Explicit MPC Approach to PMV-Based Thermal Comfort Control

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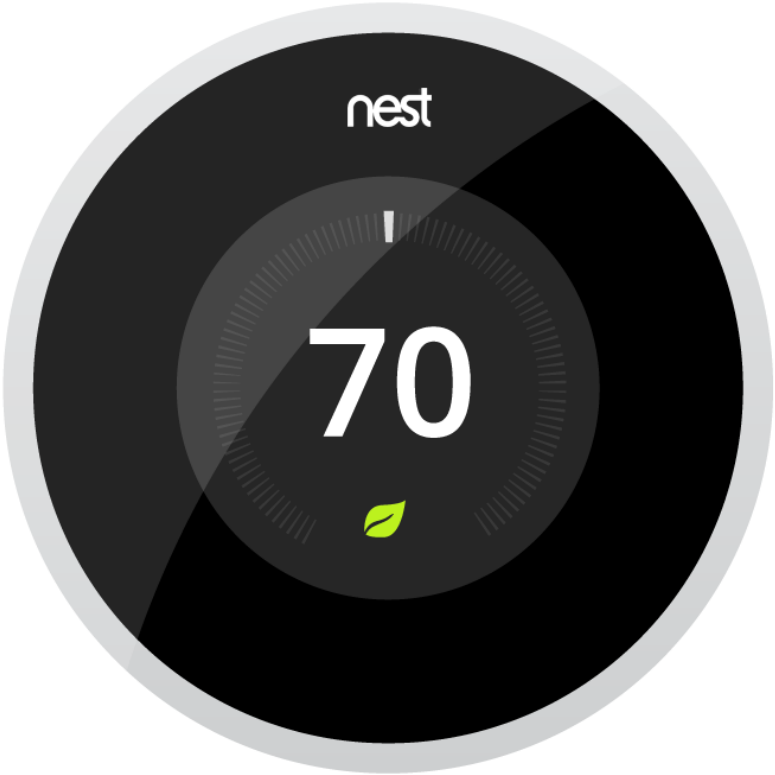
Motivation

