Explicit Stochastic MPC Approach to Building Temperature Control

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IAM

Motivation

40% of global energy use goes to HVAC

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Solution: Model Predictive Control

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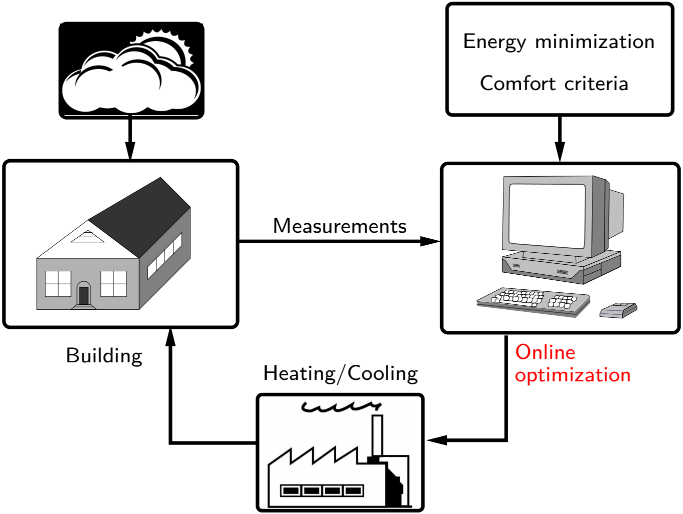
Building Temperature Control

Control objectives:

- Maintain thermal comfort in room
- Minimize cost of heating and cooling process
- Obey technological limitations

Building Temperature Control Scheme



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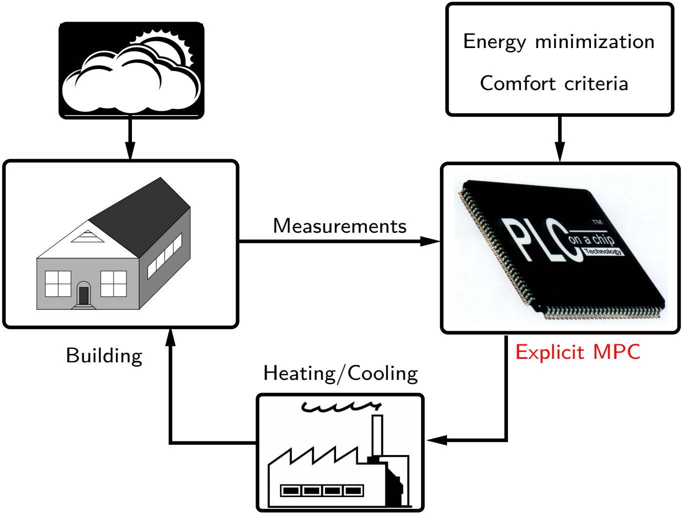


Process Description

State (Measured) Variables

x_1 – floor temperature
x_2 – internal facade temperature
x_3 – external facade temperature
x_4 – internal temperature

Controlled Variable

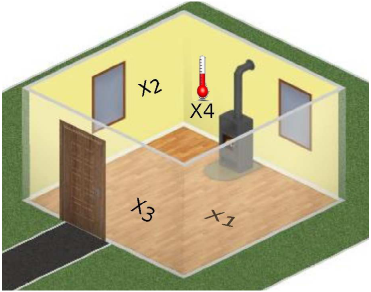
$y = x_4$

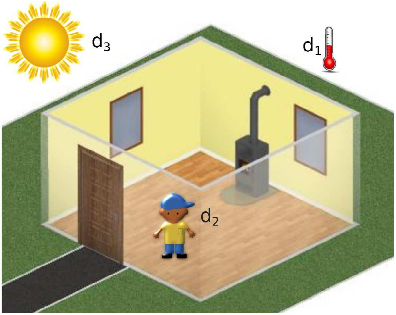
Measured Disturbances

d_1 – external temperature
d_2 – occupancy
d_3 – solar radiation

Manipulated Variable

u – heat flow





Building Model

$$x_{k+1} = Ax_k + Bu_k + Ed_k$$
$$y = Cx_k$$

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$$u_{\min} \leq u_k \leq u_{\max}$$

