Mixed-Integer SOCP Formulation of the Path Planning Problem for Heterogeneous Multi-Vehicle Systems

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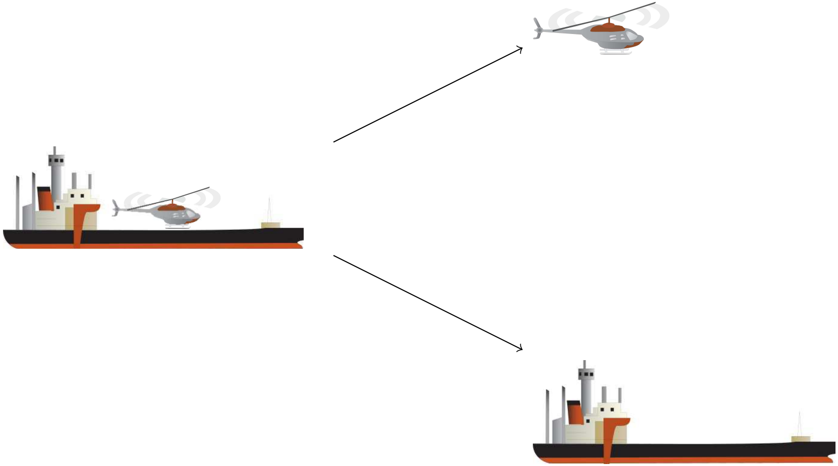
FCHPT