www.emersonclimate.com

Motivation



www.digitaltrends.com, GOOGLE

Motivation

Indoor Temperature

Motivation

Indoor Temperature

Humidity

Motivation

Indoor Temperature

Humidity

Air Speed

Motivation

Indoor Temperature

Humidity

Air Speed

Clothing

Motivation

Indoor Temperature

Humidity

Metabolic Rate

Air Speed

Clothing

Motivation

Indoor Temperature

Radiant Temperature

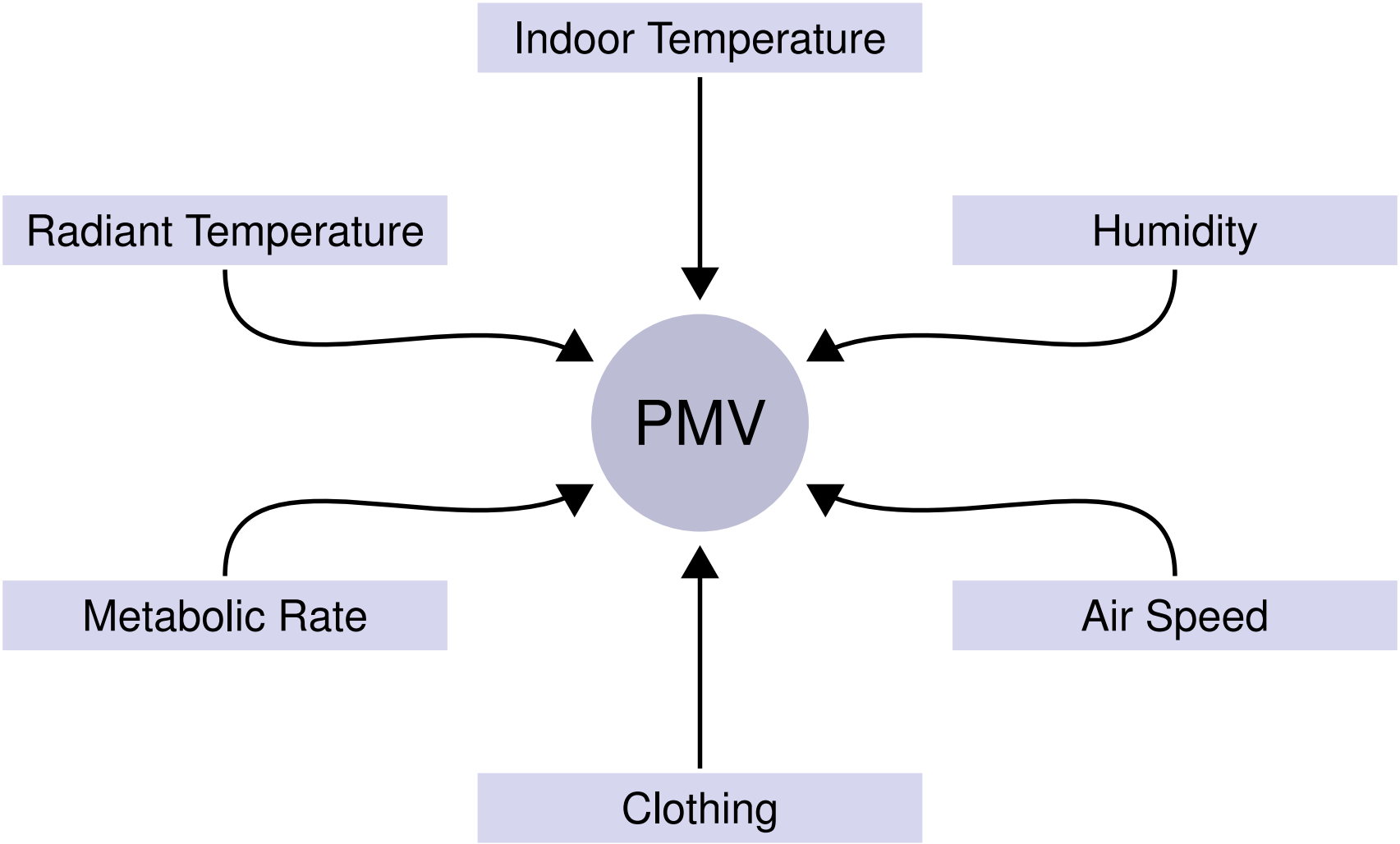
Humidity

Metabolic Rate

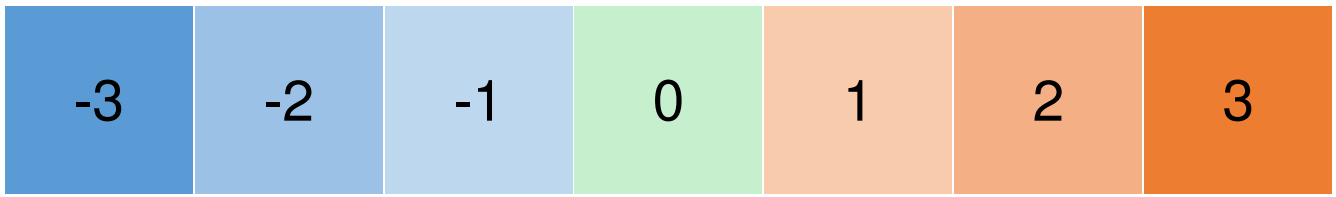
Air Speed

Clothing

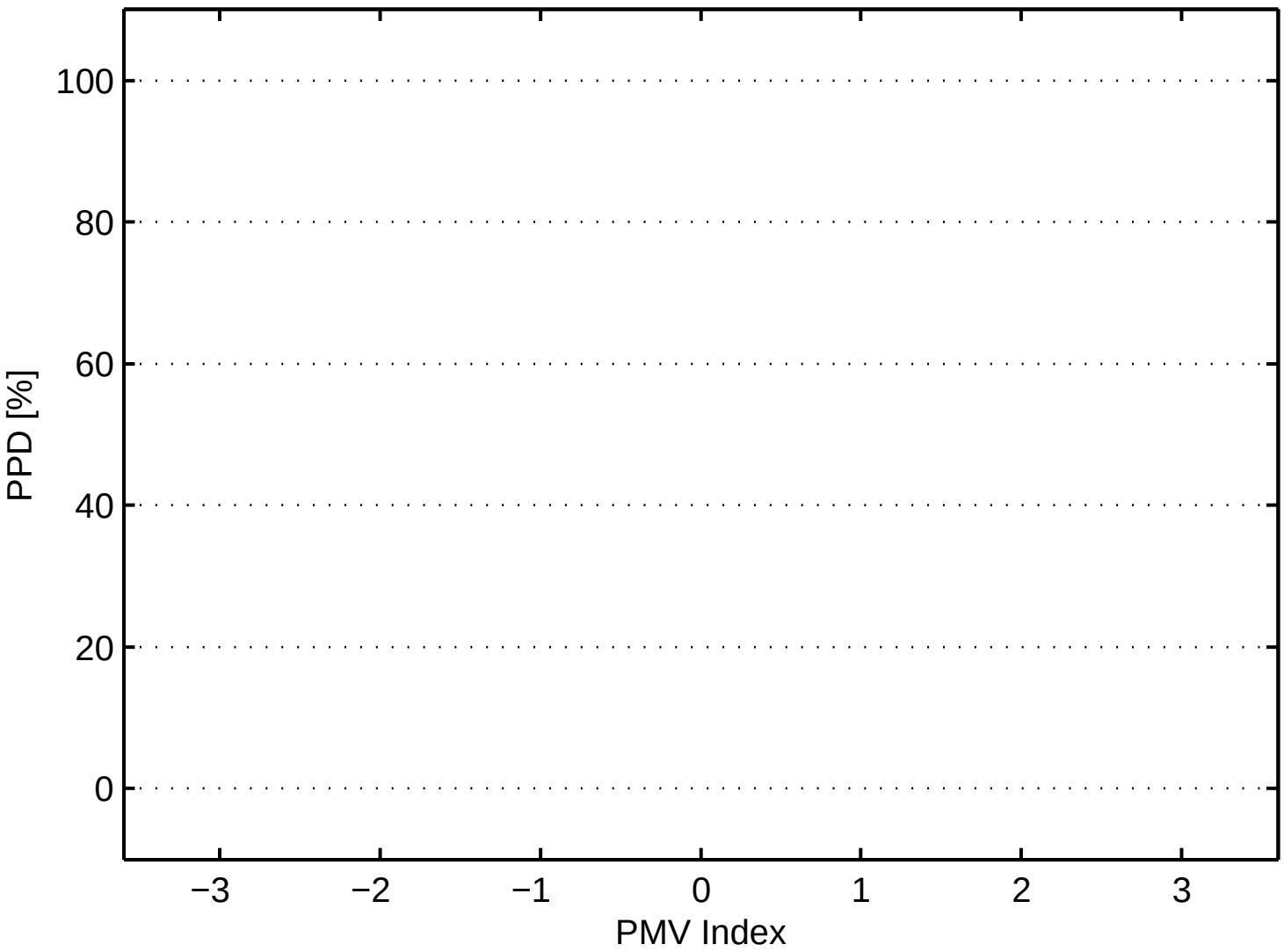
Thermal Comfort - PMV Index



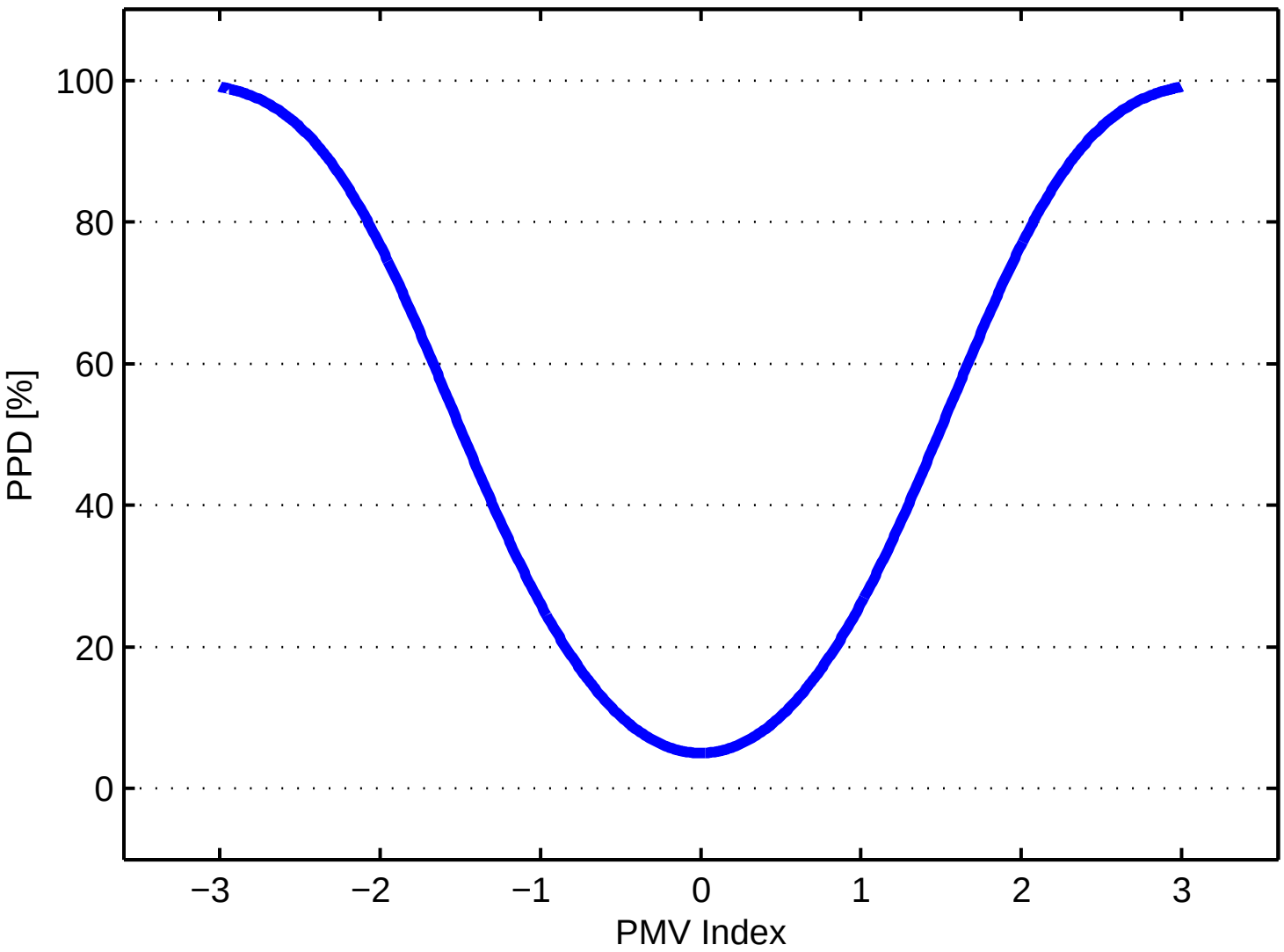
Thermal Comfort - PMV Index



Thermal Comfort - PMV Index



Thermal Comfort - PMV Index



Control Objective

Maintain PMV index within -0.2 to 0.2*

* EN ISO 7730:2006 Ergonomics of Thermal Environment

Problem Statement

Energy-Efficient Thermal Comfort Control

Problem Statement

Energy-Efficient Thermal Comfort Control

Implemented on Thermostat-Like Hardware

Problem Statement

Explicit Model Predictive Control

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$

$$u_{\min} \leq u_k \leq u_{\max}$$

$$p_k = \mathrm{PMV}(x_k)$$

$$p_{\min} \leq p_k \leq p_{\max}$$

$$x_0 = x(t),\ d_0 = d(t)$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2$$

energy consumption minimization

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$

$$u_{\min} \leq u_k \leq u_{\max}$$

$$p_k = \mathrm{PMV}(x_k)$$

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MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2$$ ← energy consumption minimization

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$ ← building model

$$u_{\min} \leq u_k \leq u_{\max}$$

$$p_k = \mathrm{PMV}(x_k)$$

