Model Predictive Control with Applications in Building Thermal Comfort Control

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STU FCHPT

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Presentation Outline

1. Building Control
2. Model Predictive Control
3. Simulation Case Studies
4. Aims of the Thesis

Presentation Outline

Building Control Motivation

Problem: EU spends 400 billion EUR/year on energy.

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Solution: MPC-based control

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Building Control Objectives

- Satisfy thermal comfort constraints
- Minimize energy consumption
- Obey technological restrictions

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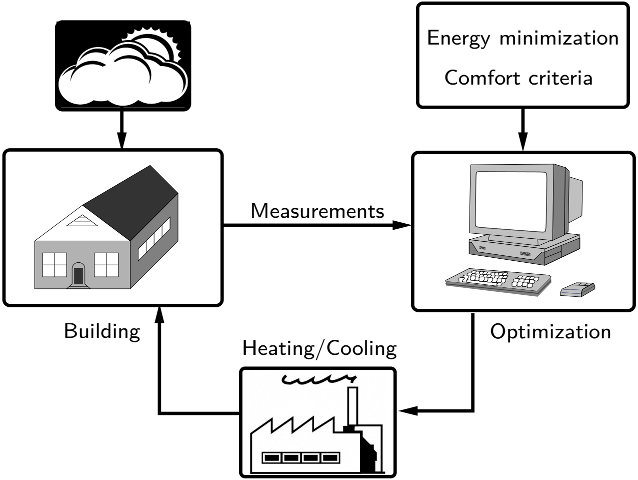
Existing Solutions

Presentation Outline

Building Thermal Control Scheme



Model Predictive Control

Objective function

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} \ell(x_k, u_k)$$

Constraints

$$x_{k+1} = Ax_k + Bu_k + Ed_0$$

$$x_{\min} \le x_k \le x_{\max}$$

$$u_{\min} \le u_k \le u_{\max}$$

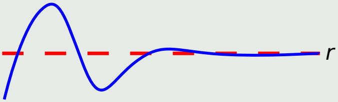
$$x_0 = x(t)$$

$$d_0 = d(t)$$

Deterministic MPC Formulations

Reference Tracking + Energy Minimization (Basic)

$$\ell(x_k, u_k) = q_x(Cx_k - r)^2 + q_u u_k^2$$



Comfort Zone Tracking + Energy Minimization (CZT)

$$\ell(s_k, u_k) = q_s s_k^2 + q_u u_k^2$$
$$\text{s.t. } r - \epsilon - s_k \leq Cx_k \leq r + \epsilon + s_k$$

Minimization of Zone Violations + Energy Minimization (Hybrid)

$$\ell(\delta_k, u_k) = q_\delta \delta_k + q_u u_k^2$$
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$$(s_k > 0) \implies (\delta_k = 1)$$

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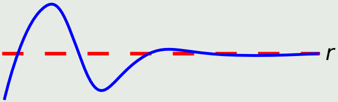

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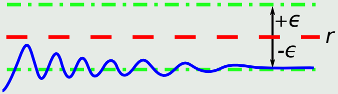

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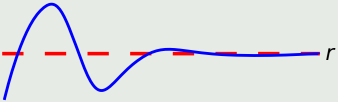

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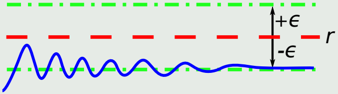

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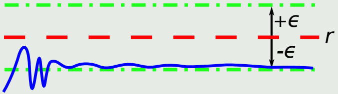
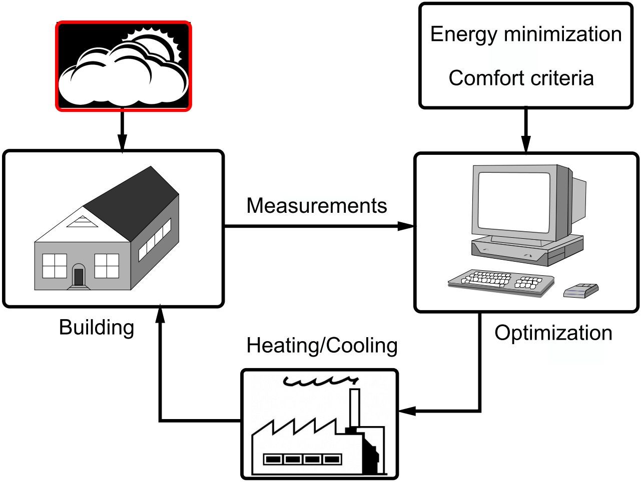


