Mixed-Integer SOCP Formulation of the Path Planning Problem for Heterogeneous Multi-Vehicle Systems

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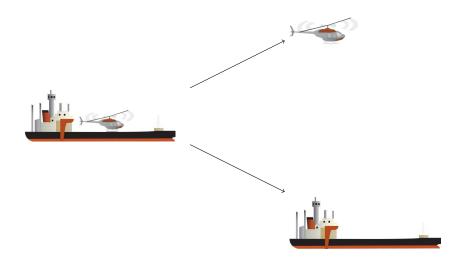
Slovak University of Technology in Bratislava, Slovakia

June 26, 2014

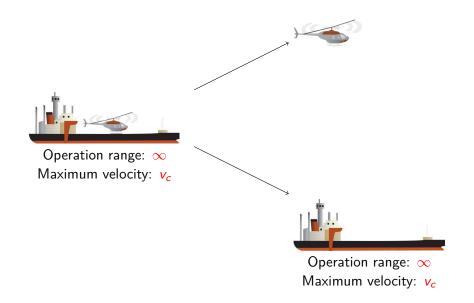




Heterogeneous Multi-Vehicle Systems



Heterogeneous Multi-Vehicle Systems



Heterogeneous Multi-Vehicle Systems

Operation range: t_{max}

Maximum velocity: $v_h > v_c$



Operation range: ∞ Maximum velocity: v_c



Operation range: ∞ Maximum velocity: v_c









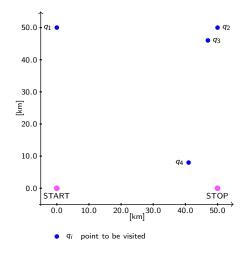




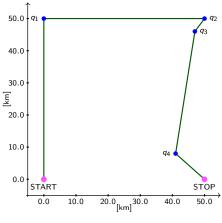




Single Vehicle

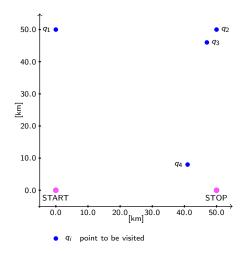


Single Vehicle



point to be visited

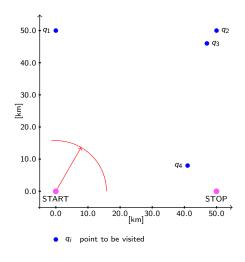
Ship speed: Total mission time: $\begin{array}{l} \textit{v}_{\textit{c}} = 18 \text{ km/h} \\ \approx 8.6 \text{ h} \end{array}$



Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$

6 / 15

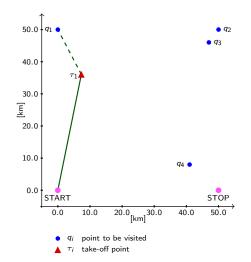
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Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$

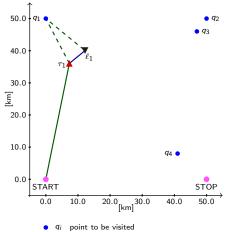
$$v_c = 18 \text{ km/h}$$

 $v_h = 90 \text{ km/h}$



Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$

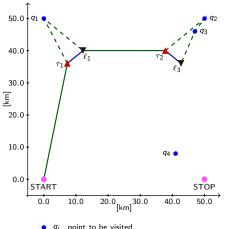
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Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$

take-off point

landing point

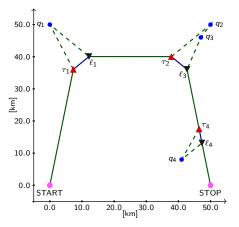


Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$

point to be visited

take-off point

landing point



Ship speed: $v_c = 18 \text{ km/h}$ Helicopter speed: $v_h = 90 \text{ km/h}$ Total mission time: $\approx 6.2 \text{ h} (8.6 \text{ h})$

qi point to be visited

 $ightharpoonup au_i$ take-off point

V ℓ_i landing point

Multi-vehicle System: Main Aim

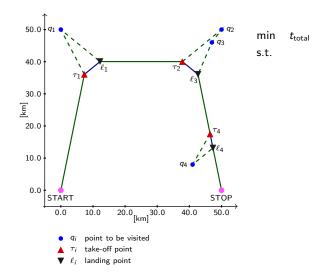
Calculate coordinates of take-off and landing points