Stochastic Model Predictive Control

$$
\begin{aligned}
\min_{u_0,\dots,u_N} \quad & \sum_{k=0}^{N} u_k^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k \\
& Cx_k \geq r - \epsilon \\
& Cx_k \leq r + \epsilon \\
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& \delta \sim \mathcal{N}(0, \sigma(t))
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Probabilistic Constraints

$$\left.\begin{array}{r} x_{k+1} = Ax_k + Bu_k + E(d_0 + k\delta) \\ Cx_k \geq r - \epsilon \end{array}\right\} \rightarrow \; g(x, u, d_0, \delta) \leq 0$$

$$\Pr(g(x, u, d_0, \delta) \leq 0) \geq 1 - \alpha$$

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Campi M. and Garrati S., 2008

$$g(x, u, d_0, \delta^{(i)}) \leq 0, \quad i = 1, \ldots, M$$

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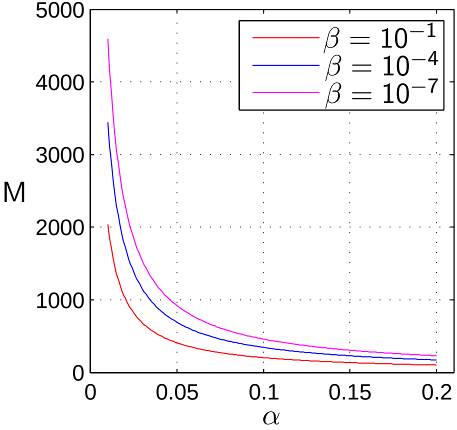
Alamo T., Tempo R. and Luque A., 2010

$$M \geq \frac{1 + (Nn_u) + \ln(1/\beta) + \sqrt{2((Nn_u) + 1)\ln(1/\beta)}}{\alpha}$$

Number of M Samples



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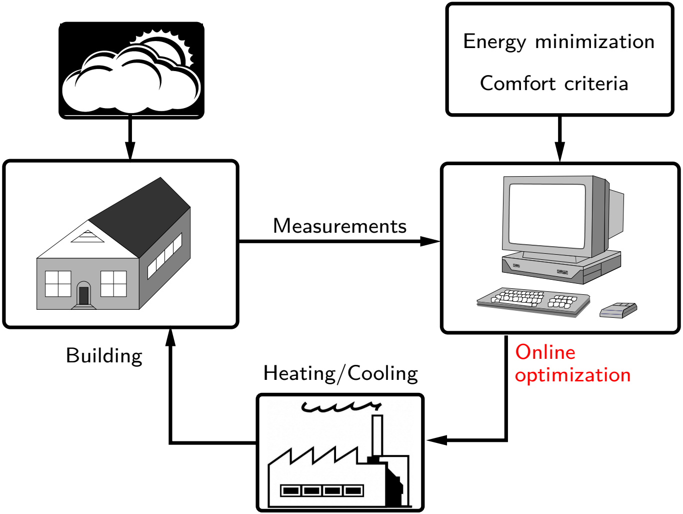

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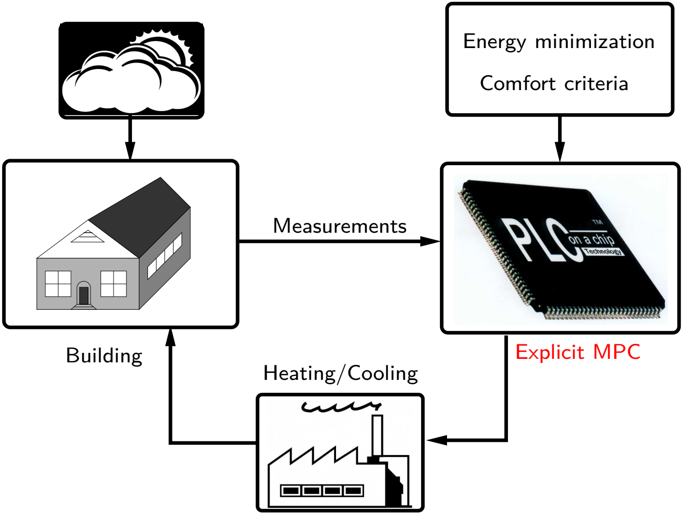