$$p_{\min} \leq p_k \leq p_{\max}$$

$$x_0 = x(t), \; d_0 = d(t)$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2$$ ← energy consumption minimization

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$ ← building model

$$u_{\min} \leq u_k \leq u_{\max}$$ ← limited control authority

$$p_k = \mathrm{PMV}(x_k)$$

$$p_{\min} \leq p_k \leq p_{\max}$$

$$x_0 = x(t), \; d_0 = d(t)$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2$$ ← energy consumption minimization

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$ ← building model

$$u_{\min} \leq u_k \leq u_{\max}$$ ← limited control authority

$$p_k = \text{PMV}(x_k)$$ ← PMV index

$$p_{\min} \leq p_k \leq p_{\max}$$

$$x_0 = x(t), \; d_0 = d(t)$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2$$ ← energy consumption minimization

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$ ← building model

$$u_{\min} \leq u_k \leq u_{\max}$$ ← limited control authority

$$p_k = \text{PMV}(x_k)$$ ← PMV index

$$p_{\min} \leq p_k \leq p_{\max}$$ ← thermal comfort zone

$$x_0 = x(t), \; d_0 = d(t)$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \quad \longleftarrow \text{energy consumption minimization}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0 \quad \longleftarrow \text{building model}$$

$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \text{limited control authority}$$

$$p_k = \mathrm{PMV}(x_k) \quad \longleftarrow \text{PMV index}$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \text{thermal comfort zone}$$

$$x_0 = x(t), \; d_0 = d(t) \quad \longleftarrow \text{measurements}$$

MPC with PMV Index

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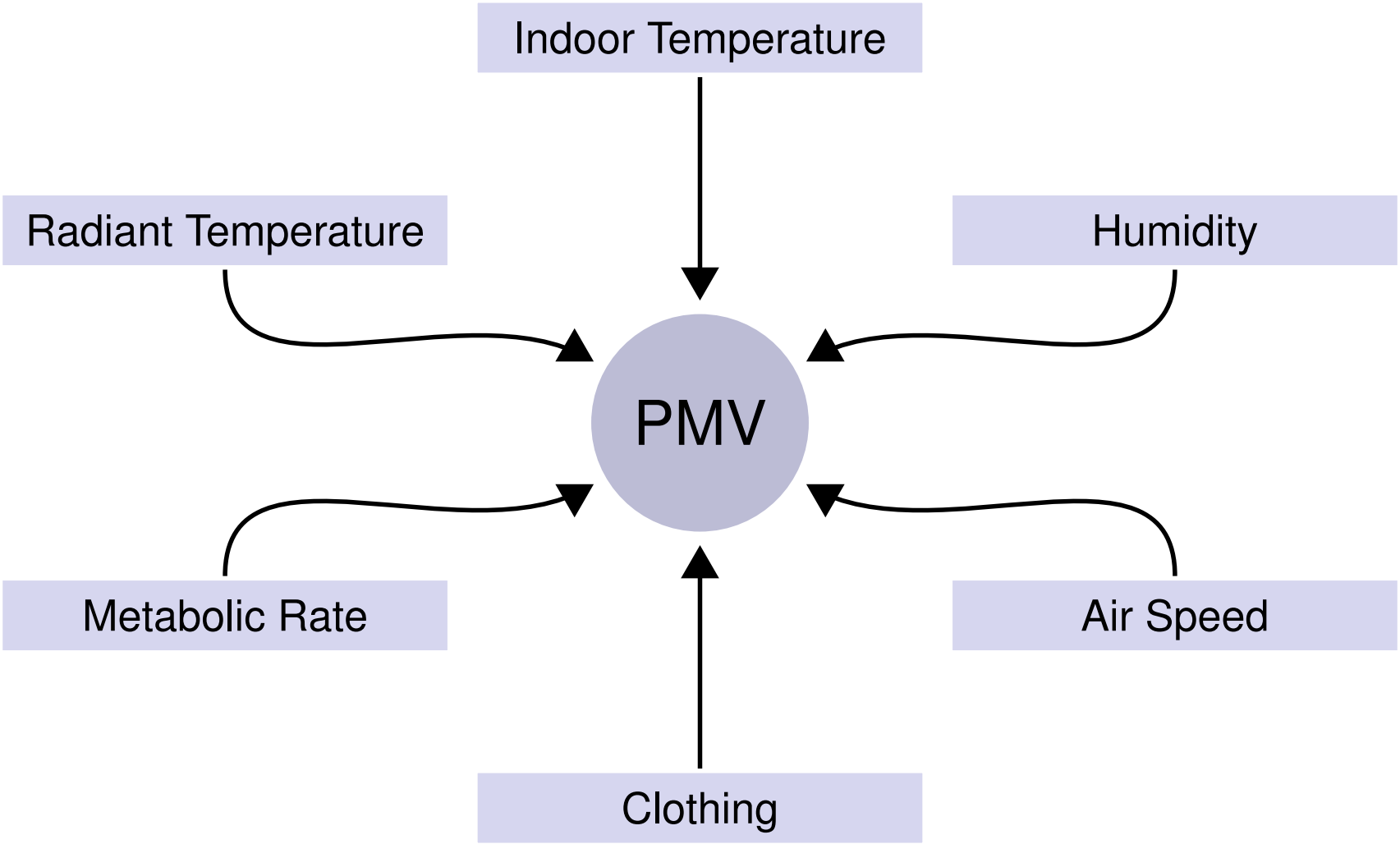
MPC with PMV Index



