat{x}_{k+1} \\ \hat{p}_{k+1} \end{bmatrix}}_{\tilde{x}_{k+1}} = \underbrace{\begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} \hat{x}_k \\ \hat{p}_k \end{bmatrix}}_{\tilde{x}_k} + \underbrace{\begin{bmatrix} B \\ \mathbf{0} \end{bmatrix}}_{\tilde{B}} u_k + \underbrace{\begin{bmatrix} E \\ \mathbf{0} \end{bmatrix}}_{\tilde{E}} d_k$$

$$\hat{y}_k = \underbrace{\begin{bmatrix} C & F \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} \hat{x}_k \\ \hat{p}_k \end{bmatrix}}_{\tilde{x}_k} + \underbrace{\begin{bmatrix} D \\ \mathbf{0} \end{bmatrix}}_{\tilde{D}} u_k$$

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$$\begin{aligned} \min_{u_0, \dots, u_{N-1}} \quad & \sum_{k=0}^{N-1} \left( \|s_k\|_{Q_s}^2 + \|u_k\|_{Q_u}^2 \right) \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k \\ & y_k = Cx_k + Du_k \\ & lb_k - s_k \leq y_k \leq ub_k + s_k \\ & \underline{u} \leq u_k \leq \bar{u} \\ & x_0 = \hat{x}(t) \\ & \forall k \in \{0, \dots, N-1\} \end{aligned}$$

$$x_1 = Ax_0 + Bu_0 + Ed_0$$

$$x_2 = A(Ax_0 + Bu_0 + Ed_0) + Bu_1 + Ed_1$$

⋮

$$x_{k+1} = A^{k+1}x_0 + \dots$$

$$\begin{bmatrix} A^k B & \dots & AB & B \end{bmatrix} \begin{bmatrix} u_0^T & \dots & u_k^T \end{bmatrix}^T + \dots$$

$$\begin{bmatrix} A^k E & \dots & AE & E \end{bmatrix} \begin{bmatrix} d_0^T & \dots & d_k^T \end{bmatrix}^T$$

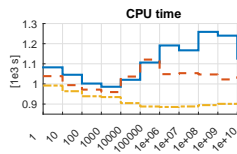
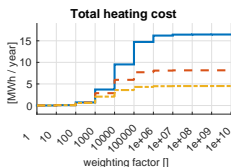
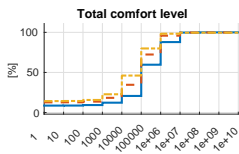
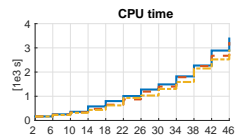
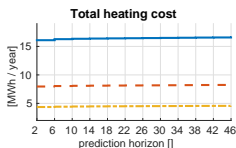
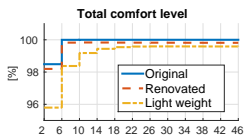
$$y_k = CA^k x_0 + \dots$$

$$C \begin{bmatrix} A^{k-1} B & \dots & AB & B \end{bmatrix} \begin{bmatrix} u_0^T & \dots & u_{k-1}^T \end{bmatrix}^T + \dots$$

$$C \begin{bmatrix} A^{k-1} E & \dots & AE & E \end{bmatrix} \begin{bmatrix} d_0^T & \dots & d_{k-1}^T \end{bmatrix}^T + Du_k + Fp_0$$

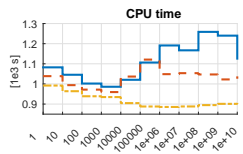
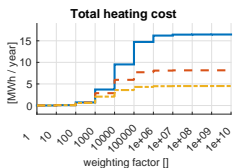
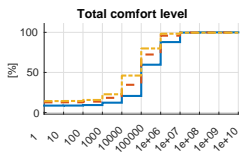
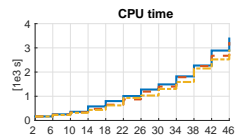
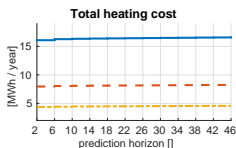
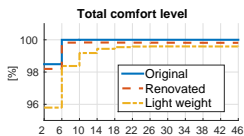
Significantly reduces the number of the optimization variables.