Weather Predictions Problem



Stochastic Model Predictive Control

Energy minimisation objective function

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} u_k^2$$

Normal distribution of disturbances

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + E(d_0 + k\theta), \\ Cx_k &\geq r - \epsilon \\ Cx_k &\leq r + \epsilon \\ u_{\min} &\leq u_k \leq u_{\max} \\ \theta &\sim \mathcal{N}(0, \sigma(t)) \end{aligned}$$

Stochastic Model Predictive Control

Energy minimisation objective function

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} u_k^2$$

Probabilistic constraints

$$x_{k+1} = Ax_k + Bu_k + E(d_0 + k\theta),$$
$$\Pr(Cx_k \geq r - \epsilon) \geq 1 - \alpha$$
$$\Pr(Cx_k \leq r + \epsilon) \geq 1 - \alpha$$
$$u_{\min} \leq u_k \leq u_{\max}$$
$$\theta \sim \mathcal{N}(0, \sigma(t))$$

Probabilistic Constraints

$$g(x, u, \theta) \leq 0$$

$$\Pr(g(x, u, \theta) \leq 0) \geq 1 - \alpha$$

Campi M. and Garrati S., 2008

$$g(x, u, \theta^{(i)}) \leq 0, \quad i = 1, \ldots, M$$

$$\Pr(\Pr(g(x, u, \theta) \leq 0) \geq 1 - \alpha) \geq 1 - \beta$$

Alamo T., Tempo R., and Luque A., 2010

$$M \geq \frac{1 + (Nn_u) + \ln(1/\beta) + \sqrt{2((Nn_u) + 1)\ln(1/\beta)}}{\alpha}$$

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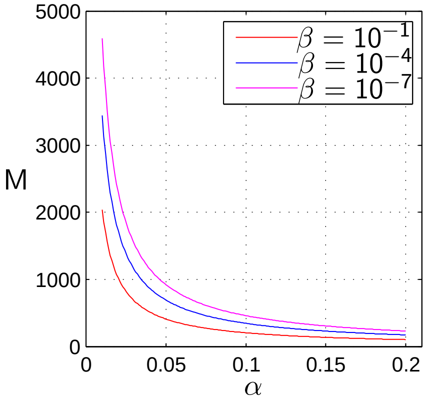
Number of M Samples



$$M \geq \frac{1 + (Nn_u) + \ln(1/\beta) + \sqrt{2((Nn_u) + 1)\ln(1/\beta)}}{\alpha}$$

Deterministic Realization of Probabilistic Constraints

Energy minimisation objective function

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} u_k^2$$

Finitely many deterministic constraints

$$x_{k+1} = Ax_k + Bu_k + E(d_0 + k\theta^{(i)}), \quad i = 1, \ldots, M$$

$$Cx_k \geq r - \epsilon$$

$$Cx_k \leq r + \epsilon$$

$$u_{\min} \leq u_k \leq u_{\max}$$

$$\theta \sim \mathcal{N}(0, \sigma(t))$$

Drgoňa, J. – Kvasnica, M. – Klaučo, M. – Fikar, M.: Explicit Stochastic MPC Approach to Building Temperature Control. Conference on Decision and Control, Florence, Italy, 2013.