Explicit Stochastic Model Predictive Control

Obtained off-line

$$\min\{U^T HU + \xi^T FU \mid GU \leq W + S\xi\}$$

$$U^*(\xi) = \begin{cases} F_1\xi + g_1 & \text{if } \xi \in \mathcal{R}_1, \\ & \vdots \\ F_R\xi + g_R & \text{if } \xi \in \mathcal{R}_R, \end{cases}$$

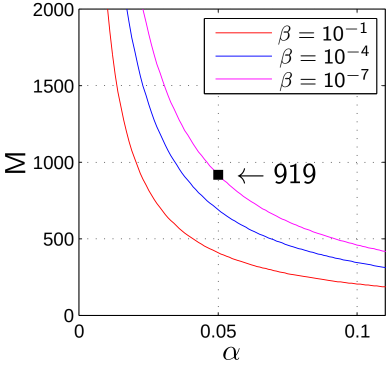
$$U = [u_0, \ldots, u_N]^T$$
$$\xi = [x(t), d(t), r(t), \delta^{(1)}, \ldots, \delta^{(M)}]^T$$

Building Stochastic MPC

$\alpha = 0.05$

$\beta = 10^{-7}$

$N = 10$



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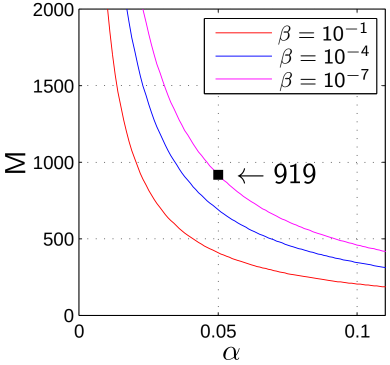

Resulting Parametric QP has:

927 parametric variables
18400 constraints

Number of Parameter and Constraints Reduction

$$\left.\begin{array}{c} a_1^T x + b_1 \leq 0 \\ \vdots \\ a_n^T x + b_n \leq 0 \end{array}\right\} \quad \rightarrow \quad \max_x \left(a_i^T x + b \right) \leq 0, \quad i = 1, \ldots, n$$

Number of Parameter and Constraints Reduction

$$x_{k+1} = Ax_k + Bu_k + E(d_0 + k\delta)$$
$$Cx_k \leq r + \epsilon$$

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$$C(Ax_k + Bu_k + Ed_0) + k \max_i \{CE\delta^{(i)}\} \leq r + \epsilon$$

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$$C(Ax_k + Bu_k + Ed_0) + k \max_i \{CE\delta^{(i)}\} \leq r + \epsilon$$

$$C(Ax_k + Bu_k + Ed_0) + k \min_i \{CE\delta^{(i)}\} \geq r - \epsilon$$

Building Stochastic MPC

$$\overline{\delta} = \text{arg max}_{\delta^{(i)}}\{CE\delta^{(i)}\}$$

$$\underline{\delta} = \text{arg min}_{\delta^{(i)}}\{CE\delta^{(i)}\}$$

Building Stochastic MPC

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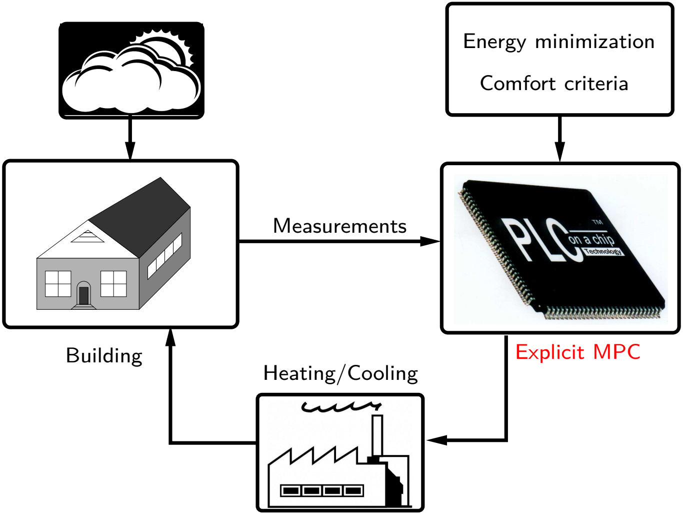
Resulting Parametric QP has:

14 parametric variables
60 constraints

Building Stochastic MPC



Building Stochastic MPC

Number of parameters: 14
Number of constraints: 60

Number of regions: 816
Time to compute: $\approx$ 6min

Control Algorithm

At each sample time T_s

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1. Measure $x(t)$, $d(t)$, $r(t)$ and obtain $\sigma(t)$

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3. Pick $\underline{\delta}$ and $\overline{\delta}$
4. Set $\xi = [x(t), d(t), r(t), \underline{\delta}, \overline{\delta}]$ and identify $\mathcal{R}_i$

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At each sample time T_s

1. Measure $x(t)$, $d(t)$, $r(t)$ and obtain $\sigma(t)$
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3. Pick $\underline{\delta}$ and $\overline{\delta}$
4. Set $\xi = [x(t), d(t), r(t), \underline{\delta}, \overline{\delta}]$ and identify $\mathcal{R}_i$
5. $u^*(t) = \tilde{F}_{i^*}\xi + \tilde{g}_{i^*}$

Simulation Scenarios

1. **Best Case** - full disturbance profile is available

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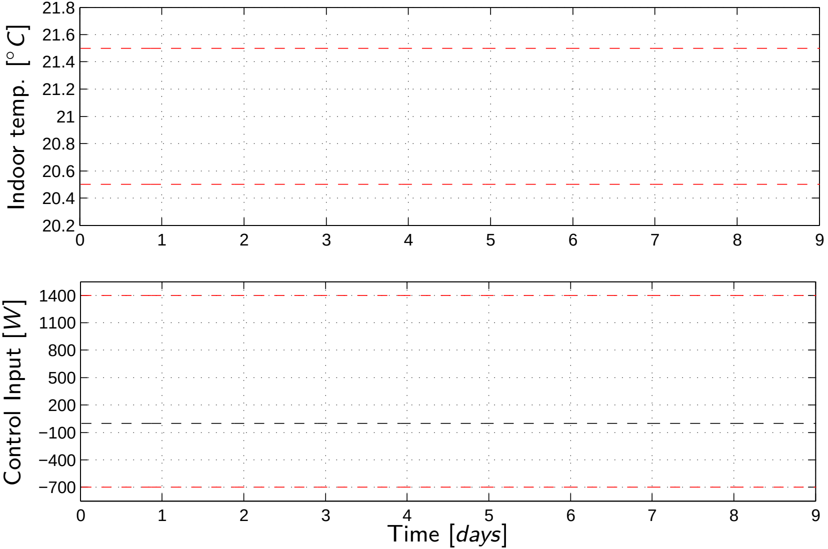
1. **Best Case** - full disturbance profile is available
2. **Worst Case** - worst possible disturbance is considered
3. **Stochastic Case** - Stochastic MPC is considered

Simulation results



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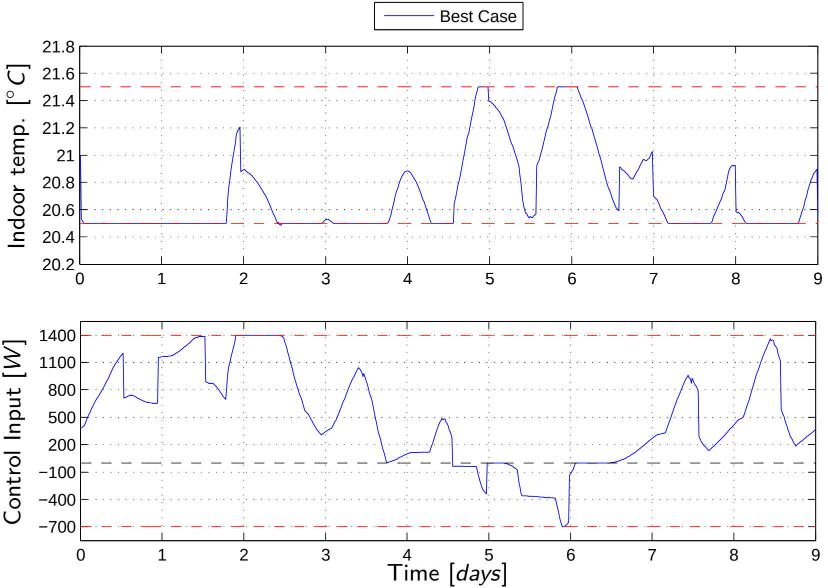

