

EDITORS

M. HUBA, S. SKOGESTAD, M. FIKAR, M. HOVD,  
T.A. JOHANSEN, B. RÓHAL-ILKIV

# SELECTED TOPICS ON CONSTRAINED AND NONLINEAR CONTROL



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**M. Huba, S. Skogestad, M. Fikar, M. Hovd,  
T. A. Johansen, B. Rohal'-Ilkiv**

Editors

**Selected Topics on  
Constrained and Nonlinear  
Control  
Textbook**

STU Bratislava – NTNU Trondheim

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## Preface

This textbook was created within the NIL-I-007-d Project “Enhancing NO-SK Cooperation in Automatic Control” (ECAC) carried out in 2009-2011 by university teams from the Slovak University of Technology in Bratislava and from the Norwegian University of Science and Technology in Trondheim. As it is already given by the project title, its primary aim was enhancing co-operation in academic research in the automatic control area in the partner institutions. This was achieved by supporting broad spectrum of activities ranging from student mobilities at the MSc. and PhD. level, staff mobilities, organization of multilateral workshop and conferences, joint development of teaching materials and publishing scientific publications. With respect to the original project proposal, the period for carrying out the foreseen activities was reasonably shortened and that made management of the all work much more demanding. Despite of this, the project has reached practically all planned outputs – this textbook represents one of them – and we believe that it really contributes to the enhancement of Slovak-Norwegian cooperation and to improvement of the educational framework at both participating universities. Thereby, I would like to thank all colleagues participating in the project activities at both participating universities and especially professor Sigurd Skogestad, coordinator of activities at the NTNU Trondheim, associate professor Katarína Žáková for managing the project activities, and professor Miroslav Fikar for the patient and voluminous work with collecting all contribution and compilation of all main three project publications (textbook, workbook, and workshop preprints).

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*Mikuláš Huba*  
*Project coordinator*



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## List of Contributors

Luboš Čirka

Institute of Information Engineering, Automation and Mathematics,  
Faculty of Chemical and Food Technology, Slovak University of Technology  
in Bratislava, e-mail: lubos.cirka@stuba.sk

Miroslav Fikar

Institute of Information Engineering, Automation and Mathematics,  
Faculty of Chemical and Food Technology, Slovak University of Technology  
in Bratislava, e-mail: miroslav.fikar@stuba.sk

Martin Herceg

Swiss Federal Institute of Technology, Zurich, e-mail: herceg@control.ee.  
ethz.ch

Morten Hovd

Department of Engineering Cybernetics, Norwegian University of Science  
and Technology, Trondheim, Norway, e-mail: morten.hovd@itk.ntnu.no

Mikuláš Huba

Institute of Control and Industrial Informatics, Faculty of Electrical  
Engineering and Information Technology, Slovak University of Technology  
in Bratislava, e-mail: mikulas.huba@stuba.sk

Johannes Jäschke

Department of Chemical Engineering, Norwegian University of Science and  
Technology, Trondheim, Norway, e-mail: jaschke@chemeng.ntnu.no

Tor A. Johansen

Department of Engineering Cybernetics, Norwegian University of Science  
and Technology, Trondheim, Norway, e-mail: tor.arne.johansen@itk.ntnu.no

Michal Kvasnica

Institute of Information Engineering, Automation and Mathematics,  
Faculty of Chemical and Food Technology, Slovak University of Technology

in Bratislava, e-mail: michal.kvasnica@stuba.sk

Martin Lauko

Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering Slovak University of Technology, Bratislava, Slovakia., e-mail: martin.lauko@stuba.sk

Tomáš Polóni

Institute of Measurement, Automation and Informatics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, e-mail: tomas.poloni@stuba.sk

Boris Rohal'-Ilkiv

Institute of Measurement, Automation and Informatics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, e-mail: boris.rohal-ilkiv@stuba.sk

Selvanathan Sivalingam

Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway, e-mail: selvanathan.sivalingam@itk.ntnu.no

Sigurd Skogestad

Department of Chemical Engineering, Norwegian University of Science and Technology, Trondheim, Norway, e-mail: skoge@chemeng.ntnu.no

Gergely Takács

Institute of Measurement, Automation and Informatics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, e-mail: gergely.takacs@stuba.sk