

# Explicit Model Predictive Control of a Fuel Cell

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# Outline

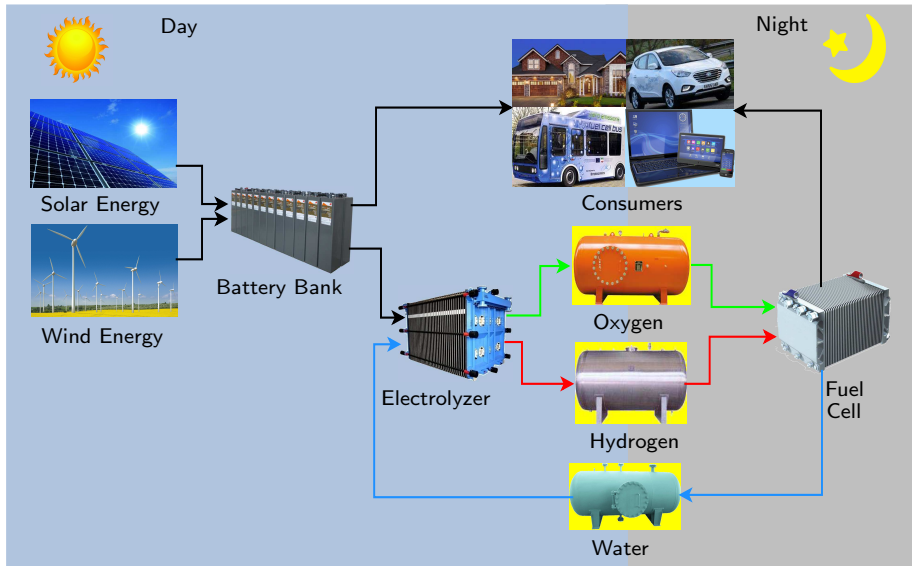
- 1 Fuel Cell
- 2 Experimental Set-up
- 3 PEM Fuel Cell Control
- 4 Fuel Cell Modeling
- 5 Experimental Results
- 6 Conclusions

# Fuel Cell

- A device which converts the chemical energy from the fuel into electric energy through a chemical reaction
- Emits heat and pure water
- Operates like a battery
- Proton Exchange Membrane (PEM)



# Fuel Cell Power System



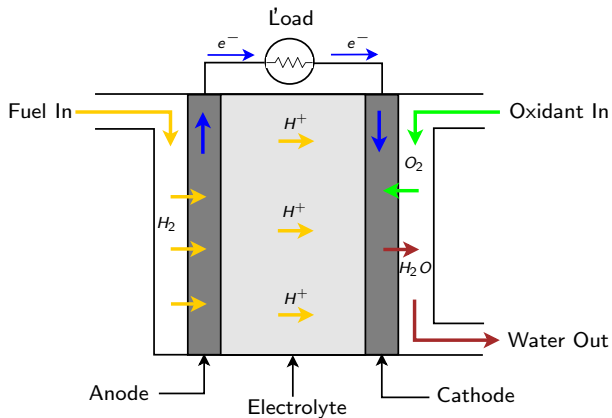
# Applications of a PEM Fuel Cell



- Properties of fuel cells
  - Renewable source of energy
  - Silent operation
  - No pollution
  - High Efficiency
  - Consumer market

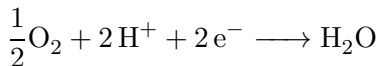
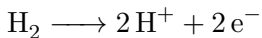
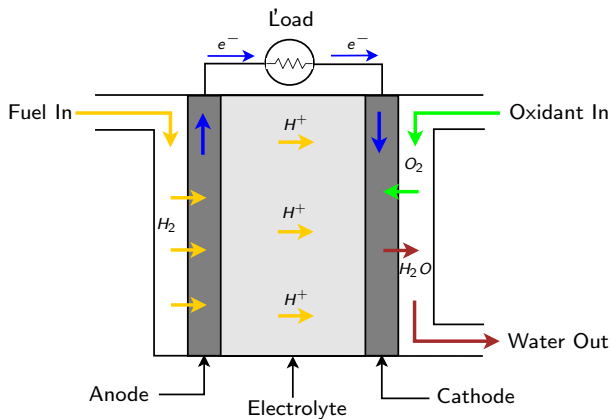
- Properties of fuel cells
  - Renewable source of energy
  - Silent operation
  - No pollution
  - High Efficiency
  - Consumer market
- Control of electrolyzer and fuel cell