MPC with PMV Index

$$\mathrm{PMV} = \left(0.303e^{-0.036M} + 0.028\right) \cdot L$$

$$\begin{aligned} L =& (M - W) - 3.05 \cdot 10^{-3}\left(5733 - 6.99(M - W) - p_a\right) - \\ & - 0.42\left((M - W) - 58.15\right) - 1.7 \cdot 10^{-5} M(5867 - p_a) - \\ & - 0.0014M(34 - T_{in}) - 3.96 \cdot 10^{-8} f_{cl}\left(K_{tcl} - K_{tr}\right) - \\ & - f_{cl} h_c (T_{cl} - T_{in}) \end{aligned}$$

$$\begin{aligned} T_{cl} =& - 0.028(M - W) - I_{cl}\Big(3.96 \cdot 10^{-8}\left(f_{cl} K_{tcl} - K_{tr}\right) + \\ & + f_{cl} h_c (T_{cl} - T_{in})\Big) + 35.7 \end{aligned}$$

$$K_{tcl} = (T_{cl} + 273.16)^4$$

$$K_{tr} = (T_r + 273.16)^4$$

MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \text{energy consumption minimization}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0 \quad \longleftarrow \text{building model}$$

$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \text{limited control authority}$$

$$p_k = \text{PMV}(x_k) \quad \longleftarrow \text{PMV index}$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \text{thermal comfort zone}$$

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MPC with PMV Index

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \text{energy consumption minimization}$$

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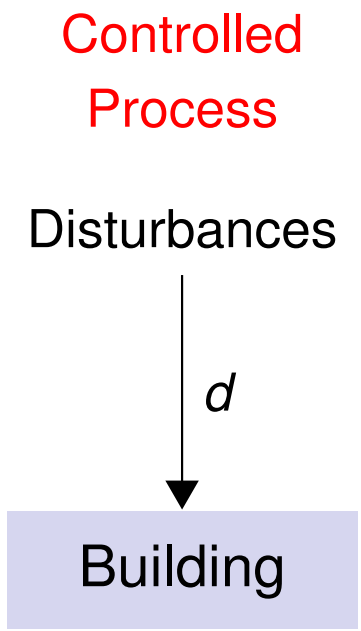
$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \text{limited control authority}$$

$$p_k = a(x_0)^T x_k + b(x_0) \quad \longleftarrow \text{PMV index}$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \text{thermal comfort zone}$$

