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**Process Modelling, Identification,
and Control**
I

**Models and dynamic characteristics of continuous
processes**

Slovak University of Technology in Bratislava

This publication deals with mathematical modelling, dynamical process characteristics and properties. The intended audience of this book includes graduate students but can be of interest of practising engineers or applied scientists that are interested in modelling, identification, and process control.

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Preface

This publication is the first part of a book that deals with mathematical modelling of processes, their dynamical properties and dynamical characteristics. The need of investigation of dynamical characteristics of processes comes from their use in process control. The second part of the book will deal with process identification, optimal, and adaptive control.

The aim of this part is to demonstrate the development of mathematical models for process control. Detailed explanation is given to state-space and input-output process models.

In the chapter Dynamical properties of processes, process responses to the unit step, unit impulse, harmonic signal, and to a random signal are explored.

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Bratislava, March 2000

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Chapter 1

Introduction

This chapter serves as an introduction to process control. The aim is to show the necessity of process control and to emphasize its importance in industries and in design of modern technologies. Basic terms and problems of process control and modelling are explained on a simple example of heat exchanger control. Finally, a short history of development in process control is given.

1.1 Topics in Process Control

Continuous technologies consist of unit processes, that are rationally arranged and connected in such a way that the desired product is obtained effectively with certain inputs.

The most important technological requirement is safety. The technology must satisfy the desired quantity and quality of the final product, environmental claims, various technical and operational constraints, market requirements, etc. The operational conditions follow from minimum price and maximum profit.

Control system is the part of technology and in the framework of the whole technology which is a guarantee for satisfaction of the above given requirements. Control systems in the whole consist of technical devices and human factor. Control systems must satisfy

- disturbance attenuation,
- stability guarantee,
- optimal process operation.

Control is the purposeful influence on a controlled object (process) that ensures the fulfillment of the required objectives. In order to satisfy the safety and optimal operation of the technology and to meet product specifications, technical, and other constraints, tasks and problems of control must be divided into a hierarchy of subtasks and subproblems with control of unit processes at the lowest level.

The lowest control level may realise *continuous-time control* of some measured signals, for example to hold temperature at *constant value*. The second control level may perform static *optimisation* of the process so that optimal values of some signals (flows, temperatures) are calculated in certain time instants. These will be set and remain constant till the next optimisation instant. The optimisation may also be performed continuously. As the unit processes are connected, their operation is *coordinated* at the third level. The highest level is influenced by *market, resources*, etc.

The fundamental way of control on the lowest level is *feedback control*. Information about process output is used to calculate control (manipulated) signal, i.e. process output is fed back to process input.

There are several other methods of control, for example feed-forward. *Feed-forward control* is a kind of control where the effect of control is not compared with the desired result. In this case we speak about *open-loop control*. If the feedback exists, *closed-loop system* results.

Process design of “modern” technologies is crucial for successful control. The design must be developed in such a way, that a “sufficiently large number of degrees of freedom” exists for the purpose of control. The control system must have the ability to operate the whole technology or the unit process in the required technology regime. The processes should be “well” controllable and the control system should have “good” information about the process, i.e. the design phase of the process should include a selection of suitable *measurements*. The use of computers in the process control enables to choose optimal structure of the technology based on claims formulated in advance. Projectants of “modern” technologies should be able to include all aspects of control in the design phase.

Experience from control praxis of “modern” technologies confirms the importance of assumptions about dynamical behaviour of processes and more complex control systems. The control centre of every “modern” technology is a place, where all information about operation is collected and where the operators have contact with technology (through keyboards and monitors of control computers) and are able to correct and interfere with technology. A good knowledge of technology and process control is a necessary assumption of qualified human influence of technology through control computers in order to achieve optimal performance.

All of our further considerations will be based upon *mathematical models of processes*. These models can be constructed from a physical and chemical nature of processes or can be abstract. The investigation of dynamical properties of processes as well as whole control systems gives rise to a need to look for effective means of differential and difference equation solutions. We will carefully examine dynamical properties of open and closed-loop systems. A fundamental part of each procedure for effective control design is the process *identification* as the real systems and their physical and chemical parameters are usually not known perfectly. We will give procedures for design of *control algorithms* that ensure effective and safe operation.