Heterogeneous Multi-Vehicle Systems



Heterogeneous Multi-Vehicle Systems

Operation range: ∞
Maximum velocity: v_c

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Maximum velocity: v_c

Heterogeneous Multi-Vehicle Systems

Operation range: ∞

Maximum velocity: v_c

Operation range: $t_{\max}$

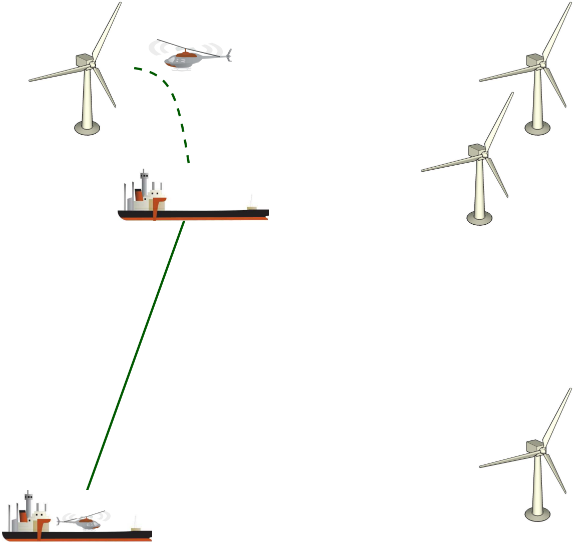
Maximum velocity: $v_h > v_c$

Operation range: ∞

Maximum velocity: v_c

Motivation

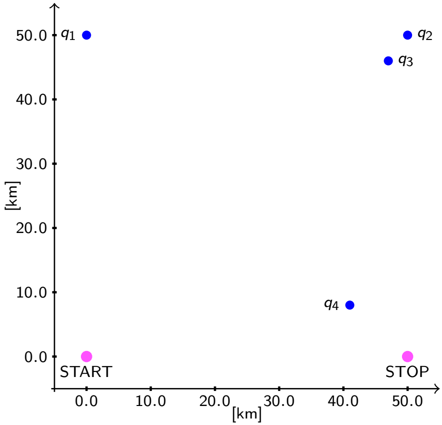
Motivation



Motivation

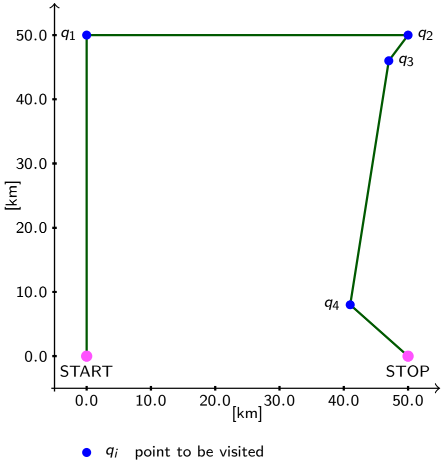
Motivation

Single Vehicle



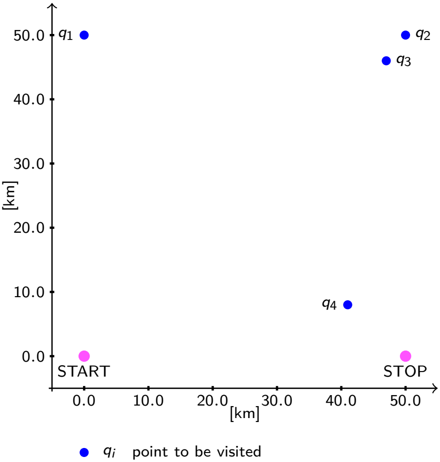
● q_i point to be visited

Single Vehicle



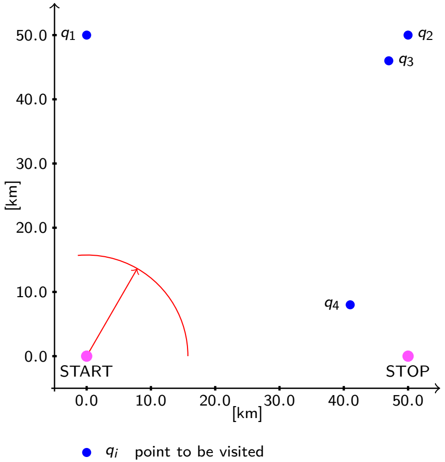
Ship speed: $v_c = 18$ km/h
Total mission time: ≈ 8.6 h

Multi-vehicle System



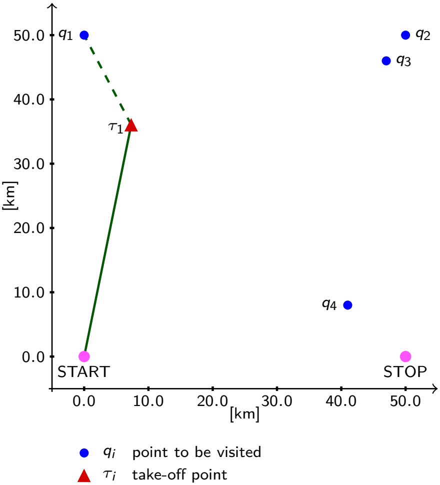
Ship speed: $v_c = 18$ km/h
Helicopter speed: $v_h = 90$ km/h

Multi-vehicle System



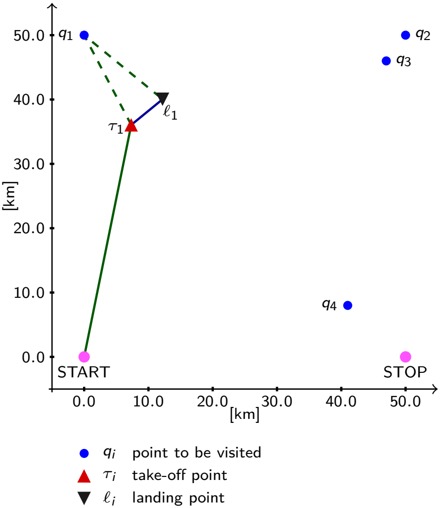
Ship speed: $v_c = 18$ km/h
Helicopter speed: $v_h = 90$ km/h

Multi-vehicle System



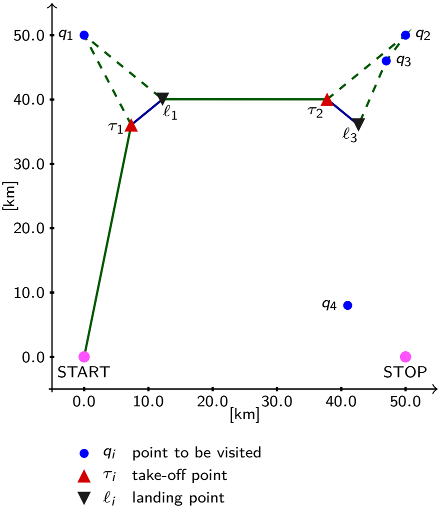
Ship speed: $v_c = 18$ km/h
Helicopter speed: $v_h = 90$ km/h