$$x_0 = x(t), \; d_0 = d(t) \quad \longleftarrow \text{measurements}$$

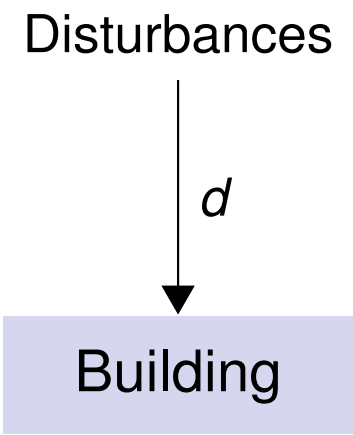
Closed-Loop System



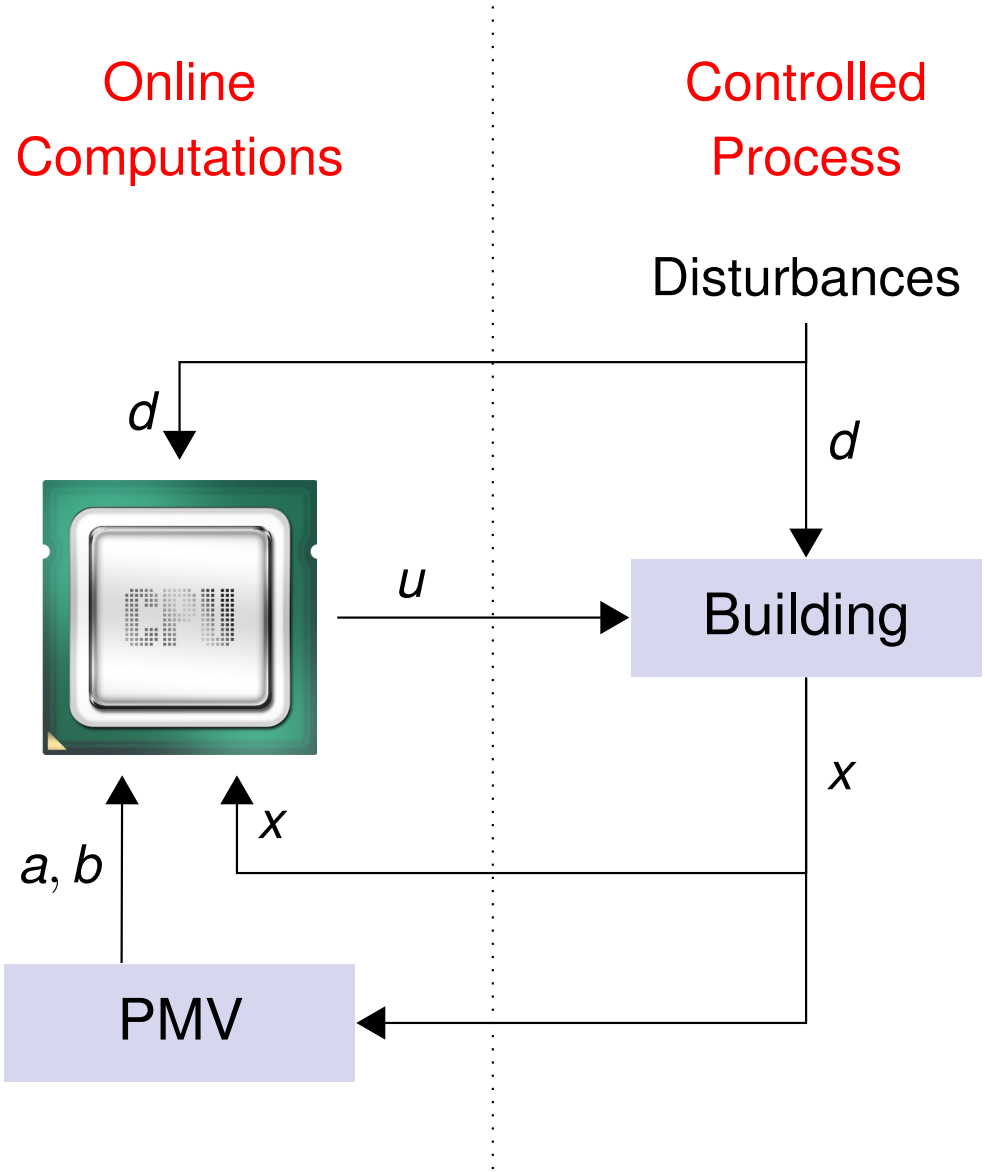
Closed-Loop System

Online Computations

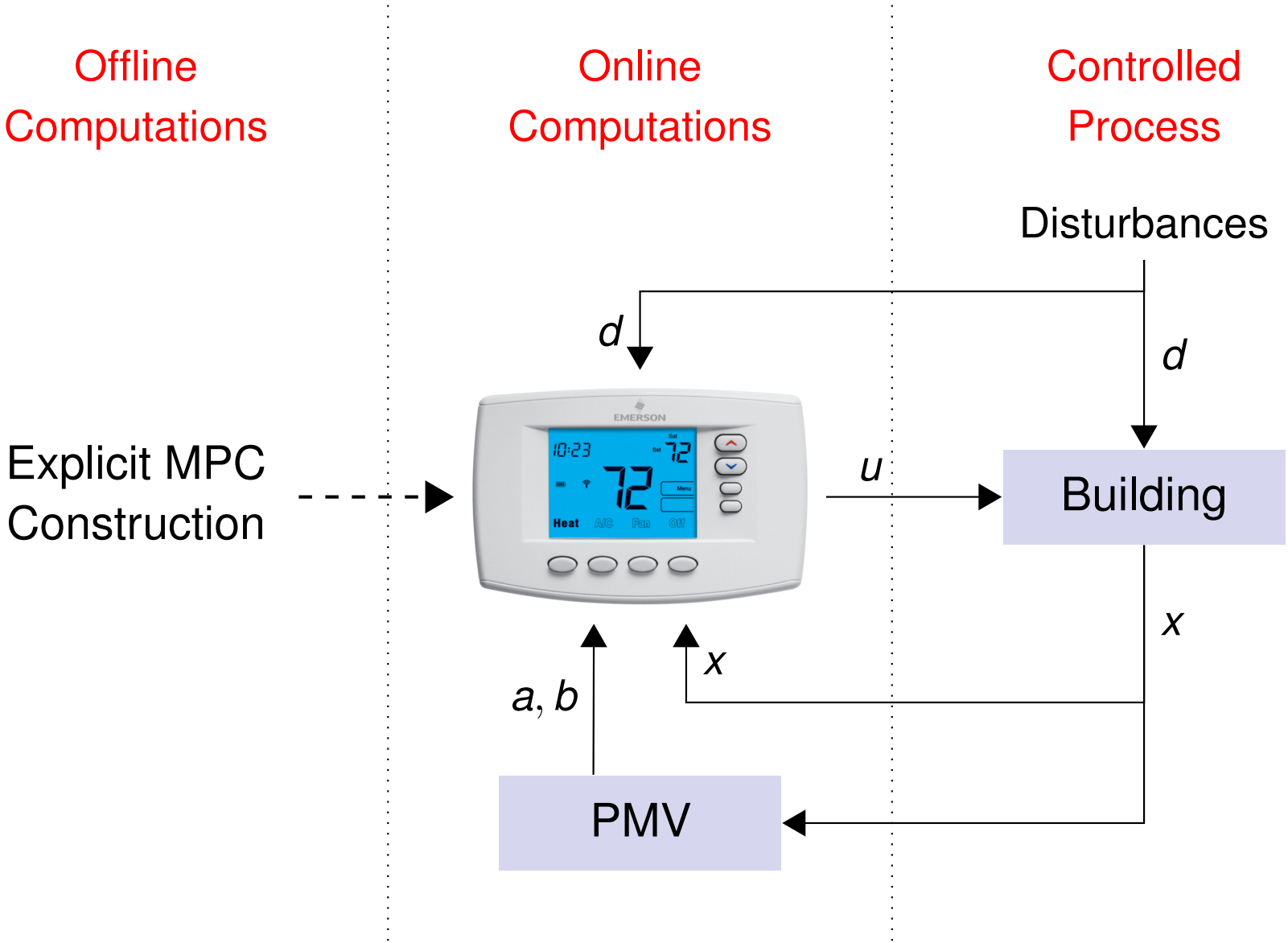
Controlled Process



Closed-Loop System



Closed-Loop System



MPC Implementation

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& p_k = a(x_0)^T x_k + b(x_0) \\
& p_{\min} \leq p_k \leq p_{\max} \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Explicit MPC

$$
u^\star(\theta) = \begin{cases} F_1\theta + g_1 & \text{if } \theta \in \mathcal{R}_1 \\ \vdots & \\ F_L\theta + g_L & \text{if } \theta \in \mathcal{R}_M \end{cases}
$$

MPC Implementation

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
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& x_0 = x(t), \; d_0 = d(t)
\end{aligned}
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Explicit MPC

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MPC Implementation

Online MPC

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\begin{aligned}
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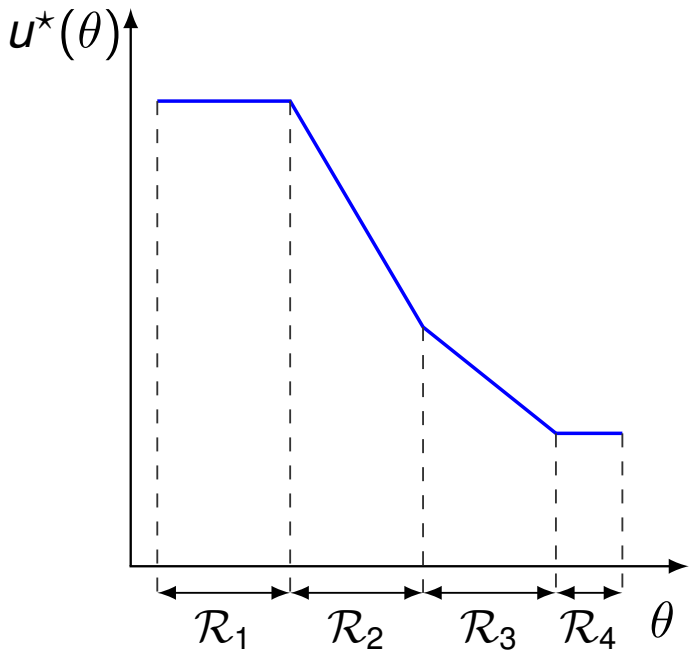
Explicit MPC



MPC Implementation

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \quad \text{convex quadratic} \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& p_k = a(x_0)^T x_k + b(x_0) \\
& p_{\min} \leq p_k \leq p_{\max} \\
& x_0 = x(t), \; d_0 = d(t)
\end{aligned}
$$

MPC Implementation

Online MPC

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \quad \text{convex quadratic}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0 \quad \longleftarrow \quad \text{linear}$$

$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \quad \text{linear}$$

$$p_k = a(x_0)^T x_k + b(x_0)$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \quad \text{linear}$$

$$x_0 = x(t), \; d_0 = d(t)$$

MPC Implementation

Online MPC

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \quad \text{convex quadratic}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0 \quad \longleftarrow \quad \text{linear}$$

$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \quad \text{linear}$$

$$p_k = a(x_0)^T (Ax_{k-1} + Bu_{k-1}) + b(x_0) \quad \longleftarrow \quad \text{non-linear}$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \quad \text{linear}$$

$$x_0 = x(t), \; d_0 = d(t)$$

MPC Implementation

Online MPC

$$\min \quad \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 \quad \longleftarrow \quad \text{convex quadratic}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0 \quad \longleftarrow \quad \text{linear}$$

$$u_{\min} \leq u_k \leq u_{\max} \quad \longleftarrow \quad \text{linear}$$

$$p_k = \ell_1^T a(x_0) + \ell_2^T x_{k-1} + \ell_3^T u_{k-1} + \ell_4 b(x_0) \quad \longleftarrow \quad \text{linear}$$

$$p_{\min} \leq p_k \leq p_{\max} \quad \longleftarrow \quad \text{linear}$$

$$x_0 = x(t), \; d_0 = d(t)$$

Control algorithm

Offline:

1. Construct explicit MPC - 279 regions (1min to construct)

Online:

1. Measure the current states and disturbances
2. Obtain linearisation coefficients for the PMV index
3. Evaluate the explicit control law
4. Apply the control action

Process Description

State (Measured) Variables

x_1 – floor temperature
x_2 – internal facade temperature
x_3 – external facade temperature
x_4 – internal temperature

Measured Disturbances

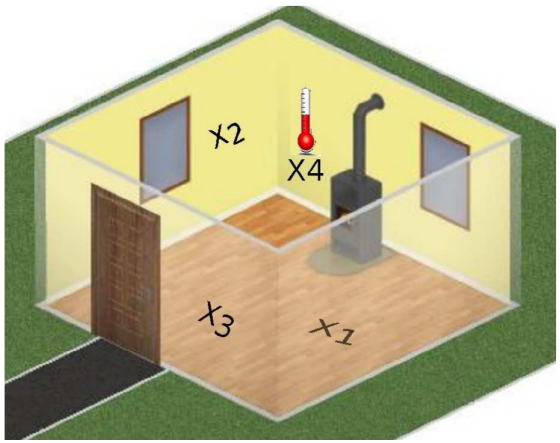
d_1 – external temperature
d_2 – occupancy
d_3 – solar radiation

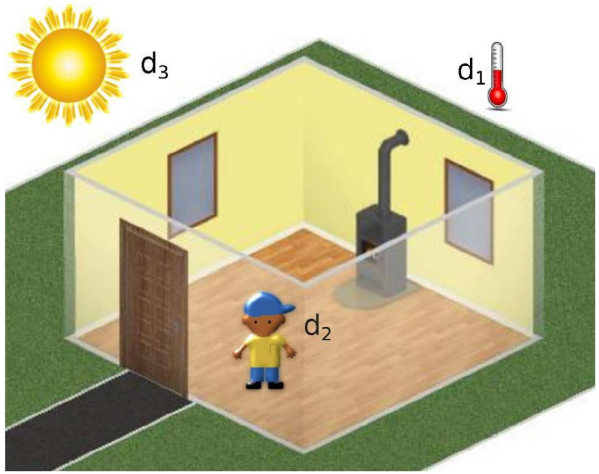
Controlled Variable

$y = x_4$

Manipulated Variable

u – heat flow

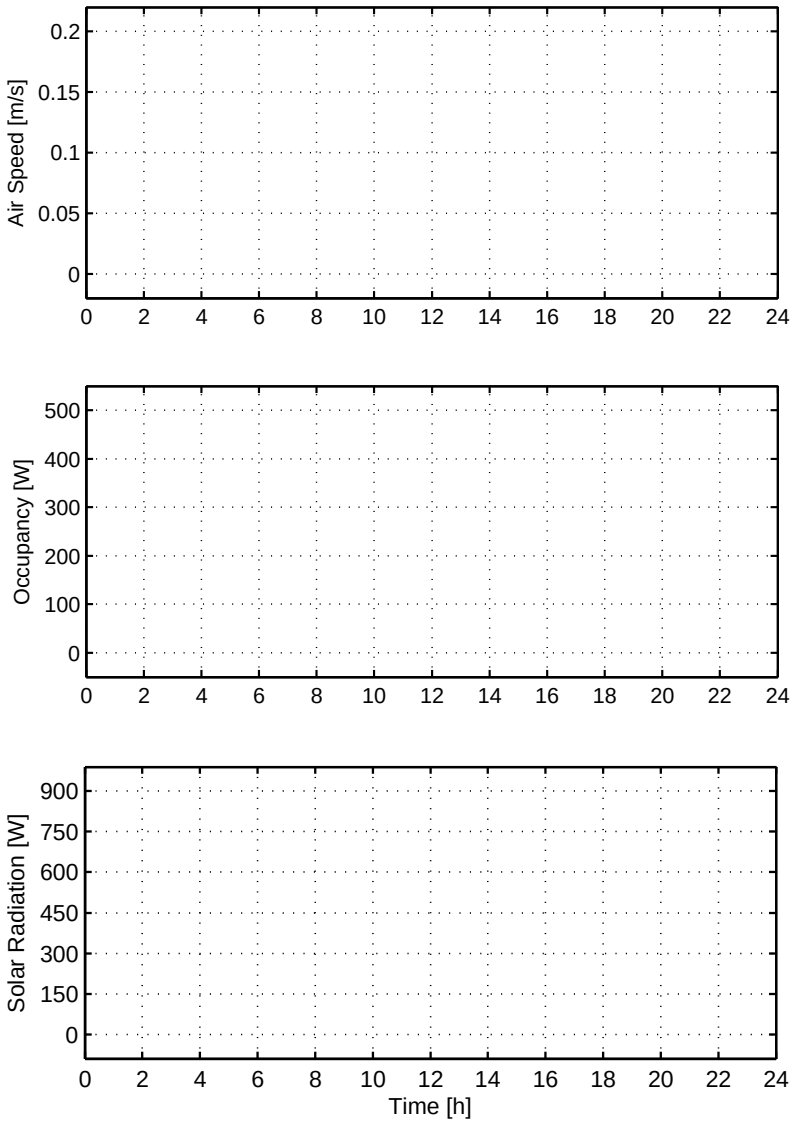




Simulation Test Scenarios

1. Temperature based control
2. PMV based control

Simulation Test Scenarios



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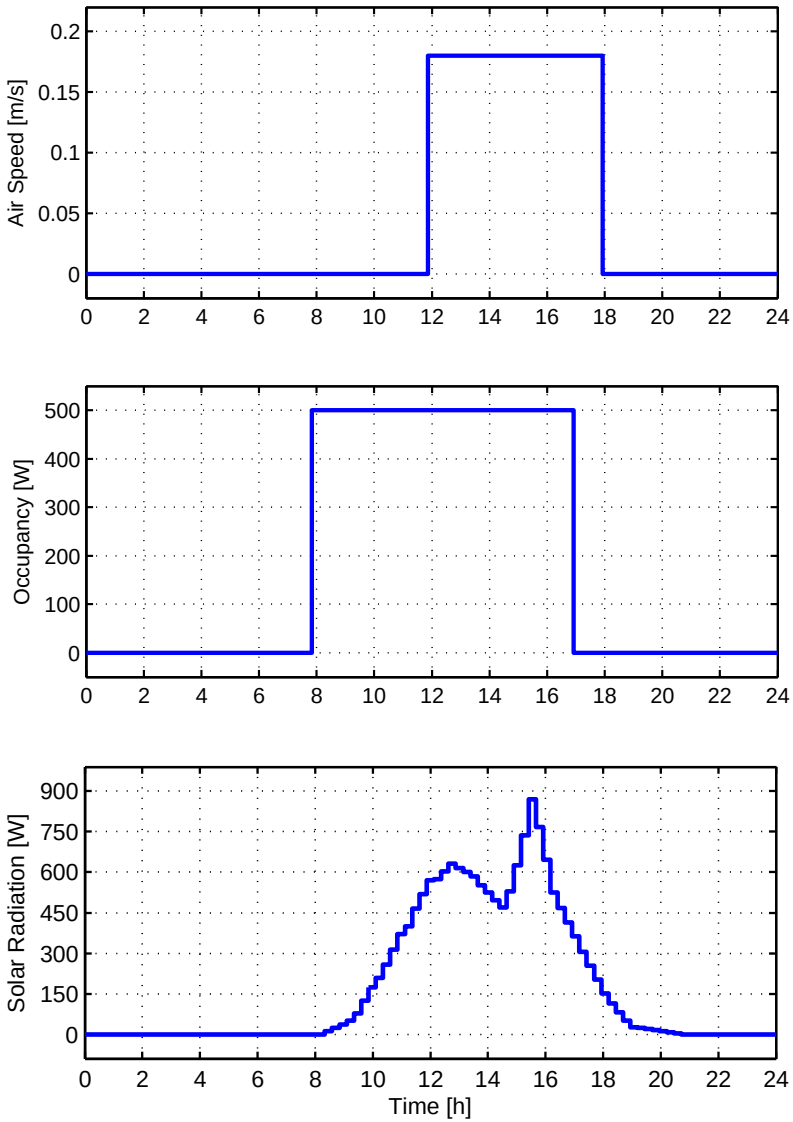