One of the ways to secure a high quality process control is to apply *adaptive control laws*. Adaptive control is characterised by gaining information about unknown process and by using the information about on-line changes to process control laws.

1.2 An Example of Process Control

We will now demonstrate problems of process dynamics and control on a simple example. The aim is to show some basic principles and problems connected with process control.

1.2.1 Process

Let us assume a heat exchanger shown in Fig. 1.2.1. Inflow to the exchanger is a liquid with a flow rate q and temperature ϑ_v . The task is to heat this liquid to a higher temperature ϑ_w . We assume that the heat flow from the heat source is independent from the liquid temperature and only dependent from the heat input ω . We further assume ideal mixing of the heated liquid and no heat loss. The accumulation ability of the exchanger walls is zero, the exchanger holdup, input and output flow rates, liquid density, and specific heat capacity of the liquid are constant. The temperature on the outlet of the exchanger ϑ is equal to the temperature inside the exchanger. The exchanger that is correctly designed has the temperature ϑ equal to ϑ_w . *The process* of heat transfer realised in the heat exchanger is defined as our *controlled system*.

1.2.2 Steady-State

The inlet temperature ϑ_v and the heat input ω are *input variables* of the process. The outlet temperature ϑ is process *output variable*. It is quite clear that every change of input variables ϑ_v, ω results in a change of output variable ϑ . From this fact follows *direction of information*

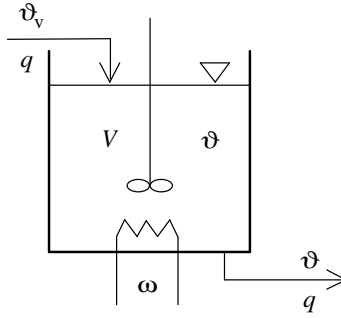


Figure 1.2.1: A simple heat exchanger.

transfer of the process. The process is in the *steady state* if the input and output variables remain constant in time t .

The heat balance in the steady state is of the form

$$q\rho c_p(\vartheta^s - \vartheta_v^s) = \omega^s \quad (1.2.1)$$

where

ϑ^s is the output liquid temperature in the steady state,

ϑ_v^s is the input liquid temperature in the steady state,

ω^s is the heat input in the steady state,

q is volume flow rate of the liquid,

ρ is liquid density,

c_p is specific heat capacity of the liquid.

ϑ_v^s is the desired input temperature. For the suitable exchanger design, the output temperature in the steady state ϑ^s should be equal to the desired temperature ϑ_w . So the following equation follows

$$q\rho c_p(\vartheta_w - \vartheta_v^s) = \omega^s. \quad (1.2.2)$$

It is clear, that if the input process variable ω^s is constant and if the process conditions change, the temperature ϑ would deviate from ϑ_w . The change of operational conditions means in our case the change in ϑ_v . The input temperature ϑ_v is then called *disturbance variable* and ϑ_w *setpoint variable*.

The heat exchanger should be designed in such a way that it can be possible to change the heat input so that the temperature ϑ would be equal to ϑ_w or be in its neighbourhood for all operational conditions of the process.

1.2.3 Process Control

Control of the heat transfer process in our case means to influence the process so that the output temperature ϑ will be kept close to ϑ_w . This influence is realised with changes in ω which is called *manipulated variable*. If there is a deviation ϑ from ϑ_w , it is necessary to adjust ω to achieve smaller deviation. This activity may be realised by a human operator and is based on the observation of the temperature ϑ . Therefore, a thermometer must be placed on the outlet of the exchanger. However, a human is not capable of high quality control. The task of the change of ω based on error between ϑ and ϑ_w can be realised automatically by some device. Such control method is called *automatic control*.