Multi-vehicle System



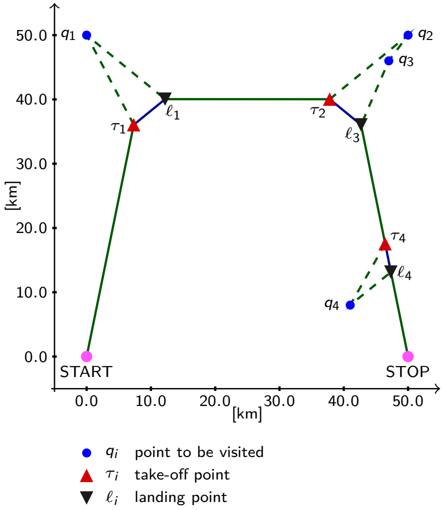
Ship speed:	$v_c = 18$ km/h
Helicopter speed:	$v_h = 90$ km/h

Multi-vehicle System



Ship speed: $v_c = 18$ km/h
Helicopter speed: $v_h = 90$ km/h

Multi-vehicle System



Ship speed:	$v_c = 18$ km/h
Helicopter speed:	$v_h = 90$ km/h
Total mission time:	≈ 6.2 h (8.6 h)

Multi-vehicle System: Main Aim

Calculate coordinates of take-off and landing points

- minimize total time of operation

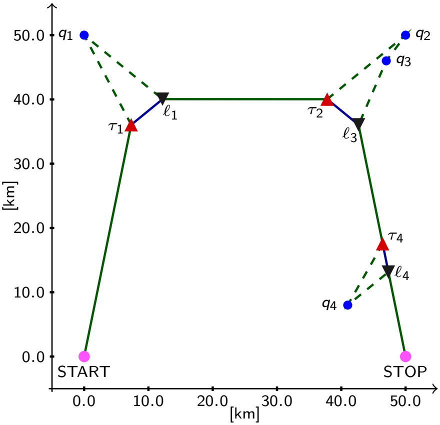
Multi-vehicle System: Main Aim

Calculate coordinates of take-off and landing points

- minimize total time of operation

- visit all way-points
- bounded helicopter flyover
- visit multiple way-points during one flyover

Problem Formulation

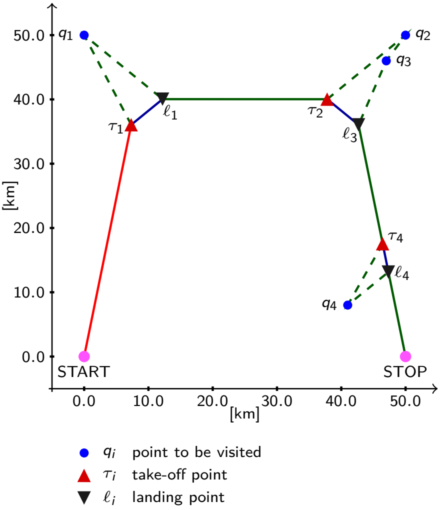


- q_i point to be visited
- τ_i take-off point
- ℓ_i landing point

min $\quad t_{\text{total}}$

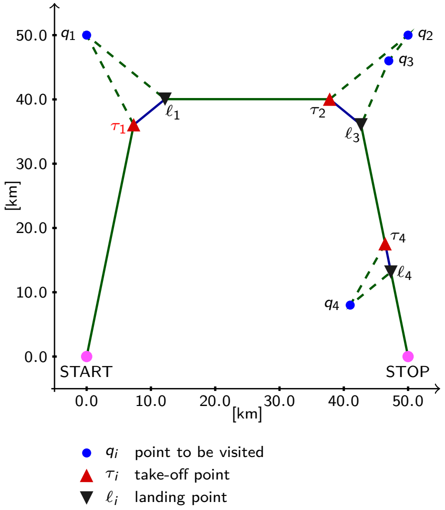
s.t.