Presentation Outline

Single Zone Building Model

State (Measured) Variables

x_1 − floor temperature
x_2 − internal facade temperature
x_3 − external facade temperature
x_4 − internal temperature

Controlled Variable

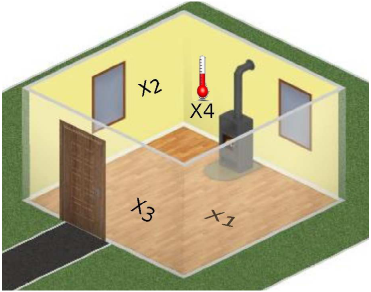
$y = x_4$

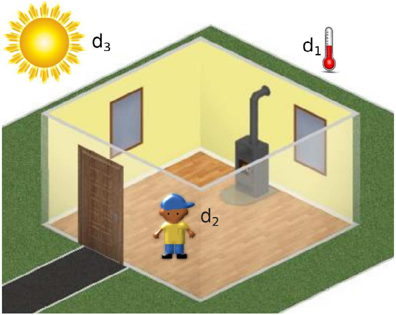
Measured Disturbances

d_1 − external temperature
d_2 − occupancy
d_3 − solar radiation

Manipulated Variable

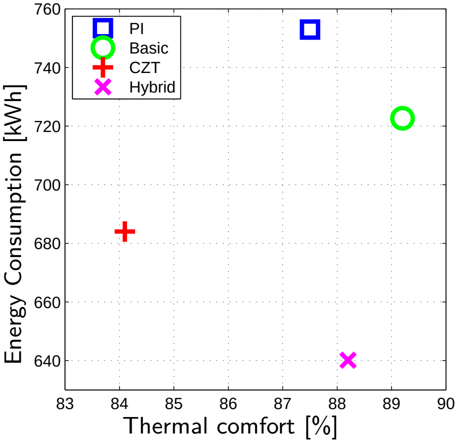
u − heat flow





Deterministic MPC Performance

	Thermal comfort [%]	Consumed energy [kWh]	Energy savings [%]
□	87.5	753.0	-
○	89.2	722.7	4.0
+	84.1	684.0	9.1
×	88.2	640.1	15.0



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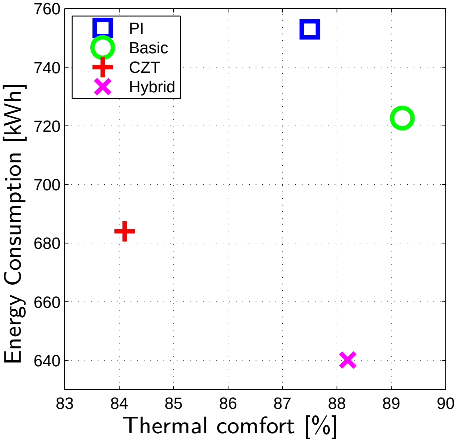
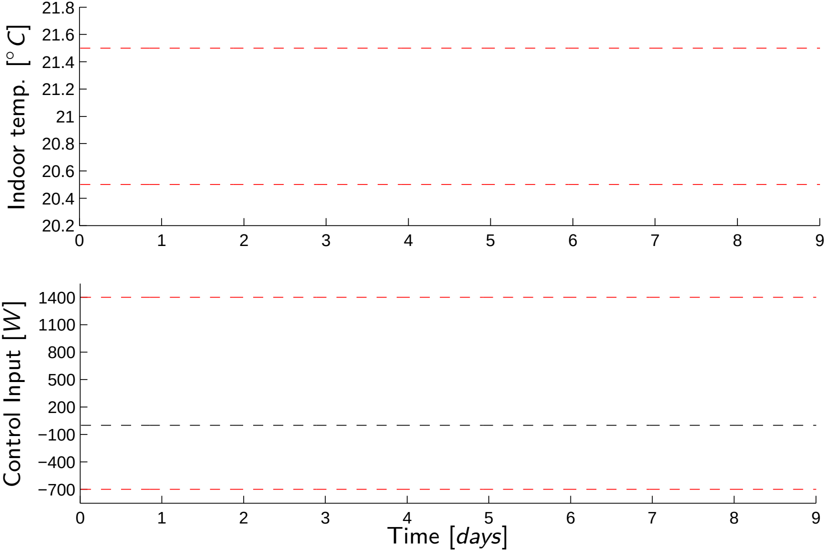