1.2.4 Dynamical Properties of the Process

In the case that the control is realised automatically then it is necessary to determine values of ω for each possible situation in advance. To make control decision in advance, the changes of ϑ as the result of changes in ω and ϑ_v must be known. The requirement of the knowledge about *process response* to changes of input variables is equivalent to knowledge about *dynamical properties of the process*, i.e. description of the process in *unsteady state*. The heat balance for the heat transfer process for a very short time Δt converging to zero is given by the equation

$$(q\rho c_p \vartheta_v dt + \omega dt) - (q\rho c_p \vartheta dt) = (V\rho c_p d\vartheta), \quad (1.2.3)$$

where V is the volume of the liquid in the exchanger. The equation (1.2.3) can be expressed in an abstract way as

$$(\text{inlet heat}) - (\text{outlet heat}) = (\text{heat accumulation})$$

The dynamical properties of the heat exchanger given in Fig. 1.2.1 are given by the differential equation

$$V\rho c_p \frac{d\vartheta}{dt} + q\rho c_p \vartheta = q\rho c_p \vartheta_v + \omega, \quad (1.2.4)$$

The heat balance in the steady state (1.2.1) may be derived from (1.2.4) in the case that $\frac{d\vartheta}{dt} = 0$. The use of (1.2.4) will be given later.

1.2.5 Feedback Process Control

As it was given above, process control may be realised either by human or automatically via *control device*. The control device performs the control actions practically in the same way as a human operator, but it is described exactly according to *control law*. The control device specified for the heat exchanger utilises information about the temperature ϑ and the desired temperature ϑ_w for the calculation of the heat input ω from formula formulated in advance. The difference between ϑ_w and ϑ is defined as *control error*. It is clear that we are trying to minimise the control error. The task is to determine the feedback control law to remove the control error optimally according to some criterion. The control law specifies the structure of the feedback controller as well as its properties if the structure is given.

The considerations above lead us to controller design that will change the heat input proportionally to the control error. This control law can be written as

$$\omega(t) = q\rho c_p(\vartheta_w - \vartheta_v^s) + Z_R(\vartheta_w - \vartheta(t)) \quad (1.2.5)$$

We speak about *proportional control* and *proportional controller*. Z_R is called the *proportional gain*. The proportional controller holds the heat input corresponding to the steady state as long as the temperature ϑ is equal to desired ϑ_w . The deviation between ϑ and ϑ_w results in nonzero control error and the controller changes the heat input proportionally to this error. If the control error has a plus sign, i.e. ϑ is greater as ϑ_w , the controller decreases heat input ω . In the opposite case, the heat input increases. This phenomenon is called *negative feedback*. The output signal of the process ϑ brings to the controller information about the process and is further transmitted via controller to the process input. Such kind of control is called *feedback control*. The quality of feedback control of the proportional controller may be influenced by the choice of controller gain Z_R . The equation (1.2.5) can be with the help of (1.2.2) written as

$$\omega(t) = \omega^s + Z_R(\vartheta_w - \vartheta(t)). \quad (1.2.6)$$

1.2.6 Transient Performance of Feedback Control

Putting the equation (1.2.6) into (1.2.4) we get

$$V\rho c_p \frac{d\vartheta}{dt} + (q\rho c_p + Z_R)\vartheta = q\rho c_p\vartheta_v + Z_R\vartheta_w + \omega^s. \quad (1.2.7)$$

This equation can be arranged as

$$\frac{V}{q} \frac{d\vartheta}{dt} + \frac{q\rho c_p + Z_R}{q\rho c_p} \vartheta = \vartheta_v + \frac{Z_R}{q\rho c_p} \vartheta_w + \frac{1}{q\rho c_p} \omega^s. \quad (1.2.8)$$

The variable $V/q = T_1$ has dimension of time and is called *time constant* of the heat exchanger. It is equal to time in which the exchanger is filled with liquid with flow rate q . We have assumed that the inlet temperature ϑ_v is a function of time t . For steady state ϑ_v^s is the input heat given as ω^s . We can determine the behaviour of $\vartheta(t)$ if ϑ_v, ϑ_w change. Let us assume that the process is controlled with feedback controller and is in the steady state given by values of $\vartheta_v^s, \omega^s, \vartheta^s$. In some time denoted by zero, we change the inlet temperature with the increment $\Delta\vartheta_v$. Idealised change of this temperature may be expressed mathematically as

$$\vartheta_v(t) = \begin{cases} \vartheta_v^s + \Delta\vartheta_v & t \geq 0 \\ \vartheta_v^s & t < 0 \end{cases} \quad (1.2.9)$$