One year performances of RBC, PID, MPC.  
Three types of 6-zone buildings, with 300 states.



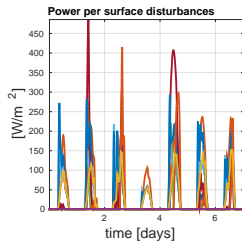
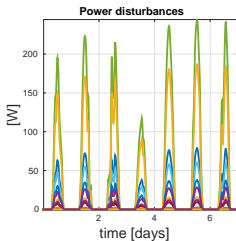
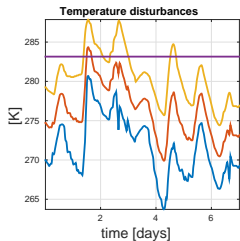
$$T_s = 900, N = 40 \text{ steps (i.e., 10 hours)}, \frac{Q_s}{Q_u} = 10^8.$$

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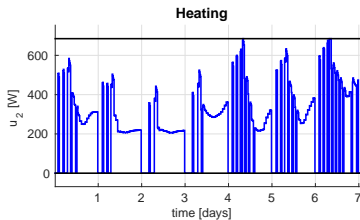
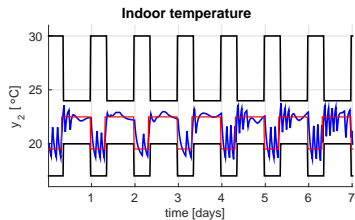
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# Disturbance Profiles



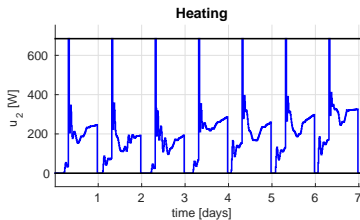
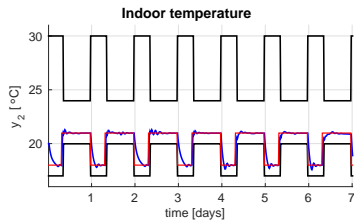
52 disturbances

# Control Profiles – RBC



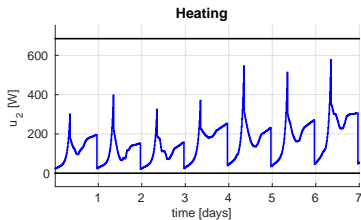
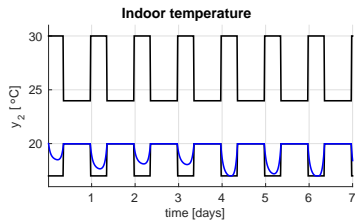
Comfort satisfaction, spanning from 93.9% to 95.2%.

# Control Profiles – PID



Comfort satisfaction, spanning from 95.6% to 99.6%.  
Energy savings around 6%.

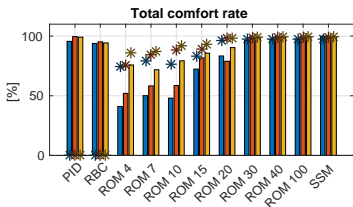
# Control Profiles – MPC



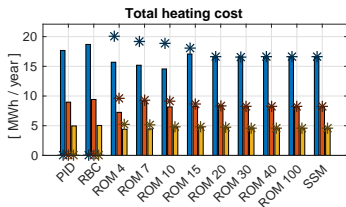
Comfort satisfaction, close to 100.0%.  
Energy savings around 13%.



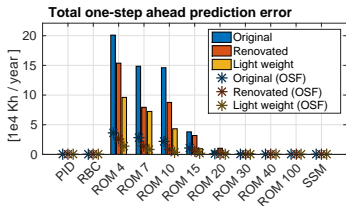
# Performance Evaluation



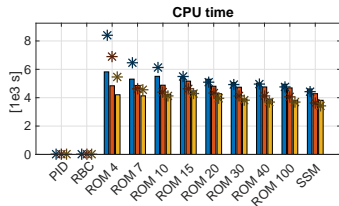
(a)



(b)



(c)



(d)

- 1 Influence of controller model accuracy on controller performance.
- 2 Minimum of 30 states was necessary for 6-rooms house.
- 3 When a dense formulation is used a CPU time becomes independent of the number of states of the controller model.
- 4 Use a controller model which emulates the real building as accurately as possible!

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