# Principle of a Fuel Cell

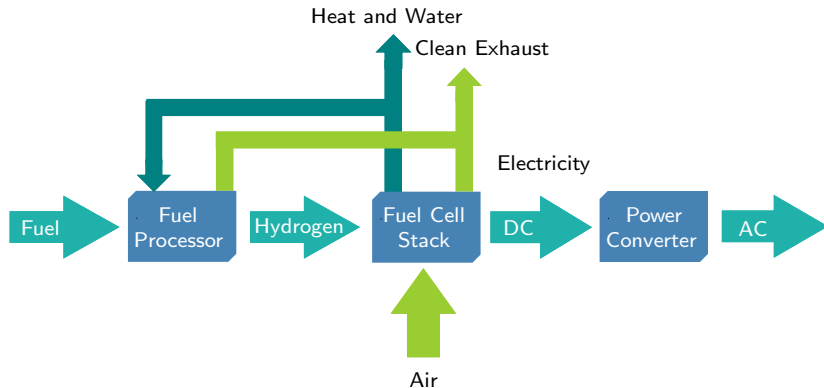




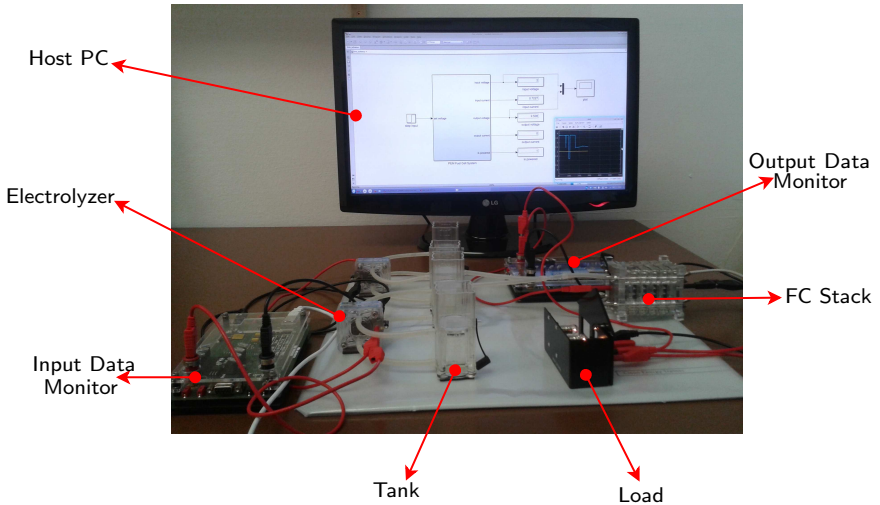
# Principle of a Fuel Cell



# Components of a Fuel Cell System




# Experimental Set-up of a PEM Fuel Cell



- Generate desired voltage from the fuel cell stack without violating the electrolyzers input voltage limits
- Challenges
  - Electrolyzer and fuel cell dynamics
  - Temperature and air flow
  - High resistance at load side
  - Small operating range of electrolyzers
- Control
  - Model predictive control
  - Disturbance modeling