Stochastic MPC Simulation Results



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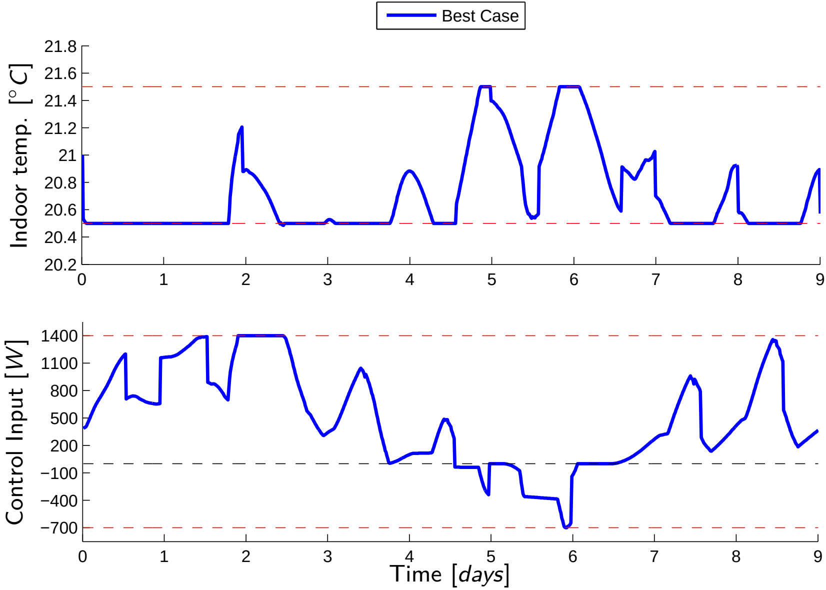

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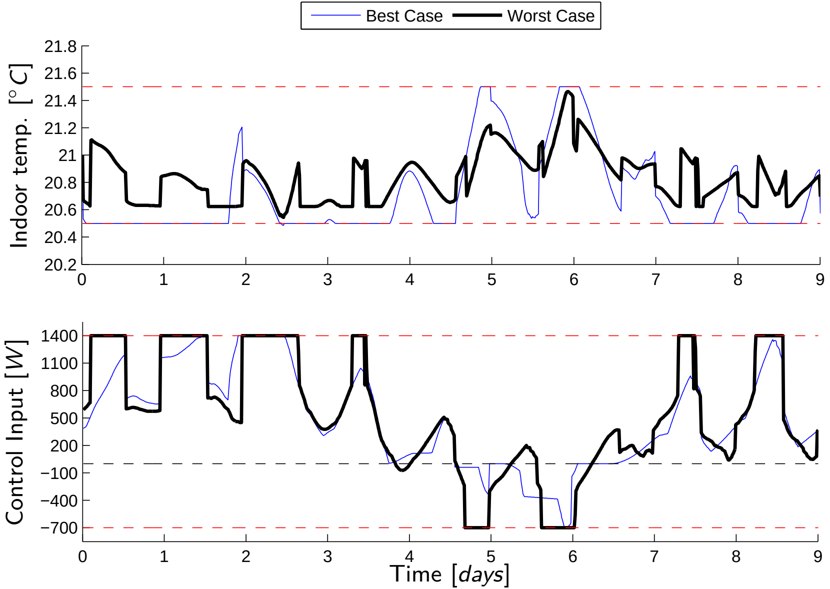