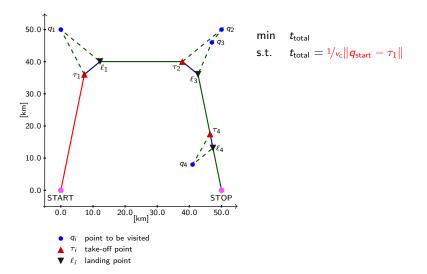
• minimize total time of operation

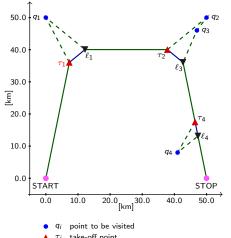
Multi-vehicle System: Main Aim

Calculate coordinates of take-off and landing points

- minimize total time of operation
- visit all way-points
- bounded helicopter flyover
- visit multiple way-points during one flyover







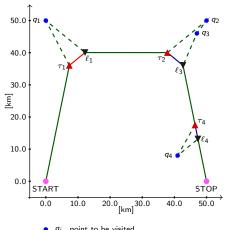
min t_{total}

 $t_{\text{total}} = 1/v_{\text{c}} ||q_{\text{start}} - \tau_1|| + f_1$ s.t.

 $f_1 \leq t_{h, \max}$

take-off point

landing point



min t_{total}

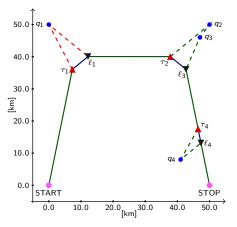
 $t_{\text{total}} = 1/v_{\text{c}} ||q_{\text{start}} - \tau_1|| + f_1$ s.t.

> $f_1 \leq t_{h,\text{max}}$ $\|\tau_1 - \ell_1\| \le v_{\rm c} f_1$

point to be visited

take-off point

landing point



min t_{total}

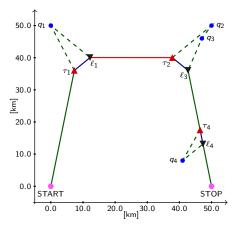
s.t. $t_{\mathsf{total}} = 1/v_{\mathsf{c}} \| q_{\mathsf{start}} - \tau_1 \| + f_1$

 $f_1 \le t_{h,\text{max}}$ $\|\tau_1 - \ell_1\| \le v_c f_1$ $\|\tau_1 - q_1\| + \|q_1 - \ell_1\| \le v_h f_1$

qi point to be visited

Δ τ; take-off point

 $\mathbf{V} \ell_i$ landing point



min t_{total}

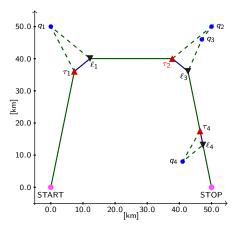
s.t. $t_{\text{total}} = \frac{1}{v_{\text{c}}} \|q_{\text{start}} - \tau_1\| + f_1 + \frac{s_1}{s_1}$

$$\begin{aligned} f_1 &\leq t_{h,\text{max}} \\ \|\tau_1 - \ell_1\| &\leq v_{\text{c}} f_1 \\ \|\tau_1 - q_1\| + \|q_1 - \ell_1\| &\leq v_{\text{h}} f_1 \\ \|\ell_1 - \tau_2\| &\leq v_{\text{c}} s_1 \end{aligned}$$

qi point to be visited

 τ_i take-off point

▼ ℓ_i landing point



min

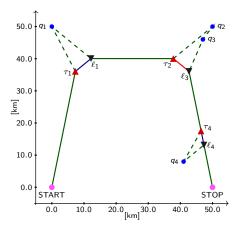
 t_{total} $t_{\text{total}} = 1/v_{\text{c}} ||q_{\text{start}} - \tau_1|| + f_1 + s_1 + f_1 + f$ s.t. $+ f_2$ $f_1 \leq t_{h,\text{max}}$ $\|\tau_1 - \ell_1\| \leq v_{\mathsf{c}} f_1$ $\|\tau_1 - q_1\| + \|q_1 - \ell_1\| \le v_h f_1$ $\|\ell_1 - \tau_2\| \leq \mathsf{v_c} \mathsf{s}_1$

 $f_2 \leq t_{h,\text{max}}$

point to be visited

take-off point

 ℓ_i landing point



min

 $t_{\rm total}$ $t_{\text{total}} = 1/v_{\text{c}} ||q_{\text{start}} - \tau_1|| + f_1 + s_1 + f_1 + f$ s.t. $+ f_2$ $f_1 \leq t_{h,\text{max}}$ $\|\tau_1 - \ell_1\| < v_{\rm c} f_1$ $\|\tau_1 - q_1\| + \|q_1 - \ell_1\| \le v_h f_1$ $\|\ell_1 - \tau_2\| \leq \mathsf{v_c} \mathsf{s}_1$

