POLYNOMIAL APPROACH TO STABLE PREDICTIVE CONTROL

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Abstract: In this article a general framework for stable predictive controllers is derived. The basis for the unification are closed-loop poles. It is shown that many existing stabilising predictive controllers can be obtained for particular settings. Several kinds of the quadratic cost function can be minimised. It is shown that for a special cost function it is possible to construct a predictive controller with the following features: (i) it produces the same control actions as a given known controller in the unconstrained case, (ii) it introduces a new degrees of freedom that can be used for constraints handling.

Keywords: Predictive control, stability, algebraic theory.

1. INTRODUCTION

Generalised Predictive Control (GPC) proposed by Clarke *et al.* (1987) has been accepted in academia and also applied widely in industry. For safety reasons, stability of the closed-loop has to be assured.

A suitable way to achieve stability of the receding horizon minimisation is to invoke the constraint on terminal states. The first generation of stable predictive control methods have used this with finite-time responses (Clarke and Scattolini, 1991; Mosca and Zhang, 1992; Rossiter and Kouvaritakis, 1993) where it is required that both the control and tracking error are finite-time responses. To improve feasibility properties, stable infinite-time responses have been used at first for output predictions (Rawlings and Muske, 1993), then also for input predictions (Rossiter *et al.*, 1996; Fikar and Engell, 1997).

All approaches guarantee stable closed-loop behaviour and there are some user parameters that are used for final tuning for the desired behaviour. These are for example horizon lengths, penalisation factors, number of degrees of freedom, etc. While this may be considered as an advantage, the result is, that the closed-loop specifications that may have been given before, are somewhat lost. The ideal predictive controller should provide the same behaviour as specified before its implementation.

The main idea of this paper is to develop a scheme for "predictification" of an existing controller.

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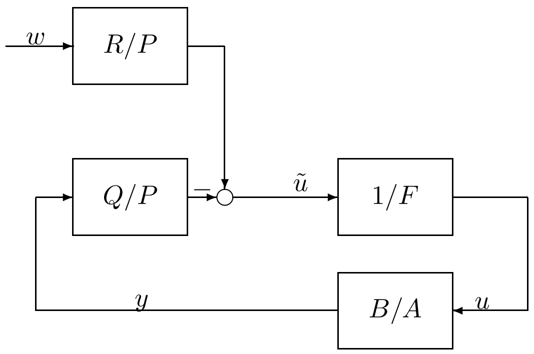

Fig. 1. 2DoF control configuration with explicit integral action

Given a controller that generates the closed-loop with stable poles, the corresponding predictive controller is developed. If additional degrees of freedom are introduced, several possible cost functions can be minimised. Among others, it is possible to construct such a predictive controller that produces the same control actions as the original controller. The second aim is to provide a unified framework for stable predictive control strategies and to show that several approaches in the literature can be recast in the proposed scheme.

This approach tries to reduce the gap between two existing approaches that are able to handle constraints: anti-windup and predictive strategies. It is predictive, but the controller may be designed without the knowledge of constraints.

1.1 *Notation*

All systems in this contribution are assumed to be single input single output, linear, time-invariant, and discrete-time. The systems are described by means of fractions of polynomials in an indeterminate z^{-1}, used in the z-transform and normally interpreted as delay operator.

For simplicity, the arguments of polynomials are omitted whenever possible - a polynomial $X(z^{-1})$ is denoted by X. We define the adjoint of a polynomial X as $X^*(z^{-1}) = X(z)$. Further, for any polynomial X, we define $\langle X \rangle$ as the coefficient of z^0, i.e. the constant term. Any polynomial X can be factored as X^+X^- where X^+ denotes its stable and X^- its totally unstable (anti-stable) part. The greatest common divisor of two polynomials X, Y is denoted by (X, Y).

2. CLOSED-LOOP SYSTEM

Let us consider a discrete-time plant with input-output representation of the form

$$Ay = Bu, \tag{1}$$

where y, u are the process output and manipulated input sequences, respectively. A and B are polynomials in z^{-1} that describe the input-output properties of the plant and $(A, B) = 1$. It is assumed that $A(0) \neq 0$ and $B(0) = 0$ (all delays are included in B).

We assume that the reference w is generated via

$$Fw = G, \tag{2}$$

where $(F, G) = 1$. A further assumption is that (A, F) are coprime, thus $(A, F) = 1$.