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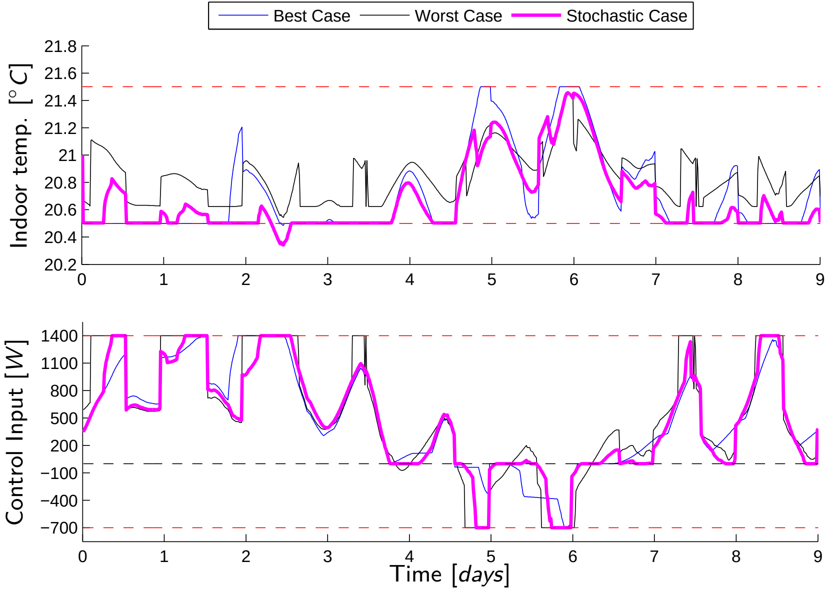
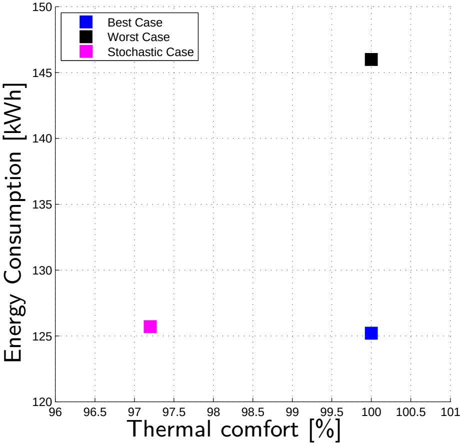


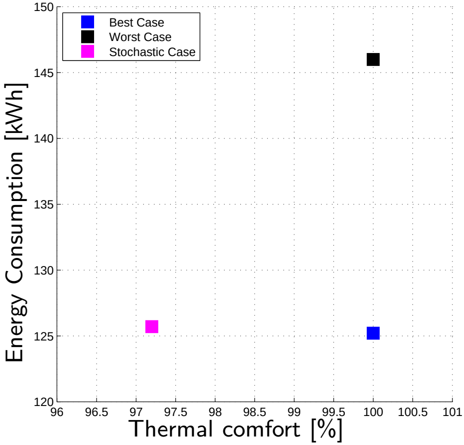
Stochastic MPC Performance

	Thermal comfort [%]	Consumed energy [kWh]
	100.0	125.2
	100.0	146.0
	97.2	125.7



Stochastic MPC Performance

	Thermal comfort [%]	Consumed energy [kWh]
■ (blue)	100.0	125.2
■ (black)	100.0	146.0
■ (magenta)	97.2	125.7



Publications

A-Class

- Klaučo, M. - Drgoňa, J. - Kvasnica, M. - Di Cairano, S.: **Building Temperature Control by Simple MPC-like Feedback Laws Learned from Closed-Loop Data.** IFAC World Congress, 2014.

IEEE Conferences

- Drgoňa, J. - Kvasnica, M.: **Comparison of MPC Strategies for Building Control.** Process Control, 2013.
- Drgoňa, J. - Kvasnica, M. - Klaučo, M. - Fikar, M.: **Explicit Stochastic MPC Approach to Building Temperature Control.** Conference on Decision and Control, 2013.

In preparation

- Drgoňa, J. - Klaučo, M. - Bendžala J. - Fikar, M.: **Model Identification and Predictive Control of a Laboratory Binary Distillation Column.** Process Control, 2015.