To know the response of the process with the feedback proportional controller for the step change of the inlet temperature means to know the solution of the differential equation (1.2.8). The process is at $t = 0$ in the steady state and the initial condition is

$$\vartheta(0) = \vartheta_w. \quad (1.2.10)$$

The solution of (1.2.8) if (1.2.9), (1.2.10) are valid is given as

$$\vartheta(t) = \vartheta_w + \Delta\vartheta_v \frac{q\rho c_p}{q\rho c_p + Z_R} \left(1 - e^{-\frac{q\rho c_p + Z_R}{q\rho c_p} \frac{q}{V} t}\right) \quad (1.2.11)$$

The response of the heat transfer process controlled with the proportional controller for the step change of inlet temperature ϑ_v given by Eq. (1.2.9) is shown in Fig. 1.2.2 for several values of the controller gain Z_R . The investigation of the figure shows some important facts. The outlet temperature ϑ converges to some new steady state for $t \rightarrow \infty$. If the proportional controller is used, *steady state error* results. This means that there exists a difference between ϑ_w and ϑ at the time $t = \infty$. The steady state error is the largest if $Z_R = 0$. If the controller gain Z_R increases, steady state error decreases. If $Z_R = \infty$, then the steady state error is zero. Therefore our first intention would be to choose the largest possible Z_R . However, this would break some other closed-loop properties as will be shown later.

If the disturbance variable ϑ_v changes with time in the neighbourhood of its steady state value, the choice of large Z_R may cause large control deviations. However, it is in our interest that the control deviations are to be kept under some limits. Therefore, this kind of disturbance requires rather smaller values of controller gain Z_R and its choice is given as a compromise between these two requirements.

The situation may be improved if the controller consists of a proportional and integral part. Such a controller may remove the steady state error even with smaller gain.

It can be seen from (1.2.11) that $\vartheta(t)$ cannot grow beyond limits. We note however that the controlled system was described by the first order differential equation and was controlled with a proportional controller.

We can make the process model more realistic, for example, assuming the accumulation ability of its walls or dynamical properties of temperature measurement device. The model and the feedback control loop as well will then be described by a higher order differential equation. The solution of such a differential equation for similar conditions as in (1.2.11) can result in ϑ growing into infinity. This case represents *unstable response* of the closed loop system. The problem of stability is usually included into the general problem of *control quality*.

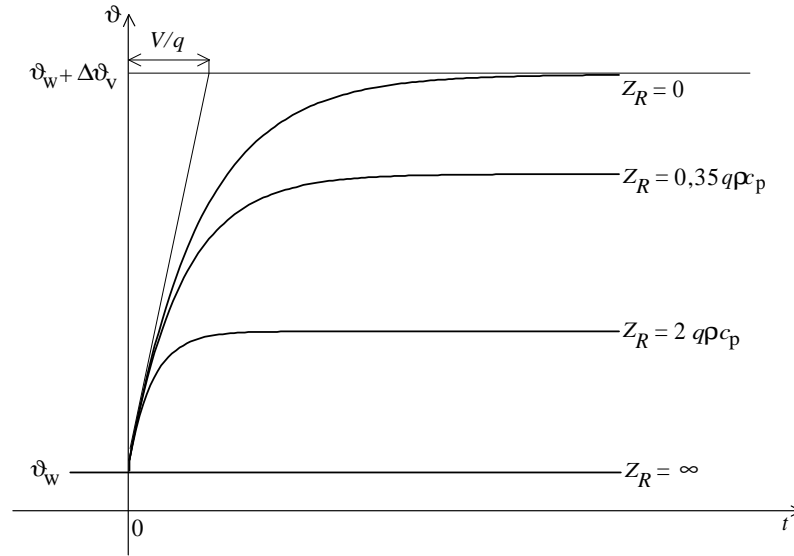


Figure 1.2.2: Response of the process controlled with proportional feedback controller for a step change of disturbance variable ϑ_v .