The 2DoF controller is another dynamical system described by the equations

$$P\tilde{u} = Rw - Qy, \;\; \tilde{u} = Fu \tag{3}$$

where P, Q, R are controller polynomials that are coprime and $P(0)$ is nonzero. In addition, an integrator forms a part of the controller to track the class of references given above. For the purpose of predictive control, the usual class of references – step changes will be assumed. Then $F = 1 - z^{-1}$, $G = 1$ and the signal $\tilde{u} = \Delta u$ is a sequence of control increments.

This description of the closed-loop configuration is shown in Fig. 1.

Given a stable closed-loop polynomial M, the minimum degree controller that internally stabilises the closed-loop system is defined uniquely and is given as follows:

Theorem 1. The minimum degree controller P, Q, R is given as a solution of two pairs of Diophantine equations that minimise the degrees Q, R

$$\begin{aligned} AFP + BQ &= M, \\ FS + BR &= M. \end{aligned} \tag{4}$$

PROOF. Kučera (1979). □

The general controller is uniquely characterised by the choice of the closed-loop poles. Some choices of M that are important in time optimal control are given below:

Theorem 2. Consider step changes in references and the following closed-loop time optimal control problems for which BIBO stability is to be guaranteed:

(1) State dead-beat: Control error is a polynomial of the smallest degree, polynomial control increments,
(2) Control dead-beat: Control increments are polynomial of the smallest degree, control error is a stable sequence,

(3) Control error dead-beat: Control error is a polynomial of the smallest degree, control increments are a stable sequence.

The corresponding closed-loop poles and minimum degrees of relevant signals with 2DoF controller are

$$
\begin{aligned}
&M_1 = 1,\ \deg(e) = \deg(B) - 1, \\
&\qquad \deg(\tilde{u}) = \deg(A), && (5)\\
&M_2 = A^+,\ \deg(\tilde{u}) = \deg(A^-), && (6)\\
&M_3 = B^+,\ \deg(e) = \deg(B^-) - 1. && (7)
\end{aligned}
$$

PROOF.

(1) (Kučera, 1979),
(2) (Fikar and Kučera, 2000),
(3) (Fikar and Unbehauen, 1999). □

The optimal pole locations are important in predictive control and have close relation to minimum possible horizons that produce stable closed-loop. It is well known that the predictive controllers have the structure of 2DoF controller. If $M = A^+$, the minimum control horizon is equal to the number of unstable system poles. Correspondingly, the minimum output horizon cannot be smaller than the number of unstable system zeros which corresponds to the case $M = B^+$.

3. PREDICTIVE CONTROLLER

Predictive controllers usually operate on the signals $\tilde{u}, e = w - y$. These are given from (1)–(4) as

$$\tilde{u} = \frac{ARG}{M},\ e = \frac{SG}{M}. \tag{8}$$

If the predictive controller is to be equivalent to the nominal controller it has to generate the signals $\tilde{u}, e$ from its internal signals $\bar{u}, \bar{e}$ by filtering through the term $1/M$. Moreover, the requirement of closed-loop stability invoked with the concept of state terminal constraints requires that the signals $\bar{u}, \bar{e}$ are polynomials of finite length. This suggests the relations

$$\tilde{u} = \frac{\bar{u}}{M}, \quad \deg(\bar{u}) = \deg(A), \tag{9}$$

$$e = \frac{\bar{e}}{M}, \quad \deg(\bar{e}) = \deg(B) - 1. \tag{10}$$

Here, the degrees of $\bar{u}, \bar{e}$ are related to the state dead-beat controller. However, further reduction of $\bar{u}, \bar{e}$ is possible. This can be achieved by decomposing the controlled system to stable and anti-stable parts. Next, state dead-beat is applied to the unstable part. The result is given in Fig. 2

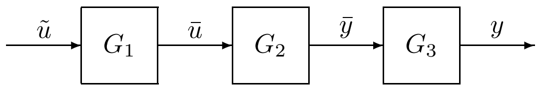

Fig. 2. System decomposition

with $G_1 = M/A^+$, $G_2 = B^-/FA^-$, $G_3 = B^+/M$, hence we have the following result:

Theorem 3. Let us define the following signals

$$\tilde{u} = \frac{A^+}{M}\bar{u}, \quad \deg(\bar{u}) = N_u, \tag{11}$$

$$y = \frac{B^+}{M}\bar{y}, \quad \bar{e} = \bar{w} - \bar{y}, \quad \deg(\bar{e}) = N. \tag{12}$$

and horizons

$$N_u = \deg(A^-), \tag{13}$$

$$N = \deg(B^-) - 1, \tag{14}$$

$$m = \max(\deg(A^-) + 1, \deg(B^-)). \tag{15}$$

The nominal controller given by the poles M is equivalent to the predictive controller with no degrees of freedom given by the set of equality constraints

$$\bar{y}_{t+N+j} = \bar{w}_{t+N},\ j = 1, \ldots, m \tag{16}$$

$$\bar{u}_{t+N_u+j} = 0,\ j = 1, \ldots, N - N_u + m, \tag{17}$$

The internal sequence of control increments $\bar{u}$ is calculated from

$$\boldsymbol{G}_1\bar{\boldsymbol{u}} = \bar{\boldsymbol{w}}_1 - \bar{\boldsymbol{f}}_1, \tag{18}$$

where

$$\bar{\boldsymbol{y}}_1 = \boldsymbol{G}_1\bar{\boldsymbol{u}} + \bar{\boldsymbol{f}}_1, \tag{19}$$

$$\bar{\boldsymbol{u}}^T = (\bar{u}_t, \bar{u}_{t+1}, \ldots, \bar{u}_{t+N_u-1}), \tag{20}$$

$$\bar{\boldsymbol{y}}_1^T = (\bar{y}_{t+N}, \bar{y}_{t+N+1}, \ldots, \bar{y}_{t+N+m-1}), \tag{21}$$

$$\bar{\boldsymbol{f}}_1^T = (\bar{f}_{t+N}, \bar{f}_{t+N+1}, \ldots, \bar{f}_{t+N+m-1}), \tag{22}$$

$$\bar{\boldsymbol{w}}_1^T = (1, \ldots, 1)M(1)/B^+(1) \tag{23}$$

$$\boldsymbol{G}_1 = \begin{pmatrix} g_N & \cdots & g_{N-m+1} \\ \vdots & \ddots & \vdots \\ g_{N+m-1} & \cdots & g_N \end{pmatrix}. \tag{24}$$

The actual control increment $\tilde{u}(t)$ is calculated from (11) and applied to the controlled system in the receding horizon manner.

PROOF. (Fikar and Unbehauen, 1999) □

3.1 *Relation to other stabilising predictive strategies*

A comparison of stable predictive control methods can be performed for the same conditions. One useful way is to consider all methods without any degrees of freedom. The incorporation of the cost function and its minimisation changes only the properties of such nominal controller.

We have the following results:

Authors	G_1	G_2	G_3
(Clarke and Scattolini, 1991; Mosca and Zhang, 1992)	1	$\dfrac{B}{AF}$	1
(Rossiter and Kouvaritakis, 1993)	$\dfrac{1}{AF}$	1	B
(Rawlings and Muske, 1993)	1	$\dfrac{B}{FA^-}$	$\dfrac{1}{A^+}$
(Rossiter *et al.*, 1996)	$\dfrac{B^+}{FA^-}$	1	$\dfrac{B^-}{A^+}$
This approach	$\dfrac{M}{A^+}$	$\dfrac{B^-}{FA^-}$	$\dfrac{B^+}{M}$

The proposed controller can be turned into any other stable predictive controller given in the table by a suitable choice of the closed-loop poles M given in the next theorem.

Theorem 4. Consider the scheme in Fig. 2 where

$$G_1 = \frac{G_{1N}}{G_{1D}}, \quad G_2 = \frac{G_{2N}}{G_{2D}}, \quad G_3 = \frac{G_{3N}}{G_{3D}}. \quad (25)$$

Assume that $(G_{1N}, G_{3D}) = M_0$, thus

$$G_{1N} = M_0\bar{G}_{1N}, G_{3D} = M_0\bar{G}_{3D}. \quad (26)$$

Then any predictive controller with no degrees of freedom and constraints (16), (17) is equivalent to the pole placement controller with poles

$$M = M_0\bar{G}_{1N}\bar{G}_{3D}. \quad (27)$$

The predictive controller is stable if and only if M is stable.

PROOF. (Fikar and Unbehauen, 1999) $\square$

4. COST MINIMISATION

The natural way of obtaining the necessary degrees of freedom to minimise a cost function and to handle constraints is an assumption that the controller can act in more than N_u steps given by (13).