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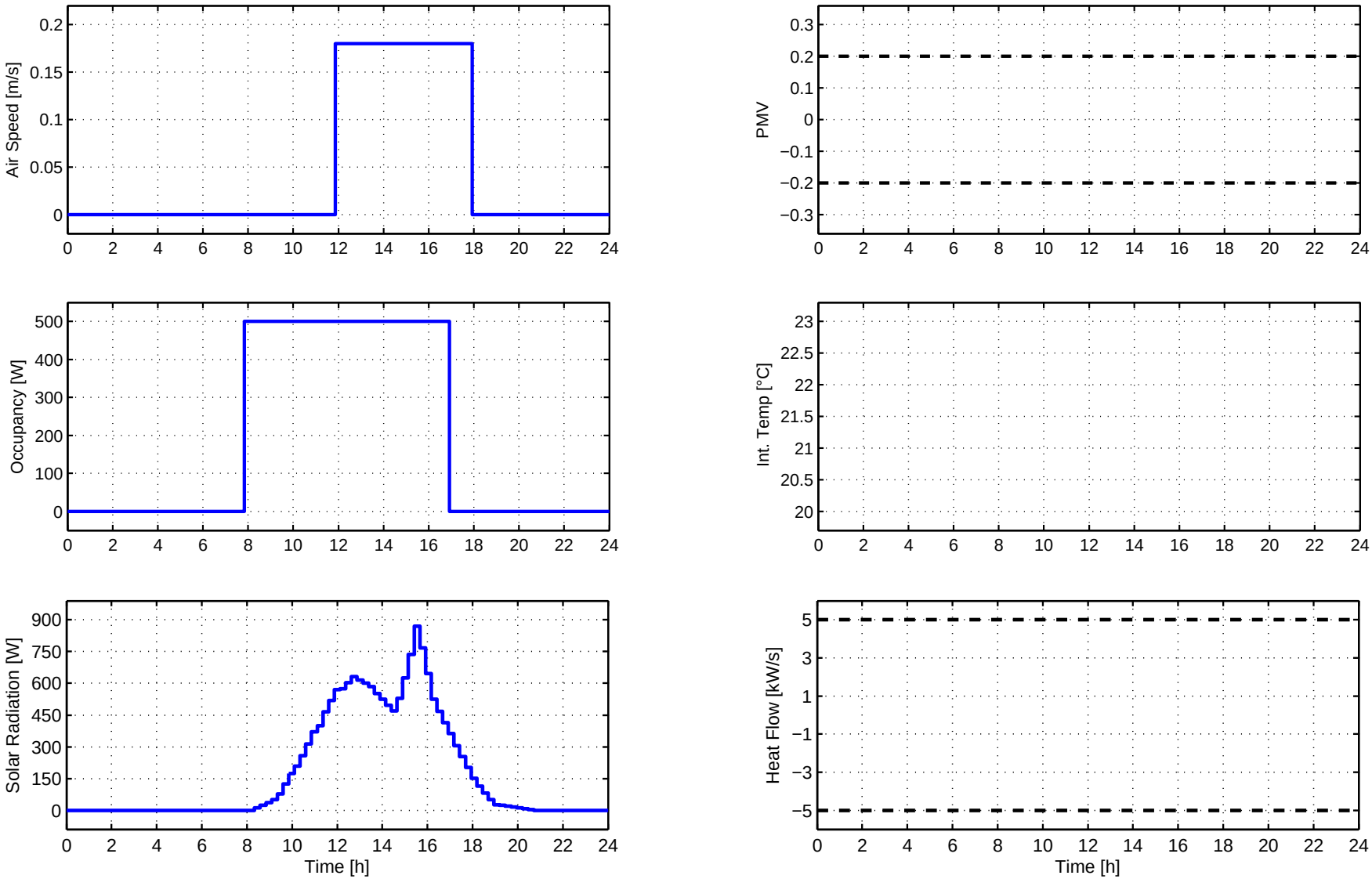

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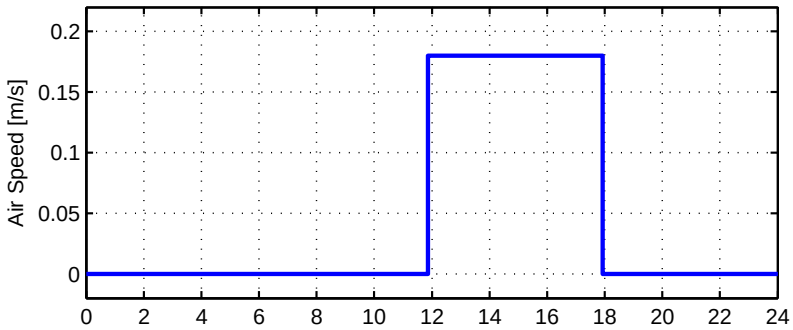