1.2.7 Block Diagram

In the previous sections the principal problems of feedback control were discussed. We have not dealt with technical issues of the feedback control implementation.

Consider again feedback control of the heat exchanger in Fig. 1.2.1. The necessary assumptions are i) to measure the outlet temperature ϑ and ii) the possibility of change of the heat input ω . We will assume that the heat input is realised by an electrical heater.

If the feedback control law is given then the feedback control of the heat exchanger may be realised as shown in Fig. 1.2.3. This scheme may be simplified for needs of analysis. Parts of the scheme will be depicted as blocks. The block scheme in Fig. 1.2.3 is shown in Fig. 1.2.4. The scheme gives physical interconnections and the information flow between the parts of the closed loop system. The signals represent physical variables as for example ϑ or instrumentation signals as for example m . Each block has its own input and output signal.

The outlet temperature is measured with a thermocouple. The thermocouple with its transmitter generates a voltage signal corresponding to the measured temperature. The dashed block represents the entire temperature controller and $m(t)$ is the input to the controller. The controller realises three activities:

1. the desired temperature ϑ_w is transformed into voltage signal m_w ,
2. the control error is calculated as the difference between m_w and $m(t)$,
3. the control signal m_u is calculated from the control law.

All three activities are realised within the controller. The controller output $m_u(t)$ in volts is the input to the electric heater producing the corresponding heat input $\omega(t)$. The properties of each block in Fig. 1.2.4 are described by algebraic or differential equations.

Block schemes are usually simplified for the purpose of the investigation of control loops. The *simplified block scheme* consists of 2 blocks: control block and controlled object. Each block of the detailed block scheme must be included into one of these two blocks. Usually the simplified control block realises the control law.

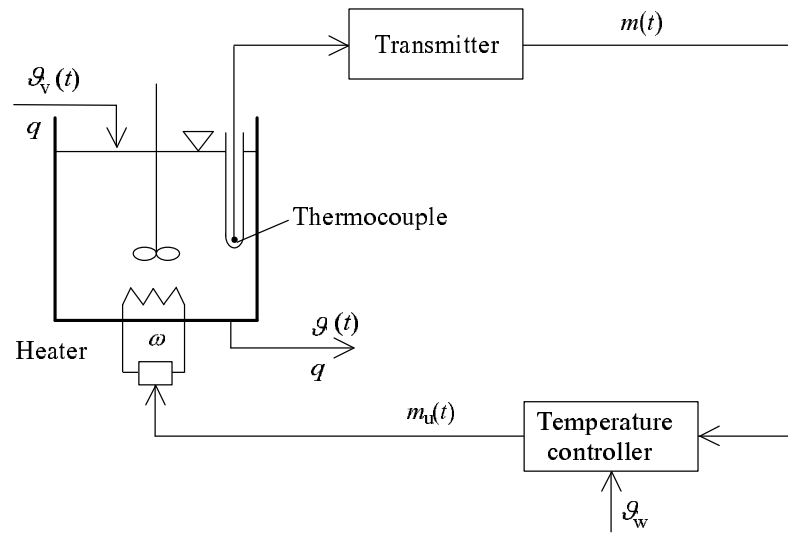


Figure 1.2.3: The scheme of the feedback control for the heat exchanger.