Let us define the following vectors and matrices

$$\bar{\boldsymbol{e}} = \bar{\boldsymbol{w}} - \bar{\boldsymbol{y}}, \quad (28)$$

$$\bar{\boldsymbol{y}}^T = (\bar{y}_{t+1}, \bar{y}_{t+2}, \dots, \bar{y}_{t+N-1}), \quad (29)$$

$$\bar{\boldsymbol{f}}^T = (\bar{f}_{t+1}, \bar{f}_{t+2}, \dots, \bar{f}_{t+N-1}), \quad (30)$$

$$\bar{\boldsymbol{w}}^T = (\bar{w}_{t+1}, \bar{w}_{t+2}, \dots, \bar{w}_{t+N-1}), \quad (31)$$

$$\boldsymbol{G} = \begin{pmatrix} g_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ g_{N-1} & \dots & g_1 \end{pmatrix}, \quad (32)$$

with

$$\bar{\boldsymbol{y}} = \boldsymbol{G}\bar{\boldsymbol{u}} + \bar{\boldsymbol{f}}. \quad (33)$$

Theorem 5. Consider the predictive controller described in the Theorem 3. Introduce the number of degrees of freedom $n > 0$ and define the horizons

$$\bar{N}_u = N_u + n, \ \bar{N} = N + n. \quad (34)$$

Let us minimise the following cost function with $\boldsymbol{W}_e > 0$ and/or $\boldsymbol{W}_u > 0$

$$J = \bar{\boldsymbol{e}}^T\boldsymbol{W}_e\bar{\boldsymbol{e}} + \bar{\boldsymbol{u}}^T\boldsymbol{W}_u\bar{\boldsymbol{u}} \quad (35)$$

subject to the equality constraints (16), (17) and possible inequality constraints

$$\boldsymbol{A}\bar{\boldsymbol{u}} \geq \boldsymbol{b}. \quad (36)$$

If the optimisation problem is feasible then the predictive controller is stabilising for any n.

PROOF. Provided the signals $\bar{u}, \bar{e}$ are polynomials, the cost function J forms the standard Lyapunov function for which stability proofs are available. $\square$

If $n > 0$ the future reference signal $w(t), \dots, w(t+n)$ is assumed to be known in advance. If this is not the case, one may simply set $w(t+k) = w(t)$.

For the actual implementation of the controller consider constrained optimisation problem with inequality constraints on $(\tilde{u}, e, \dots)$. At first, it is necessary to transform the real signals into the internal ones. Their relation is given in polynomial form in (11), (12). This can be rewritten to matrix notation as

$$\boldsymbol{T}_{M,e}\boldsymbol{e} = \boldsymbol{T}_{B^+}\bar{\boldsymbol{e}}, \quad \boldsymbol{T}_{M,u}\tilde{\boldsymbol{u}} = \boldsymbol{T}_{A^+}\bar{\boldsymbol{u}} \quad (37)$$

where $\boldsymbol{T}_X$ is a Toeplitz matrix containing the coefficients of the polynomial X. The dimensions are $\boldsymbol{T}_{M,e}[N+n+k \times N+n+k]$, $\boldsymbol{T}_{B^+}[N+n+k \times N+n]$, $\boldsymbol{T}_{M,u}[N_u+n+k \times N_u+n+k]$, and $\boldsymbol{T}_{A^+}[N_u+n+k \times N_u+n]$.

The resulting quadratic programming problem is defined as

$$\begin{aligned} \min_{\bar{\boldsymbol{u}}} J \ = \ & -2(\bar{\boldsymbol{w}} - \bar{\boldsymbol{f}})^T\boldsymbol{W}_e\boldsymbol{G}\bar{\boldsymbol{u}} \\ & + \bar{\boldsymbol{u}}^T\left(\boldsymbol{G}^T\boldsymbol{W}_e\boldsymbol{G} + \boldsymbol{W}_u\right)\bar{\boldsymbol{u}} \\ & \text{subject to} \\ & \boldsymbol{G}_1\bar{\boldsymbol{u}} = \bar{\boldsymbol{w}}_1 - \bar{\boldsymbol{f}}_1 \\ & \boldsymbol{A}\bar{\boldsymbol{u}} \geq \boldsymbol{b} \end{aligned} \quad (38)$$

4.1 *LQ cost*

Let us consider the following cost

$$J_1 = \sum_{i=1}^{\infty} e_{t+i}^2 + \lambda \sum_{i=1}^{\infty} \tilde{u}_{t+i-1}^2. \tag{39}$$