# Implementation of a Real-time MPC



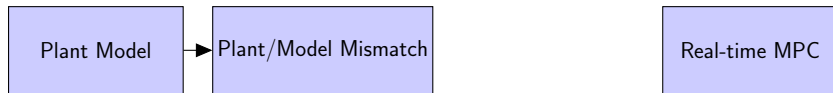
Real-time MPC

# Implementation of a Real-time MPC

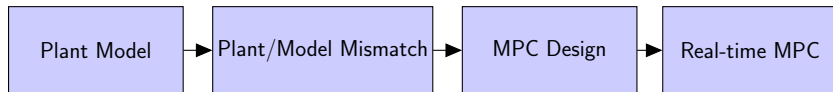
Plant Model

Real-time MPC

# Implementation of a Real-time MPC

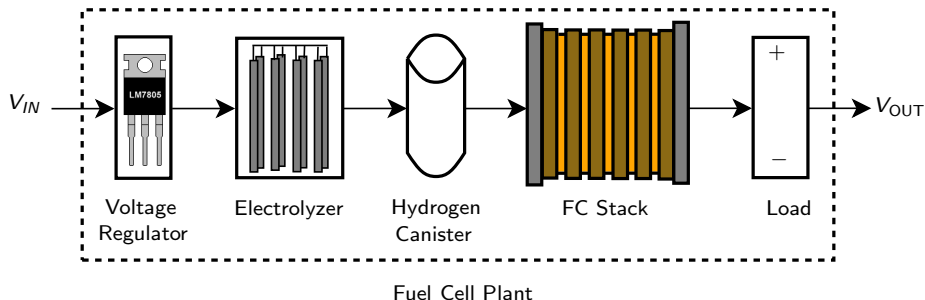


# Implementation of a Real-time MPC



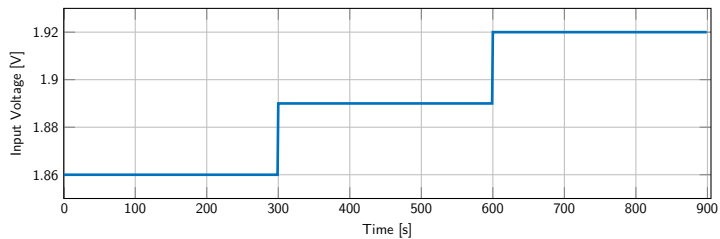


# PEM Fuel Cell Modeling

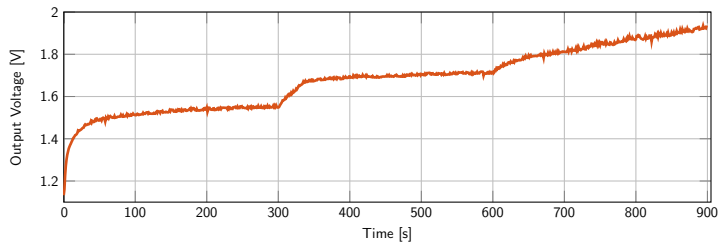
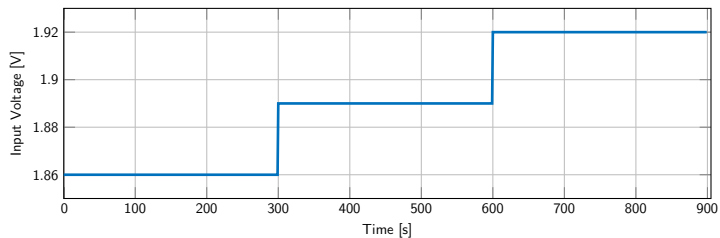


- Input: Input voltage ( $V_{IN}$ )
- Output: Output voltage ( $V_{OUT}$ )
- Input Constraints: 1.8-2.12 V

# Model Identification



# Model Identification



Model Order	Fit To Estimated Data	FPE	MSE
1	95.6 %	$3.9 \times 10^{-5}$	$3.9 \times 10^{-5}$
2	96.5 %	$2.4 \times 10^{-5}$	$2.3 \times 10^{-5}$
3	96.6 %	$2.3 \times 10^{-5}$	$2.2 \times 10^{-5}$
4	96.7 %	$2.2 \times 10^{-5}$	$2.1 \times 10^{-5}$
5	96.8 %	$2.1 \times 10^{-5}$	$2.0 \times 10^{-5}$

Identified fuel cell model:

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k$$

- Sources of model/plant mismatch
  - Temperature and air flow
  - Data monitor
- Augmented design model

$$x_{k+1} = Ax_k + Bu_k$$

$$d_{k+1} = d_k$$

$$y_k = Cx_k + d_k$$

- Luenberger observer

$$\begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_{k+1} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_k + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + L(y_{m,k} - \hat{y}_k)$$

$$\hat{y}_k = [C \quad I] \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}_k$$

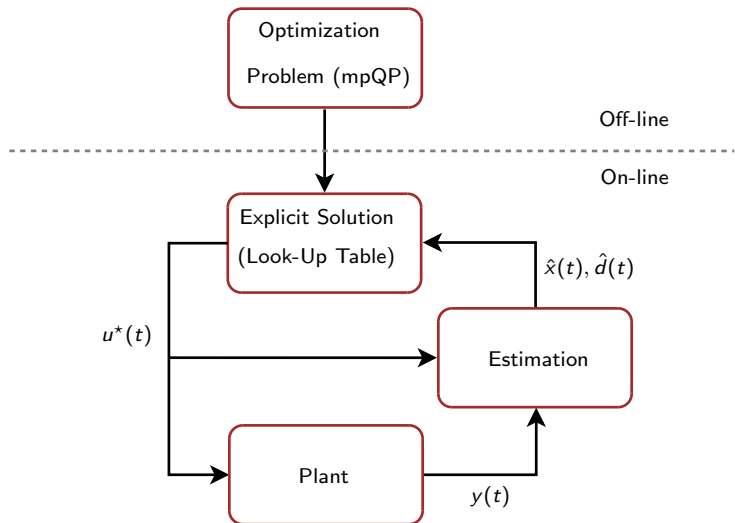
$$\begin{aligned} \min_{u_0, \dots, u_{N-1}} \quad & \sum_{k=0}^{N-1} (y_k - y_r)^T Q (y_k - y_r) + \Delta u_k^T R \Delta u_k \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k & k = 0, \dots, N-1 \\ & y_k = Cx_k & k = 0, \dots, N-1 \\ & \Delta u_k = u_k - u_{k-1} & k = 0, \dots, N-1 \\ & u_k \in \mathcal{U} & k = 0, \dots, N-1 \\ & x_0 = x(t) \end{aligned}$$