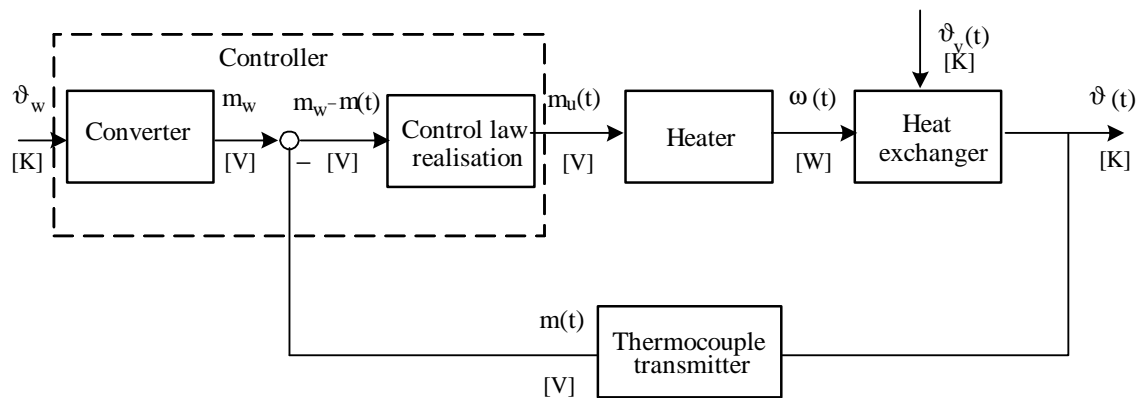


Figure 1.2.4: The block scheme of the feedback control of the heat exchanger.

1.2.8 Feedforward Control

We can also consider another kind of the heat exchanger control when the disturbance variable ϑ_v is measured and used for the calculation of the heat input ω . This is called *feedforward control*. The effect of control is not compared with the expected result. In some cases of process control it is necessary and/or suitable to use a combination of feedforward and feedback control.

1.3 Development of Process Control

The history of automatic control began about 1788. At that time J. Watt developed a revolution controller for the steam engine. An analytic expression of the influence between controller and controlled object was presented by Maxwell in 1868. Correct mathematical interpretation of automatic control is given in the works of Stodola in 1893 and 1894. E. Routh in 1877 and Hurwitz in 1895 published works in which stability of automatic control and stability criteria were dealt with. An important contribution to the stability theory was presented by Nyquist (1932). The works of Oppelt (1939) and other authors showed that automatic control was established as an independent scientific branch.

Rapid development of discrete time control began in the time after the second world war. In continuous time control, the theory of transformation was used. The transformation of sequences defined as Z -transform was introduced independently by Cypkin (1950), Ragazzini and Zadeh (1952).

A very important step in the development of automatic control was the state-space theory, first mentioned in the works of mathematicians as Bellman (1957) and Pontryagin (1962). An essential contribution to state-space methods belongs to Kalman (1960). He showed that the linear-quadratic control problem may be reduced to a solution of the Riccati equation. Parallel to the optimal control, the stochastic theory was being developed.

It was shown that automatic control problems have an algebraic character and the solutions were found by the use of polynomial methods (Rosenbrock, 1970).

In the fifties, the idea of *adaptive control* appeared in journals. The development of adaptive control was influenced by the theory of dual control (Feldbaum, 1965), parameter estimation (Eykhoff, 1974), and recursive algorithms for adaptive control (Cypkin, 1971).

The above given survey of development in automatic control also influenced development in process control. Before 1940, processes in the chemical industry and in industries with similar processes, were controlled practically only manually. If some controller were used, these were only very simple. The technologies were built with large tanks between processes in order to attenuate the influence of disturbances.

In the fifties, it was often uneconomical and sometimes also impossible to build technologies without automatic control as the capacities were larger and the demand of quality increased. The controllers used did not consider the complexity and dynamics of controlled processes.

In 1960-s the process control design began to take into considerations dynamical properties and bindings between processes. The process control used knowledge applied from astronautics and electrotechnics.

The seventies brought the demands on higher quality of control systems and integrated process and control design.

In the whole process control development, knowledge of processes and their *modelling* played an important role.

The development of process control was also influenced by the development of computers. The first ideas about the use of *digital computers* as a part of control system emerged in about 1950. However, computers were rather expensive and unreliable to use in process control. The first use was in *supervisory control*. The problem was to find the optimal operation conditions in the sense of static optimisation and the mathematical models of processes were developed to solve this task. In the sixties, the continuous control devices began to be replaced with digital equipment, the so called *direct digital process control*. The next step was an introduction of mini and microcomputers

in the seventies as these were very cheap and also small applications could be equipped with them. Nowadays, the computer control is decisive for quality and effectivity of all modern technology.

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