Aims of the Thesis

1. **Performance evaluation of MPC usage in building automation**
2. **Development of efficient MPC strategies**
3. **Experimental validation on laboratory or real devices**

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Opponents Questions

Question 1

Je uvedený rozsah (4.2) pre zachovanie linearity modelu tepelnej pohody budovy skutočne reálny? (str. 51)

$$-30^\circ C \leq x_i \leq 50^\circ C, \; i = 1, \ldots, 4,$$

- **Lineárny model**: MATLAB toolbox Hamlab/ISE
- **Nelineárne modely**: Energy+, TRNSYS
- **Komunikačné mosty**: BCVTB, MLE+, OpenBuild
- **Význam ohaničení**: Explicitné MPC a ohraničenie prehľadávaného parametrického priestoru

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Question 2

Poruchová veličina θ je trojrozmerným vektorom, deklarovaná hustota pravdepodobnosti (4.7) by potom mala byť združeným rozdelením pravdepodobnosti a σ variančnou maticou - ako by ste ju volili? (str. 54)

$$\theta \sim \mathcal{N}(0, \sigma(t))$$

- **Aplikácia**: Uvažujeme stochastické modelovanie len jednej poruchovej veličiny - vonkajšej teploty
- **Problém**: Výberová kovariančná matica je a priori neznáma
- **Riešenie**: Výpočet kovariančej matice z historicky nameraných dát

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Ako sleduje (stúpajúci alebo klesajúci) trend budúceho vývoja poruchových veličín podľa (4.8) svoje doterajšie historické dáta? (str. 54)

$$d(t + kT_s) = d(t) + k\theta,\ \ k = 1, \ldots, N,$$

- **Charakter**: Aditívna neurčitosť.
  $d(t)$ - meranie poruchy s konštantnou predikciou na N
  θ - z $1 - \alpha$ intervalu spoľahlivosti rozdelenia $\mathcal{N}(0, \sigma(t))$.

- Zjednodušenie výrazu:

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Question 4

Podľa akej argumentácie bola vybratá perióda vzorkovania Ts na hodnote 444 sekúnd? (str. 55)

- $T_{90} \approx 10000s \approx 2.8hod$
- $T_{90}/20$ ako celočíselný násobok periódy vzorkovania modelu z toolboxu ISE, kde $Ts = 222s$
- $Ts = 888s$ pri stochastických návrhoch MPC

Question 4

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Question 5

Používaným štandardom riadenia tepelnej pohody v budovách je ekvitermická regulácia, teda uvažovanie doprednej väzby od vonkajšej teploty. Prečo pri PI riadení nebola táto väzba v srovnávacej simulácii uvažovaná? (str. 65)

- Návrh PI regulátora z perpektívy teórie riadenia

Question 5

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- Návrh PI regulátora z perpektívy teórie riadenia

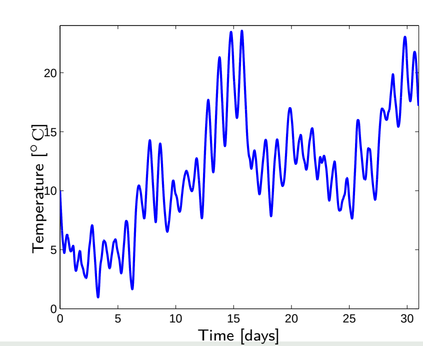
Discussion - Backup Slides

Simulation Study Deterministic MPC

Closed Loop Simulation Parameters

- Prediction horizon: $N = 10$
- Sampling time: $T_s = 444$ sec
- Simulation time: $T_{sim} = 31$ days
- Initial indoor temperature: $x_4 = 10^\circ C$
- No weather predictions