The cost function can be rewritten by substituting (11), (12) as

$$\begin{aligned} J_1 &= \langle e^* e \rangle + \lambda \langle \tilde{u}^* \tilde{u} \rangle \\ &= \langle \bar{e}^* \frac{B_+^* B_+}{M^* M} \bar{e} \rangle + \lambda \langle \bar{u}^* \frac{A_+^* A_+}{M^* M} \bar{u} \rangle. \end{aligned} \tag{40}$$

The polynomials $\bar{e}, \bar{u}$ can be transformed into constant column vectors by

$$\begin{aligned} \bar{e}(z^{-1}) &= (1 \; z^{-1} \; \dots \; z^{-(N+n)}) \bar{\boldsymbol{e}} \\ &= \boldsymbol{P}_{N+n}(z^{-1}) \bar{\boldsymbol{e}}, \end{aligned} \tag{41}$$

$$\bar{u}(z^{-1}) = \boldsymbol{P}_{N_u+n}(z^{-1}) \bar{\boldsymbol{u}}. \tag{42}$$

This gives the standard form of (35)

$$\begin{aligned} J_1 &= \bar{\boldsymbol{e}}^T \langle \frac{B_+^* B_+}{M^* M} \boldsymbol{P}_{N+n}^* \boldsymbol{P}_{N+n} \rangle \bar{\boldsymbol{e}} \\ &+ \bar{\boldsymbol{u}}^T \langle \lambda \frac{A_+^* A_+}{M^* M} \boldsymbol{P}_{N_u+n}^* \boldsymbol{P}_{N_u+n} \rangle \bar{\boldsymbol{u}}. \end{aligned} \tag{43}$$

For the calculation of the weighting matrices consider the following separation into causal and strictly uncausal terms (consider $\boldsymbol{W}_e$ only)

$$\frac{B_+^* B_+}{M^* M} \boldsymbol{P}_{N+n}^* \boldsymbol{P}_{N+n} = \frac{\boldsymbol{X}_e}{M} + \frac{\boldsymbol{Y}_e^*}{M^*}, \; \langle \boldsymbol{Y}_e^* \rangle = \boldsymbol{0}, \tag{44}$$

which can be rewritten as a bilateral Diophantine equation

$$M^* \boldsymbol{X}_e + M \boldsymbol{Y}_e^* = B_+^* B_+ \boldsymbol{P}_{N+n}^* \boldsymbol{P}_{N+n}, \; \langle \boldsymbol{Y}_e^* \rangle = \boldsymbol{0}. \tag{45}$$

Hence, the weight $\boldsymbol{W}_e$ (and analogously also for $\boldsymbol{W}_u$) is given as

$$\boldsymbol{W}_e = \langle \boldsymbol{X}_e \rangle. \tag{46}$$

Although the choice of the polynomial M can be arbitrary, the preferred approach is to calculate it as closed-loop poles minimising J_1. Hence

$$\lambda (AF)^* (AF) + B^* B = M^* M. \tag{47}$$

4.2 *Cost function for nominal controller*

Let us now consider the "constant" predictive controller, e.g. the task is to construct a controller with $n > 0$ such that in the unconstrained case its control actions are those of the nominal controller with $n = 0$. To make both controllers compatible, it is assumed that the future setpoint is constant.

To force the controller with $n > 0$ to be the same as with $n = 0$, the equality constraints $\bar{e}(t + N + i + 1) = 0$ have to be respected. However, this must be the property of the unconstrained optimum rather than strict requirement in the design. One possibility of the conversion of the equality constraints to an optimisation problem is

$$\bar{J} = \sum_{i=N+1}^{N+n} \bar{e}_{t+i}^2, \quad \Rightarrow \boldsymbol{W}_e = \begin{pmatrix} \boldsymbol{0}_N & \\ & \boldsymbol{I} \end{pmatrix}. \tag{48}$$

The weighting matrix is positive definite ($\boldsymbol{W}_e = \boldsymbol{I}$) and equivalently the corresponding predictive controller stable only if $N = 0$. According to Theorem 3 this applies to minimum phase systems.