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 \min_{u_0, \dots, u_{N-1}} \quad & \sum_{k=0}^{N-1} (y_k - y_r)^T Q (y_k - y_r) + \Delta u_k^T R \Delta u_k \\
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 & y_k = Cx_k + d_k && k = 0, \dots, N-1 \\
 & \Delta u_k = u_k - u_{k-1} && k = 0, \dots, N-1 \\
 & u_k \in \mathcal{U} && k = 0, \dots, N-1 \\
 & d_{k+1} = d_k && k = 0, \dots, N-1 \\
 & x_0 = \hat{x}(t) \\
 & d_0 = \hat{d}(t)
 \end{aligned}$$

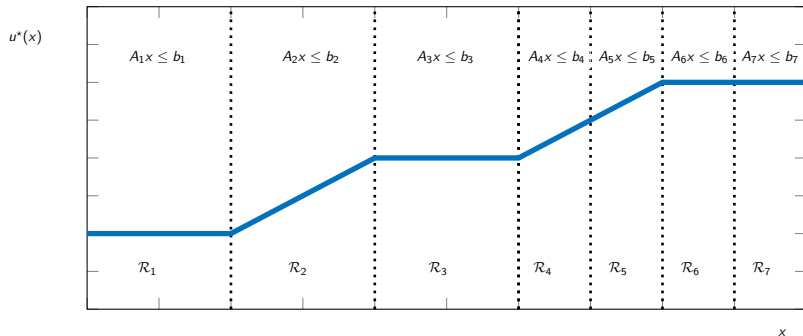
Optimization  
Problem (mpQP)



# Explicit MPC

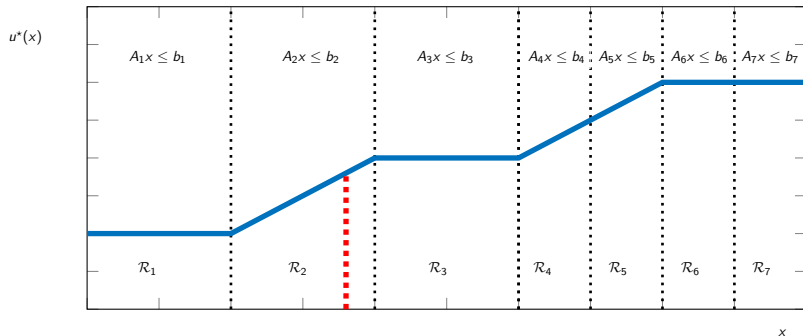


# Sequential Search



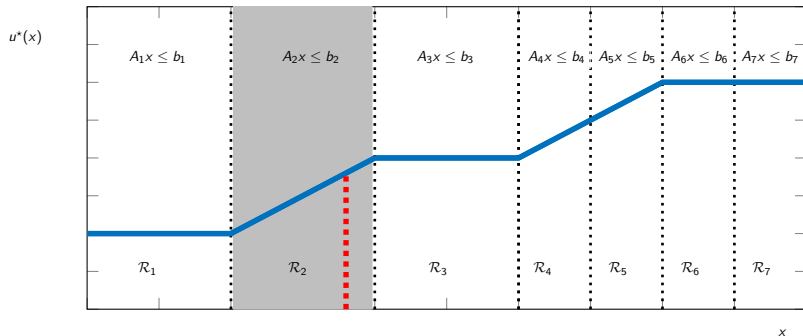
$$u^*(x) = \begin{cases} F_1x_0 + g_1 & \text{if } x_0 \in \mathcal{R}_1 \\ \vdots & \\ F_7x_0 + g_7 & \text{if } x_0 \in \mathcal{R}_7 \end{cases}$$