Problem Formulation



$$\begin{aligned} \text{min} \quad & t_{\text{total}} \\ \text{s.t.} \quad & t_{\text{total}} = {}^{1}\!/_{v_c} \left\| q_{\text{start}} - \tau_1 \right\| \end{aligned}$$

Problem Formulation

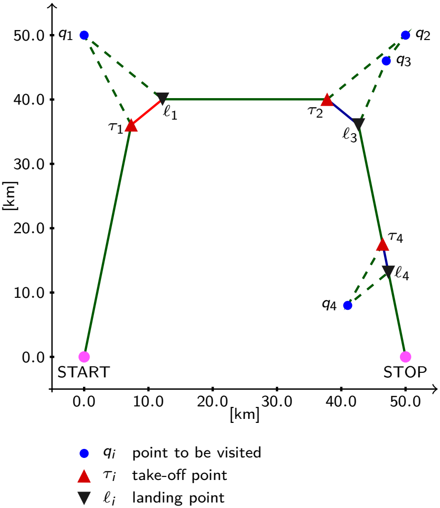


$$\min \quad t_{\text{total}}$$

$$\text{s.t.} \quad t_{\text{total}} = {}^{1}\!/_{v_c} \|q_{\text{start}} - \tau_1\| + f_1$$

$$f_1 \leq t_{h,\max}$$

Problem Formulation



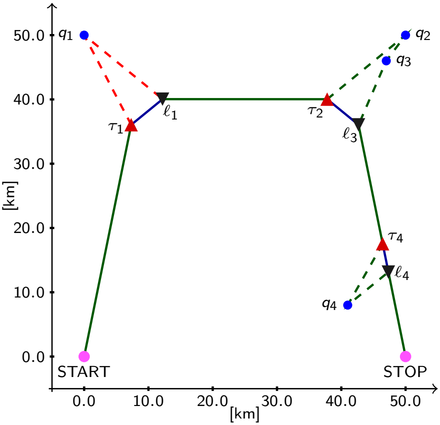
min t_{total}

s.t. $t_{\text{total}} = {}^{1}/_{v_c} \| q_{\text{start}} - \tau_1 \| + f_1$

$f_1 \leq t_{h,\max}$

$\| \tau_1 - \ell_1 \| \leq v_c f_1$

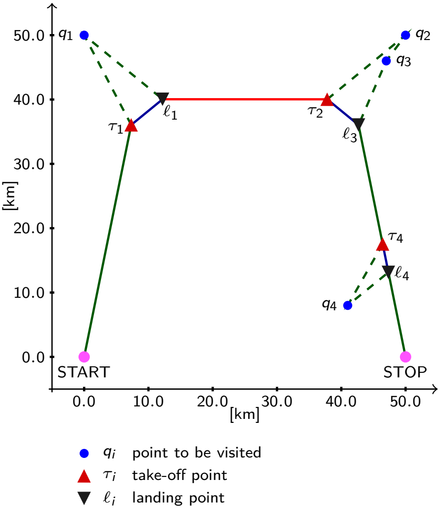
Problem Formulation



- q_i point to be visited
- τ_i take-off point
- ℓ_i landing point

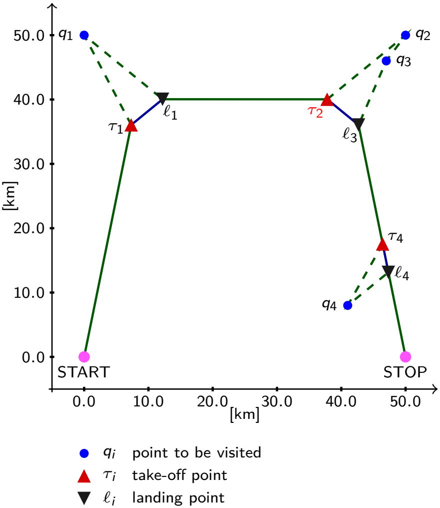
$$\begin{aligned} \text{min} \quad & t_{\text{total}} \\ \text{s.t.} \quad & t_{\text{total}} = {}^{1}/_{v_c} \| q_{\text{start}} - \tau_1 \| + f_1 \\ & f_1 \leq t_{h,\max} \\ & \| \tau_1 - \ell_1 \| \leq v_c f_1 \\ & \| \tau_1 - q_1 \| + \| q_1 - \ell_1 \| \leq v_h f_1 \end{aligned}$$