Evolution of External Temperature



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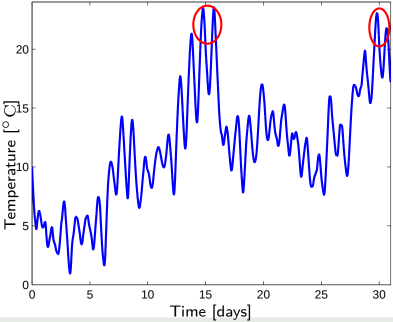
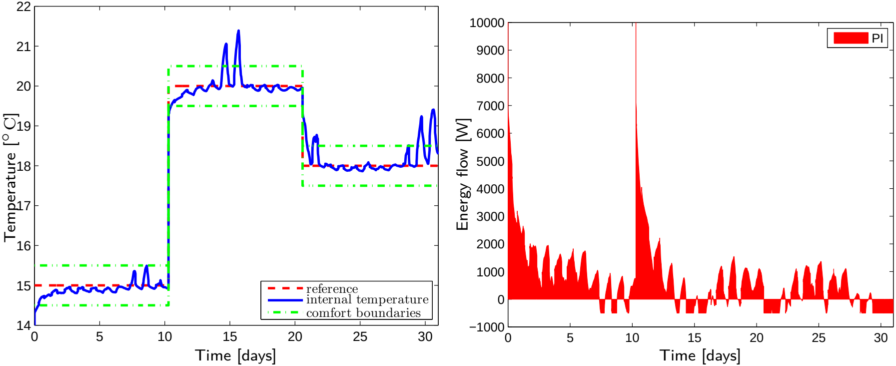
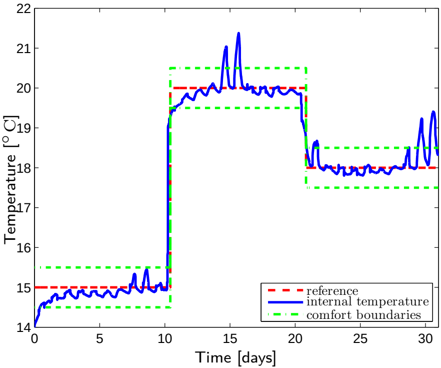


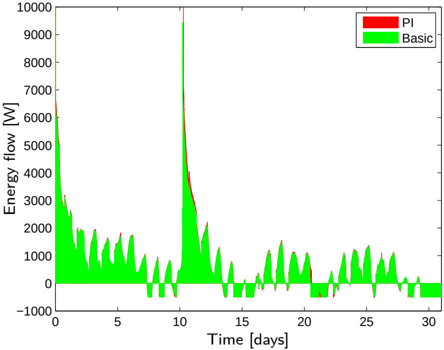
PI Controller



Control strategy	PI
Thermal comfort [%]	87.5
Energy consumption [kWh]	753.0
Energy savings [%]	-

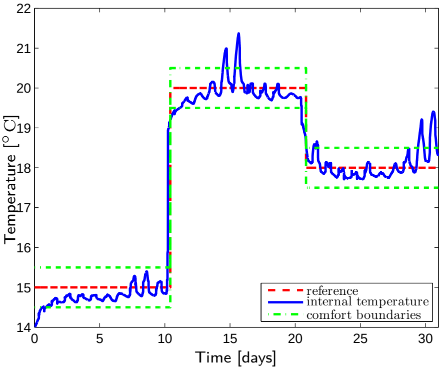
Reference Tracking (Basic)

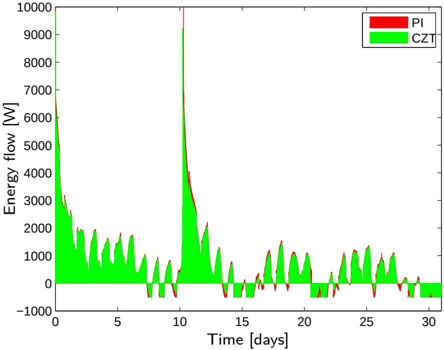




Control strategy	PI	Basic
Thermal comfort [%]	87.5	89.2
Energy consumption [kWh]	753.0	722.7
Energy savings [%]	-	4.0

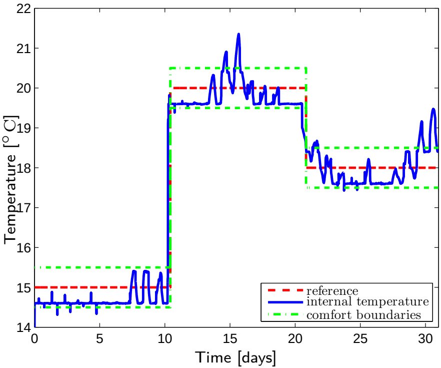
Comfort Zone Tracking (CZT)