# Sequential Search



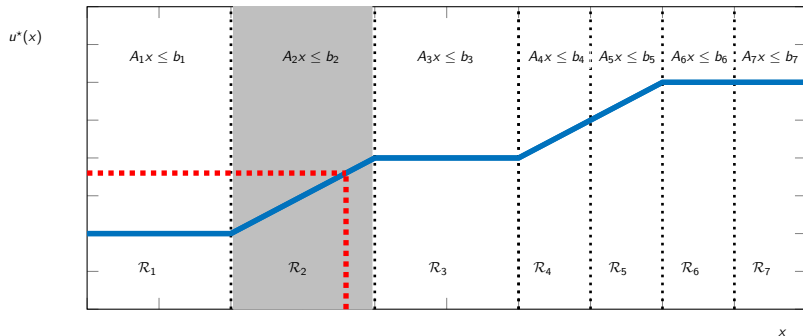
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# Sequential Search



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# Sequential Search



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# Explicit MPC : Pros and Cons

- Pros

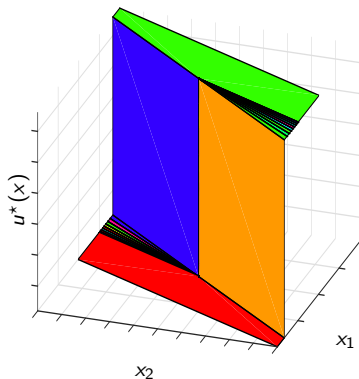
- 😊 Simple implementation: small code, division-free
- 😊 Predictable execution: exact worst-case runtime and memory
- 😊 Verifiable performance: closed-loop stability, feasibility, and safety

- Cons

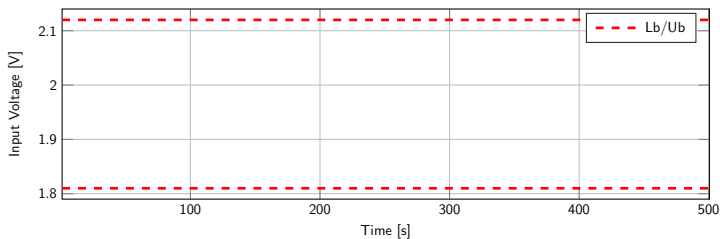
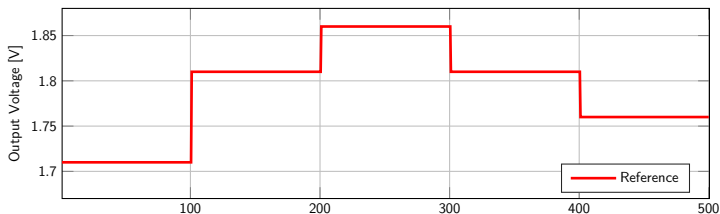
- 😞 Only for small scale problems
- 😞 Explicit solutions can be very complex
- 😞 Reducing complexity requires sacrificing performance

# Implementation of a Explicit MPC

- Prediction horizon: 10
- Number of regions: 519
- Computational time: 3.58 s

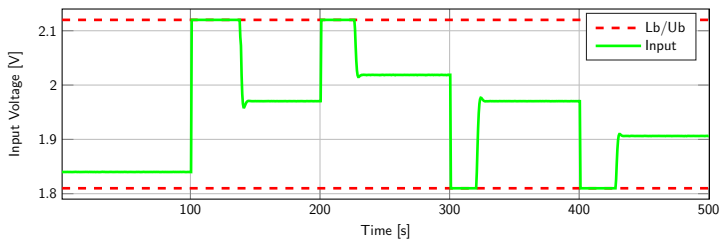
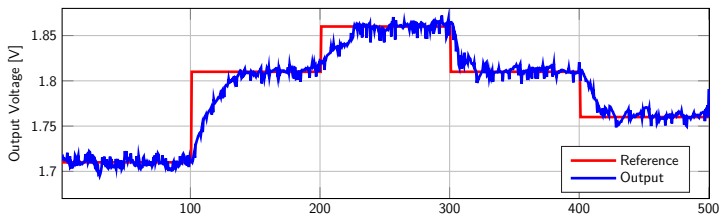


# Experimental Results





# Experimental Results



- Identification of fuel cell model
- Disturbance modeling
- Model predictive control design
- Verification of explicit MPC on fuel cell system