Problem Formulation



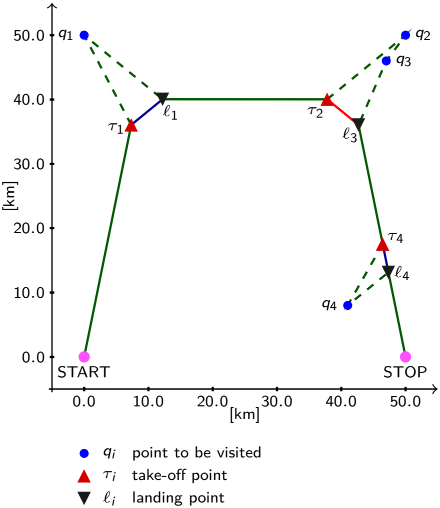
$$
\begin{aligned}
\text{min} \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^{1}/_{v_{\text{c}}} \| q_{\text{start}} - \tau_1 \| + f_1 + \color{red}{s_1} \\
& \\
& f_1 \leq t_{h,\text{max}} \\
& \| \tau_1 - \ell_1 \| \leq v_{\text{c}} f_1 \\
& \| \tau_1 - q_1 \| + \| q_1 - \ell_1 \| \leq v_{\text{h}} f_1 \\
& \color{red}{\| \ell_1 - \tau_2 \| \leq v_{\text{c}} s_1}
\end{aligned}
$$

Problem Formulation



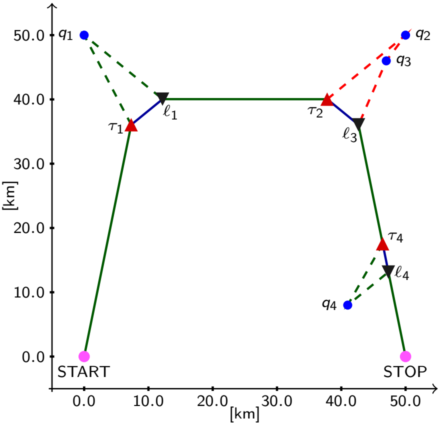
$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^{1}/_{v_c} \| q_{\text{start}} - \tau_1 \| + f_1 + s_1 + \\
& \qquad + {\color{red} f_2} \\
& f_1 \leq t_{h,\max} \\
& \| \tau_1 - \ell_1 \| \leq v_c f_1 \\
& \| \tau_1 - q_1 \| + \| q_1 - \ell_1 \| \leq v_h f_1 \\
& \| \ell_1 - \tau_2 \| \leq v_c s_1 \\
& {\color{red} f_2 \leq t_{h,\max}}
\end{aligned}
$$

Problem Formulation



$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^1/_{v_c} \|q_{\text{start}} - \tau_1\| + f_1 + s_1 + \\
& \quad + f_2 \\
& f_1 \le t_{h,\max} \\
& \|\tau_1 - \ell_1\| \le v_c f_1 \\
& \|\tau_1 - q_1\| + \|q_1 - \ell_1\| \le v_h f_1 \\
& \|\ell_1 - \tau_2\| \le v_c s_1 \\
& f_2 \le t_{h,\max} \\
& \|\tau_2 - \ell_2\| \le v_c f_2
\end{aligned}
$$