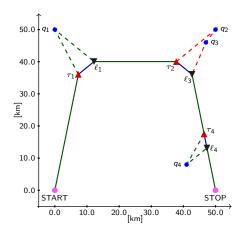
 $f_2 \leq t_{h,\max}$

 $\|\tau_2 - \ell_2\| < v_c f_2$

point to be visited

take-off point

 ℓ_i landing point



min

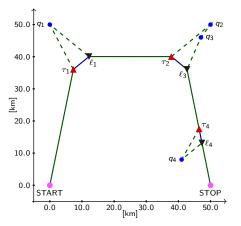
$$\begin{array}{ll} \min & t_{\text{total}} \\ \text{s.t.} & t_{\text{total}} = {}^{1}\!/{}_{v_{\text{c}}} \| q_{\text{start}} - \tau_{1} \| + f_{1} + s_{1} + \\ & + f_{2} \\ & f_{1} \leq t_{h,\text{max}} \\ & \|\tau_{1} - \ell_{1} \| \leq v_{\text{c}} f_{1} \\ & \|\tau_{1} - q_{1} \| + \|q_{1} - \ell_{1} \| \leq v_{\text{h}} f_{1} \\ & \|\ell_{1} - \tau_{2} \| \leq v_{\text{c}} s_{1} \\ & f_{2} \leq t_{h,\text{max}} \\ & \|\tau_{2} - \ell_{2} \| \leq v_{\text{c}} f_{2} \\ & \|\tau_{2} - q_{2} \| + \end{array}$$

 $+\|q_2-q_3\|+\|q_2-\ell_3\|< v_h f_2$

point to be visited

 τ_i take-off point

 ℓ_i landing point



point to be visited

 τ_i take-off point

 ℓ_i landing point

min s.t.

 t_{total}

$$t_{ ext{total}} = \frac{1}{v_c} \|q_{ ext{start}} - \tau_1\| + f_1 + s_1 + f_2 + \ldots + \frac{1}{v_c} \|q_{ ext{stop}} - \ell_4\|$$

$$f_1 \leq t_{h,\mathsf{max}}$$

 $\| au_1 - \ell_1\| < \mathsf{v_c} f_1$

$$\|\tau_1 - \ell_1\| \le v_c h$$

 $\|\tau_1 - q_1\| + \|q_1 - \ell_1\| \le v_h f_1$

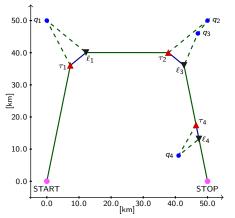
$$\|\ell_1 - \tau_2\| \le v_{\rm c} s_1$$

$$f_2 \leq t_{h,\max}$$

$$\|\tau_2 - \ell_2\| \le \mathsf{v}_\mathsf{c} \mathsf{f}_2$$

$$\| au_2 - q_2\| +$$

$$+\|q_2-q_3\|+\|q_2-\ell_3\|\leq v_h f_2$$



q_i point to be visited

 $ightharpoonup au_i$ take-off point

 $\forall \ell_i$ landing point

 $\min t_{total}$

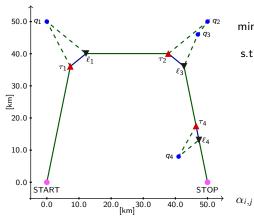
s.t.
$$\begin{aligned} t_{\mathsf{total}} &= 1/\mathsf{v}_\mathsf{c} \| q_{\mathsf{start}} - \tau_1 \| + \sum f_i + \\ &+ \sum \mathsf{s}_i + 1/\mathsf{v}_\mathsf{c} \| q_{\mathsf{stop}} - \ell_n \| \\ \alpha_{i,j} f_i &\leq t_{h,\mathsf{max}} \\ \alpha_{i,j} \| \tau_i - \ell_j \| &\leq \mathsf{v}_\mathsf{c} f_i \\ \alpha_{i,j} \| \tau_i - q_i \| + d_{i,j} + \| q_j - \ell_j \| &\leq \mathsf{v}_\mathsf{h} f_i \\ \alpha_{i,j} \| \ell_i - \tau_{i+1} \| &\leq \mathsf{v}_\mathsf{c} \mathsf{s}_i \end{aligned}$$

$$d_{i,j} = \sum_{k=i}^{j-1} ||q_k - q_{k+1}||$$

$$\alpha \in \{0, 1\}^{n \times n}$$

$$\alpha_{i,j} = 1 \Rightarrow \mathsf{fly} \mathsf{through} \; q_i, q_{i+1}, \dots, q_j$$