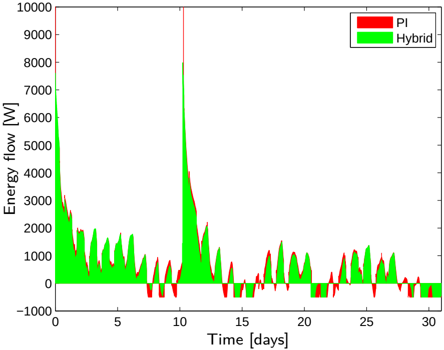




Control strategy	PI	Basic	CZT
Thermal comfort [%]	87.5	89.2	84.1
Energy consumption [kWh]	753.0	722.7	684.0
Energy savings [%]	-	4.0	9.1

Minimization of Comfort Zone Violations (Hybrid)





Control strategy	PI	Basic	CZT	Hybrid
Thermal comfort [%]	87.5	89.2	84.1	88.2
Energy consumption [kWh]	753.0	722.7	684.0	640.1
Energy savings [%]	-	4.0	9.1	15.0

Number of Parameter and Constraints Reduction

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + E(d_0 + k\theta) \\
Cx_k &\leq r + \epsilon
\end{aligned}
$$

Number of Parameter and Constraints Reduction

$$C(Ax_k + Bu_k + E(d_0 + k\theta^{(i)})) \leq r + \epsilon$$

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Explicit Stochastic MPC

$$\overline{\theta} = \text{arg max}_{\theta^{(i)}}\{CE\theta^{(i)}\}$$

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Explicit Stochastic MPC

Previous Parametric QP has:

927 parametric variables
18400 constraints

Explicit Stochastic MPC

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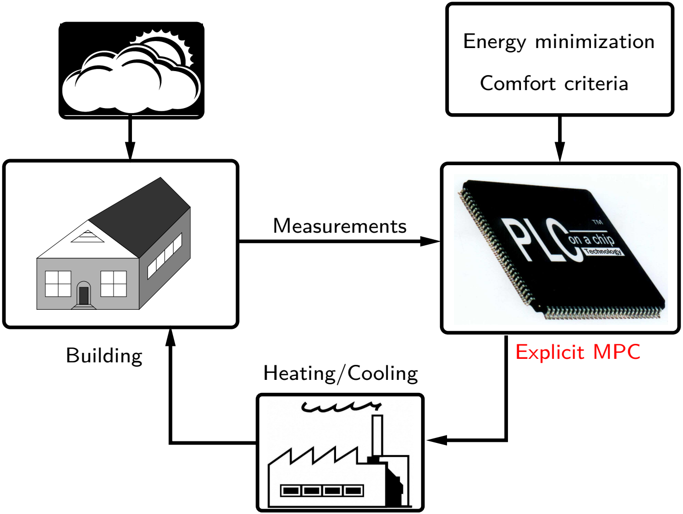
Resulting Parametric QP has:

14 parametric variables
60 constraints

Explicit Stochastic MPC



Stochastic MPC Solution Time

Number of parameters: 14
Number of constraints: 60

Number of regions: 816
Time to compute: $\approx$ 6min

Explicit Stochastic MPC Control Algorithm

At each sample time T_s

1. Measure $x(t)$, $d(t)$, $r(t)$ and obtain $\sigma(t)$
2. Generate M samples $\theta^{(1)}, \ldots, \theta^{(M)}$
3. Pick $\underline{\theta}$ and $\overline{\theta}$
4. Set $\xi = [x(t), d(t), r(t), \underline{\theta}, \overline{\theta}]$ and identify $\mathcal{R}_i$
5. $u^*(t) = \tilde{F}_{i^*}\xi + \tilde{g}_{i^*}$

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