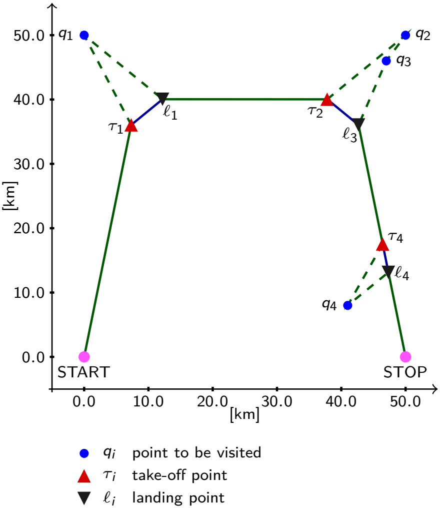
Problem Formulation



- q_i point to be visited
- τ_i take-off point
- ℓ_i landing point

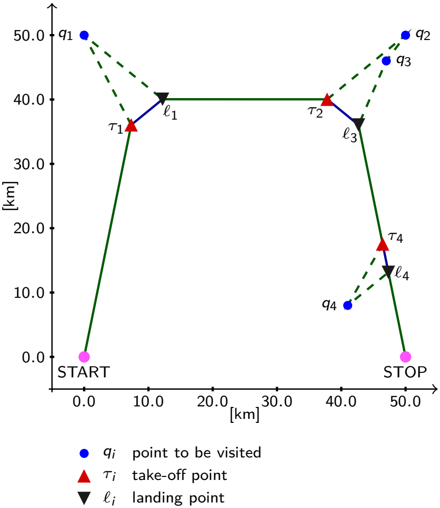
$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^{1}/_{v_c} \|q_{\text{start}} - \tau_1\| + f_1 + s_1 + \\
& \quad + {\color{red} f_2} \\
& f_1 \leq t_{h,\max} \\
& \|\tau_1 - \ell_1\| \leq v_c f_1 \\
& \|\tau_1 - q_1\| + \|q_1 - \ell_1\| \leq v_h f_1 \\
& \|\ell_1 - \tau_2\| \leq v_c s_1 \\
& f_2 \leq t_{h,\max} \\
& \|\tau_2 - \ell_2\| \leq v_c f_2 \\
& {\color{red} \|\tau_2 - q_2\| +} \\
& {\color{red} + \|q_2 - q_3\| + \|q_2 - \ell_3\| \leq v_h f_2}
\end{aligned}
$$

Problem Formulation



$$
\begin{aligned}
\text{min} \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^1/_{v_c}\|q_{\text{start}} - \tau_1\| + f_1 + s_1 + \\
& \quad + f_2 + \ldots + {}^1/_{v_c}\|q_{\text{stop}} - \ell_4\| \\
& f_1 \leq t_{h,\max} \\
& \|\tau_1 - \ell_1\| \leq v_c f_1 \\
& \|\tau_1 - q_1\| + \|q_1 - \ell_1\| \leq v_h f_1 \\
& \|\ell_1 - \tau_2\| \leq v_c s_1 \\
& f_2 \leq t_{h,\max} \\
& \|\tau_2 - \ell_2\| \leq v_c f_2 \\
& \|\tau_2 - q_2\| + \\
& + \|q_2 - q_3\| + \|q_2 - \ell_3\| \leq v_h f_2 \\
& \vdots
\end{aligned}
$$

Mixed Integer Nonlinear Problem Formulation

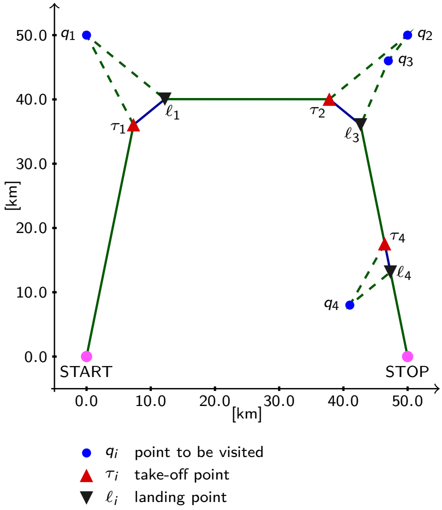


$$\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^1\!/_{v_c}\|q_{\text{start}} - \tau_1\| + \sum f_i + \\
& \qquad + \sum s_i + {}^1\!/_{v_c}\|q_{\text{stop}} - \ell_n\| \\
& \alpha_{i,j} f_i \leq t_{h,\max} \\
& \alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i \\
& \alpha_{i,j}\|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| \leq v_h f_i \\
& \alpha_{i,j}\|\ell_j - \tau_{j+1}\| \leq v_c s_i \\
& d_{i,j} = \sum_{k=i}^{j-1}\|q_k - q_{k+1}\| \\
& \alpha \in \{0,1\}^{n\times n}
\end{aligned}$$