Garone, Determe, Naldi, CDC 2012



point to be visited

take-off point landing point

min t_{total}

s.t.
$$t_{\text{total}} = \frac{1}{\nu_c} \| q_{\text{start}} - \tau_1 \| + \sum_i f_i + \sum_i s_i + \frac{1}{\nu_c} \| q_{\text{stop}} - \ell_n \|$$

$$lpha_{i,j} f_i \leq t_{h, \mathsf{max}}$$
 $lpha_{i,j} \| \tau_i - \ell_j \| \leq v_{\mathsf{c}} f_i$

$$\frac{\alpha_{i,j}\|\tau_i - q_i\| \leq v_{c}\tau_i}{\alpha_{i,j}\|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| \leq v_h f_i}$$

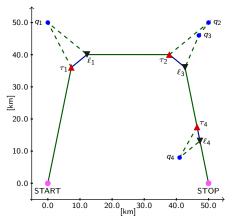
$$\alpha_{i,j}\|\ell_j- au_{j+1}\|\leq v_{\mathsf{c}}s_i$$

$$d_{i,j} = \sum_{k=i}^{j-1} ||q_k - q_{k+1}||$$

$$\alpha \in \{0,1\}^{n \times n}$$

$$lpha_{i,j} = 1 \Rightarrow \mathsf{fly} \; \mathsf{through} \; q_i, q_{i+1}, \dots, q_j$$

Garone, Determe, Naldi, CDC 2012



qi point to be visited

 $ightharpoonup au_i$ take-off point

 $\nabla \ell_i$ landing point

 $\min t_{total}$

s.t.
$$egin{aligned} t_{\mathsf{total}} &= 1/\mathsf{v_c} \|q_{\mathsf{start}} - au_1\| + \sum f_i + \\ &+ \sum s_i + 1/\mathsf{v_c} \|q_{\mathsf{stop}} - \ell_n\| \\ lpha_{i,i} f_i &< t_{h,\mathsf{max}} \end{aligned}$$

$$\alpha_{i,j} \| \tau_i - \ell_j \| \leq v_{\rm c} f_i$$

$$\alpha_{i,j} \| r_i - q_i \| + d_{i,j} + \| q_j - \ell_j \| \leq v_h f_i$$

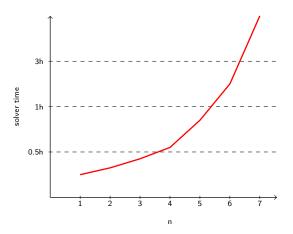
$$\alpha_{i,j} \|\ell_j - \tau_{j+1}\| \leq \mathsf{v}_\mathsf{c} \mathsf{s}_i$$

$$d_{i,j} = \sum_{k=i}^{j-1} ||q_k - q_{k+1}||$$

$$\alpha \in \{0, 1\}^{n \times n}$$

$$lpha_{i,j} = 1 \Rightarrow \mathsf{fly} \; \mathsf{through} \; q_i, q_{i+1}, \dots, q_j$$