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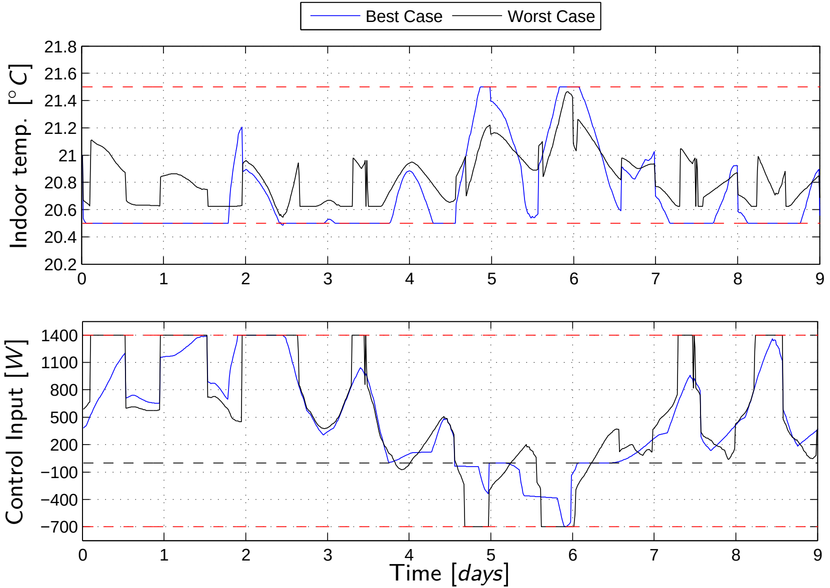

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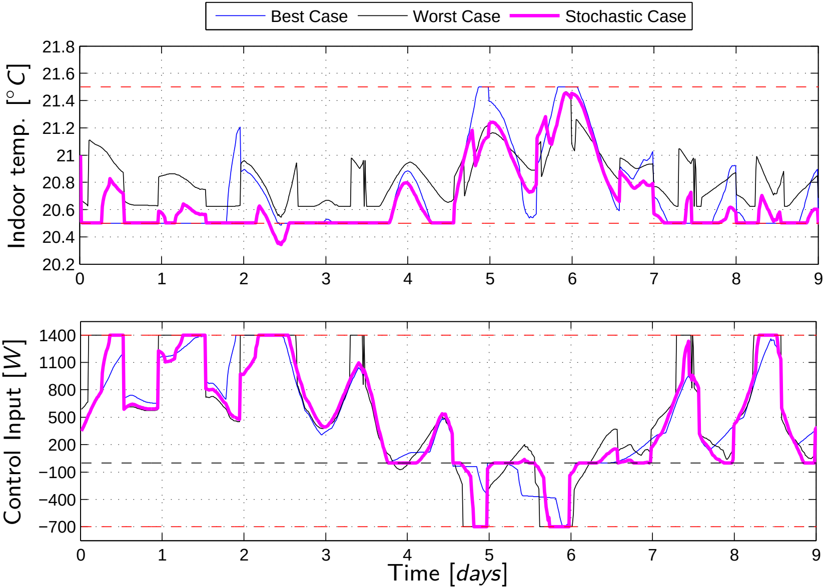
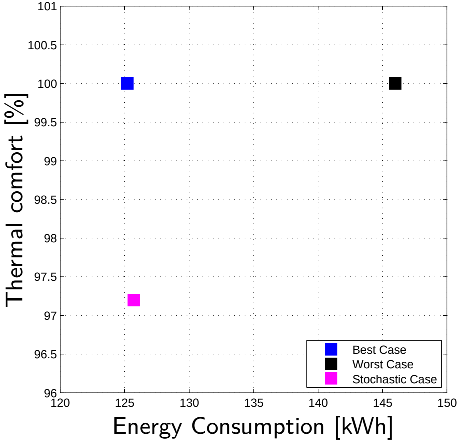


Comparison

	Thermal comfort [%]	Consumed energy [kWh]
	100.0	125.2
	100.0	146.0
	97.2	125.7



Conclusions

- Explicit formulation of MPC for building temperature control
- Implementation of probabilistic constraints
- Parameter and constraints reduction