$\alpha_{i,j} = 1 \Rightarrow$ fly through $q_i, q_{i+1}, \ldots, q_j$

Garone, Determe, Naldi, CDC 2012

Mixed Integer Nonlinear Problem Formulation



$$\min \ t_{\text{total}}$$

s.t.

$$t_{\text{total}} = {}^1/_{v_c}\|q_{\text{start}} - \tau_1\| + \sum f_i + + \sum s_i + {}^1/_{v_c}\|q_{\text{stop}} - \ell_n\|$$

$$\alpha_{i,j} f_i \leq t_{h,\max}$$

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

$$\alpha_{i,j}\|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| \leq v_h f_i$$

$$\alpha_{i,j}\|\ell_j - \tau_{j+1}\| \leq v_c s_i$$

$$d_{i,j} = \sum_{k=i}^{j-1}\|q_k - q_{k+1}\|$$

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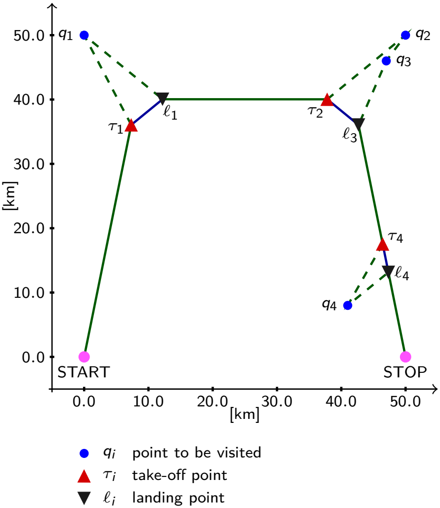

$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^{1}/_{v_c}\|q_{\text{start}} - \tau_1\| + \sum f_i + \\
& \qquad + \sum s_i + {}^{1}/_{v_c}\|q_{\text{stop}} - \ell_n\| \\
& \alpha_{i,j} f_i \leq t_{h,\max} \\
& \alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i \\
& \alpha_{i,j}\|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| \leq v_h f_i \\
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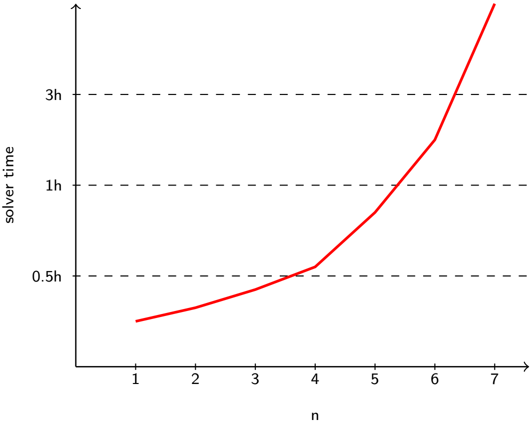
Mixed Integer Nonlinear Problem Formulation



Mixed Integer SOCP Formulation

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

Mixed Integer SOCP Formulation

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Williams H.P., *Model Building in Mathematical Programming*, 1993