To retain positive definiteness also for non-minimum phase systems consider at first the "cheap control" ISE cost

$$\bar{J} = \sum_{i=1}^{\infty} e^2(t+i). \tag{49}$$

It is well known that its minimisation corresponds to the closed-loop poles M_1 given by the spectral factorisation equation

$$B^* B = M_1^* M_1. \tag{50}$$

To counteract their effect, the following signal is created

$$\tilde{e} = \frac{B^+}{M_1} \bar{e} \tag{51}$$

and the cost minimised is

$$J_2 = \sum_{i=1}^{\infty} \tilde{e}^2(t+i). \tag{52}$$

Notice that $\tilde{e} = \bar{e}$ for minimum phase systems. The weighting matrix $\boldsymbol{W}_e$ is obtained from the Diophantine equation

$$\frac{B_+^* B_+}{M_1^* M_1} \boldsymbol{P}_{N+n}^* \boldsymbol{P}_{N+n} = \frac{\boldsymbol{X}_e}{M_1} + \frac{\boldsymbol{Y}_e^*}{M_1^*}, \; \langle \boldsymbol{Y}_e^* \rangle = \boldsymbol{0} \tag{53}$$

and

$$\boldsymbol{W}_e = \langle \boldsymbol{X}_e \rangle, \boldsymbol{W}_u = \boldsymbol{0}. \tag{54}$$

5. DISCUSSION

In this section we will show some of the properties of the proposed algorithm by means of simulations. Control of the following discrete system is considered

$$G = \frac{B^+ B^-}{A^+ A^-} = \frac{[(1 + 0.25z^{-1})][z^{-1}(1 + 4z^{-1})]}{\left[(1 + 0.2z^{-1})^2\right](1 + 5z^{-1})}. \tag{55}$$

The simulation example shows the comparison between known and unknown setpoint trajectory. The closed-loop poles were set to minimise the LQ cost with $\lambda = 0.5$. Figure 3 shows the results of the nominal LQ controller with $n = 0$ and predictive controller minimising LQ cost J_1 with $n = 8$ and with knowledge about the setpoint change at $t = 10$. Compare also the results in Fig. 4 where

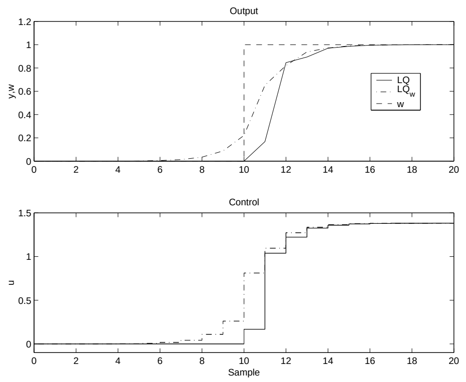

Fig. 3. LQ control with minimisation of J_1

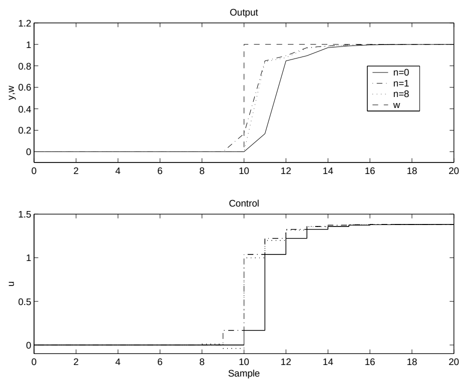

Fig. 4. LQ control with minimisation of J_2

the cost minimised was J_2. If the future setpoints are not known, any trajectory for $n > 0$ is the same as for $n = 0$ (solid line). If the future setpoint trajectory is known, the trajectories for $n > 0$ differ and are shown with dash-dotted line ($n = 1$). For $n > 1$ the trajectories are practically the same as for $n = 8$ (dotted line). Here, the effort of the optimisation is to improve the nominal LQ controller as if the setpoint trajectory was unknown. It depends on concrete conditions to choose the preferred approach.

6. CONCLUSIONS

This article discusses some new results in linear predictive control. At first, a general framework for stable predictive controllers was derived and it was shown that several predictive methods can be obtained for special choice of closed-loop poles. This general framework is based on the problem formulation with no degrees of freedom. The minimal horizons for stable predictive control were related to the number of unstable poles and zeros of the controlled system.

Next, it was discussed how to add degrees of freedom that may be used in the cost minimisation and constraint handling. This is implemented in the standard way by enlarging the number of available future control increments. As the number of optimised variables is finite, several possible cost function formulations were proposed. Here, we solved infinite LQ cost as well as the cost that forces the predictive controller to be equal to nominal pole placement controller in the unconstrained case, thus effectively provides a mechanism for transforming an existing controller into corresponding predictive controller with ability to handle the constraints. Such mechanism has a close relationship to anti-windup strategies.

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