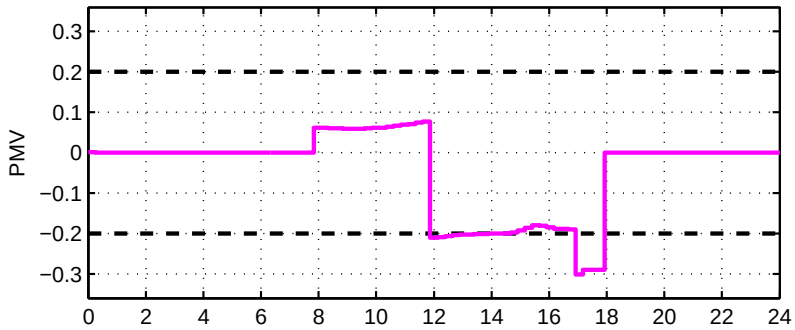

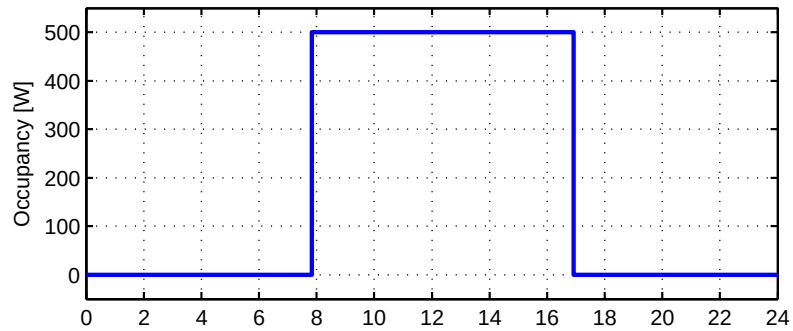

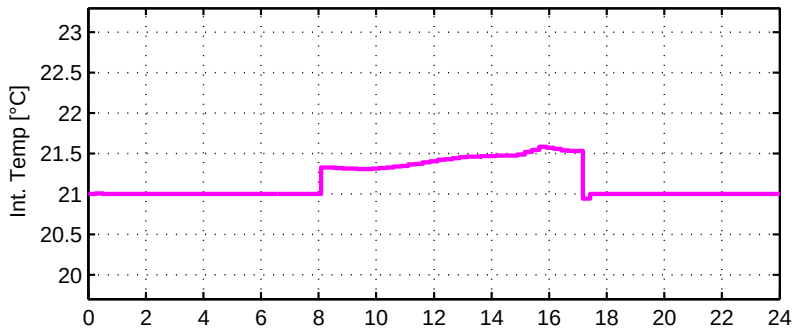

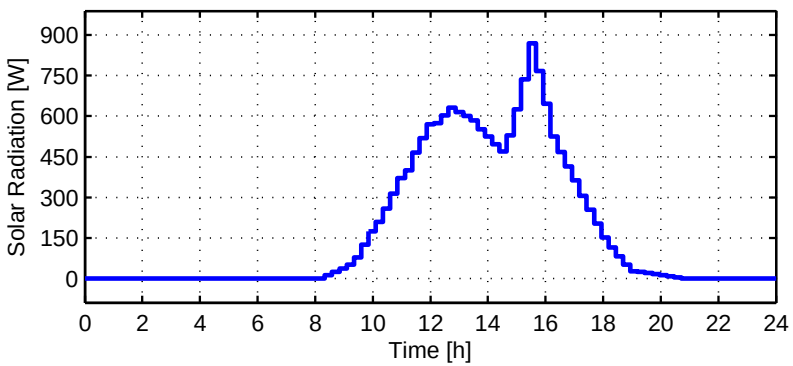

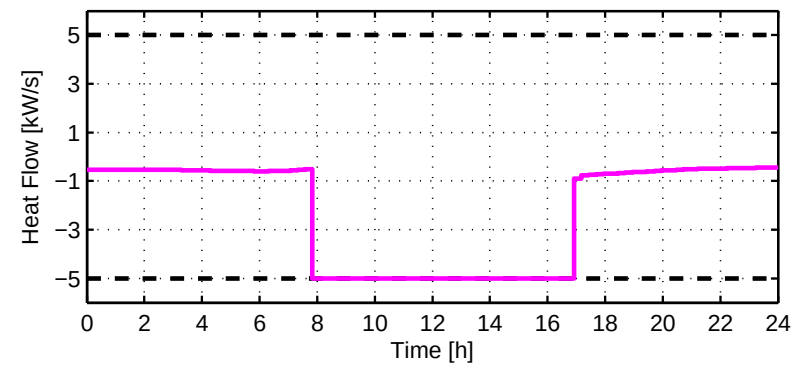

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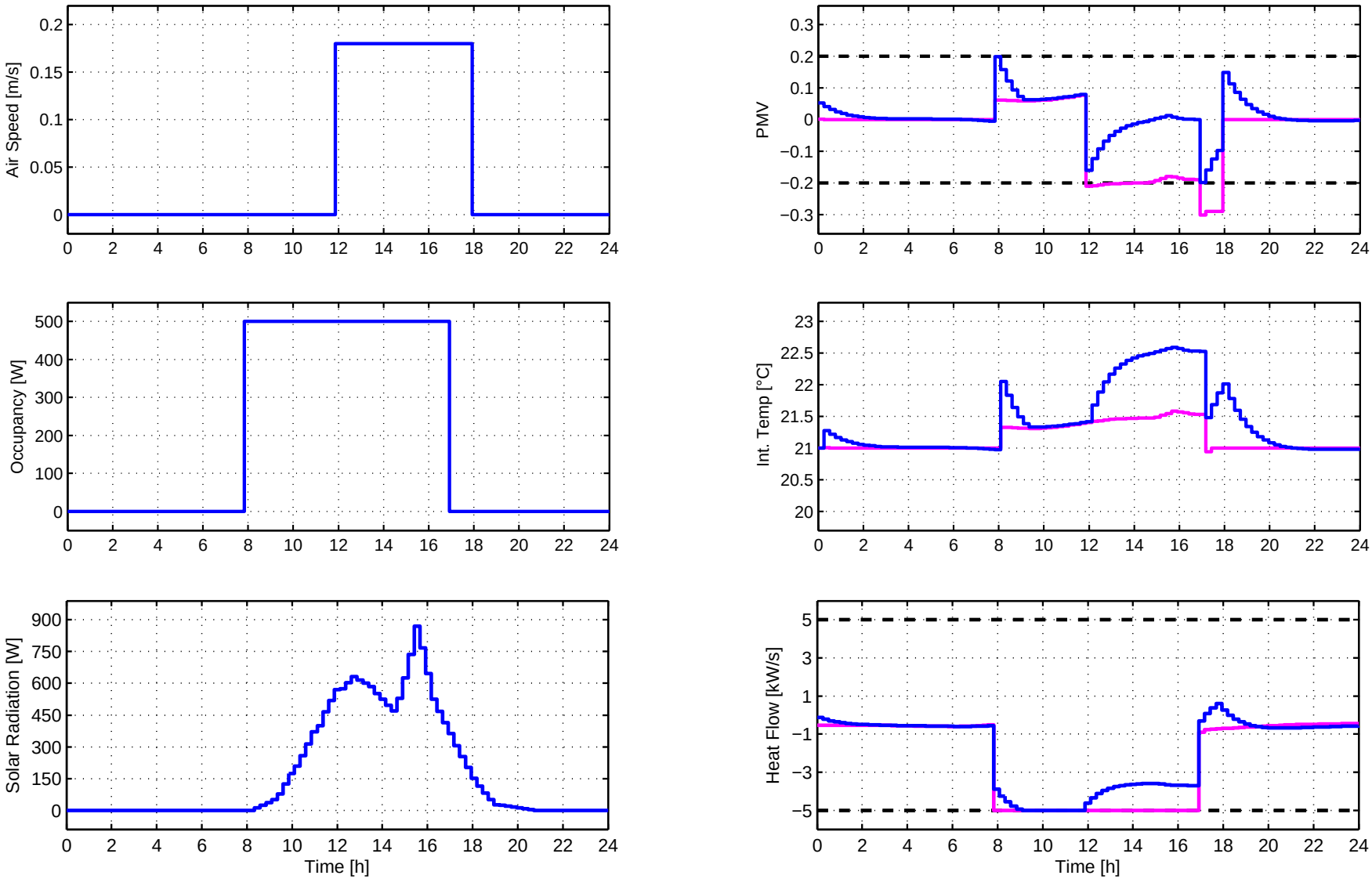

Conclusions

1. PMV index
2. Online linearisation
3. Explicit MPC on PLC