Garone, Determe, Naldi, CDC 2012



$$\alpha_{i,j} \| \tau_i - \ell_j \| \le \mathsf{v}_\mathsf{c} \mathsf{f}_i$$

$$\alpha_{i,j} \| \tau_i - \ell_j \| \le \mathsf{v}_\mathsf{c} \mathsf{f}_i$$

Williams H.P., Model Building in Mathematical Programming, 1993

$$\alpha_{i,j} \| \tau_i - \ell_j \| \le v_{\mathsf{c}} f_i$$

Williams H.P., Model Building in Mathematical Programming, 1993

$$(\alpha_{i,j} = 1) \Rightarrow \|\tau_i - \ell_j\| - v_c f_i \le 0$$

$$\alpha_{i,j} \| \tau_i - \ell_j \| \le v_{\mathsf{c}} f_i$$

Williams H.P., Model Building in Mathematical Programming, 1993

$$(\alpha_{i,j}=1)\Rightarrow \| au_i-\ell_j\|-v_{\mathsf{c}}f_i\leq 0$$

$$\| au_i - \ell_j\| - v_{\mathsf{c}} f_i \leq M(1 - \alpha_{i,j})$$

$$\alpha_{i,j} \| \tau_i - \ell_j \| \le v_{\mathsf{c}} f_i$$

Williams H.P., Model Building in Mathematical Programming, 1993

$$(\alpha_{i,j}=1)\Rightarrow \| au_i-\ell_j\|-v_{\mathsf{c}}f_i\leq 0$$

$$\|\tau_i - \ell_j\| - v_{\mathsf{c}} f_i \le M(1 - \alpha_{i,j})$$

$$M = \max_{\tau_i, \ell_i, f_{i,i}} (\|\tau_i - \ell_j\| - v_{\mathsf{c}} f_{i,j})$$

$$\begin{aligned} & \text{min} \quad t_{\text{total}} \\ & \text{s.t.} \quad \begin{aligned} & t_{\text{total}} &= \frac{1}{\nu_c} \| q_{\text{start}} - \tau_1 \| + \sum_j f_i + \\ & \quad + \sum_j s_i + \frac{1}{\nu_c} \| q_{\text{stop}} - \ell_n \| \\ & \alpha_{i,j} f_i \leq t_{h,\text{max}} \\ & \alpha_{i,j} \| \tau_i - \ell_j \| \leq \nu_c f_i \\ & \alpha_{i,j} \| \tau_i - q_i \| + d_{i,j} + \| q_j - \ell_j \| \leq \nu_h f_i \\ & \alpha_{i,j} \| \ell_j - \tau_{j+1} \| \leq \nu_c s_i \\ & d_{i,j} &= \sum_{k=i}^{j-1} \| q_k - q_{k+1} \| \\ & \alpha \in \left\{0,1\right\}^{n \times n} \end{aligned}$$

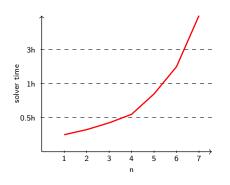
$$\alpha_{i,i} = 1 \Rightarrow \text{fly through } q_i, q_{i+1}, \dots, q_i$$

Garone, Determe, Naldi, CDC 2012

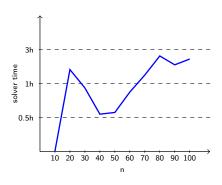
$$\begin{aligned} & \text{min} & \ t_{\text{total}} \\ & \text{s.t.} & \ t_{\text{total}} = \frac{1}{\nu_{\text{c}}} \| q_{\text{start}} - \tau_1 \| + \sum f_i + \\ & + \sum s_i + \frac{1}{\nu_{\text{c}}} \| q_{\text{stop}} - \ell_n \| \\ & f_i - t_{h, \text{max}} \leq M(1 - \alpha_{i,j}) \\ & \| \tau_i - \ell_j \| - \nu_{\text{c}} f_i \leq M(1 - \alpha_{i,j}) \\ & \| \tau_i - q_i \| + d_{i,j} + \| q_j - \ell_j \| - \nu_{\text{h}} f_i \leq M(1 - \alpha_{i,j}) \\ & \| \ell_j - \tau_{j+1} \| - \nu_{\text{c}} s_i \leq M(1 - \alpha_{i,j}) \\ & d_{i,j} = \sum_{k=i}^{j-1} \| q_k - q_{k+1} \| \\ & \alpha \in \{0,1\}^{n \times n} \end{aligned}$$

 $\alpha_{i,j} = 1 \Rightarrow \mathsf{fly} \mathsf{through} \; q_i, q_{i+1}, \dots, q_i$

$$\alpha_{i,j} \| \tau_i - \ell_j \| \le v_{\mathsf{c}} f_i$$



$$\underbrace{\|\tau_i - \ell_j\| - v_c f_i}_{\text{SOCP}} \leq \underbrace{M(1 - \alpha_{i,j})}_{\text{Mixed Int.}}$$



13 / 15

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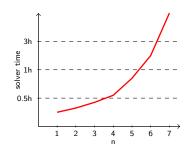
Assumptions and Restrictions

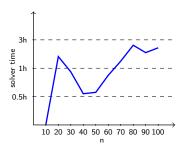
- Movement at constant speed
- Refueling is instantaneous
- Order of points is known

Same as in Garone, Determe, Naldi, CDC 2012

Wrap Up

- Path planing for multi-vehicle system
- SOCP mixed integer formulation
- Large scale applications





15 / 15

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