Mixed Integer SOCP Formulation

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

Williams H.P., *Model Building in Mathematical Programming*, 1993

$$(\alpha_{i,j} = 1) \Rightarrow \|\tau_i - \ell_j\| - v_c f_i \leq 0$$

Mixed Integer SOCP Formulation

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

Williams H.P., *Model Building in Mathematical Programming*, 1993

$$(\alpha_{i,j} = 1) \Rightarrow \|\tau_i - \ell_j\| - v_c f_i \leq 0$$

$$\|\tau_i - \ell_j\| - v_c f_i \leq M(1 - \alpha_{i,j})$$

Mixed Integer SOCP Formulation

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

Williams H.P., *Model Building in Mathematical Programming*, 1993

$$(\alpha_{i,j} = 1) \Rightarrow \|\tau_i - \ell_j\| - v_c f_i \leq 0$$

$$\|\tau_i - \ell_j\| - v_c f_i \leq M(1 - \alpha_{i,j})$$

$$M = \max_{\tau_i, \ell_j, f_{i,j}} (\|\tau_i - \ell_j\| - v_c f_{i,j})$$

Mixed Integer Nonlinear Problem Formulation

$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = {}^1\!/_{v_\text{c}} \|q_{\text{start}} - \tau_1\| + \sum f_i + \\
& \qquad + \sum s_i + {}^1\!/_{v_\text{c}} \|q_{\text{stop}} - \ell_n\| \\
& \alpha_{i,j} f_i \leq t_{h,\max} \\
& \alpha_{i,j} \|\tau_i - \ell_j\| \leq v_\text{c} f_i \\
& \alpha_{i,j} \|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| \leq v_\text{h} f_i \\
& \alpha_{i,j} \|\ell_j - \tau_{j+1}\| \leq v_\text{c} s_i \\
& d_{i,j} = \sum_{k=i}^{j-1} \|q_k - q_{k+1}\| \\
& \alpha \in \{0,1\}^{n \times n}
\end{aligned}
$$

$\alpha_{i,j} = 1 \Rightarrow$ fly through $q_i, q_{i+1}, \ldots, q_j$

Garone, Determe, Naldi, CDC 2012

Mixed Integer SOCP Formulation

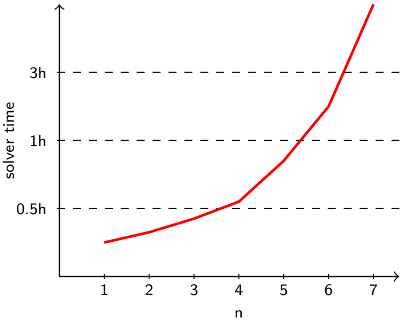
$$
\begin{aligned}
\min \quad & t_{\text{total}} \\
\text{s.t.} \quad & t_{\text{total}} = 1/v_c \|q_{\text{start}} - \tau_1\| + \sum f_i + \\
& \qquad + \sum s_i + 1/v_c \|q_{\text{stop}} - \ell_n\| \\
& f_i - t_{h,\max} \leq M(1 - \alpha_{i,j}) \\
& \|\tau_i - \ell_j\| - v_c f_i \leq M(1 - \alpha_{i,j}) \\
& \|\tau_i - q_i\| + d_{i,j} + \|q_j - \ell_j\| - v_h f_i \leq M(1 - \alpha_{i,j}) \\
& \|\ell_j - \tau_{j+1}\| - v_c s_i \leq M(1 - \alpha_{i,j}) \\
& d_{i,j} = \sum_{k=i}^{j-1} \|q_k - q_{k+1}\| \\
& \alpha \in \{0, 1\}^{n \times n}
\end{aligned}
$$

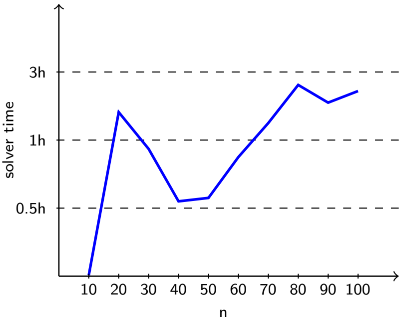
$$\alpha_{i,j} = 1 \Rightarrow \text{fly through } q_i, q_{i+1}, \ldots, q_j$$

Mixed Integer SOCP Formulation

$$\alpha_{i,j}\|\tau_i - \ell_j\| \leq v_c f_i$$

$$\underbrace{\|\tau_i - \ell_j\| - v_c f_i}_{\text{SOCP}} \leq \underbrace{M(1 - \alpha_{i,j})}_{\text{Mixed Int.}}$$





Assumptions and Restrictions

- Movement at constant speed
- Refueling is instantaneous
- Order of points is known

Same as in Garone, Determe, Naldi, CDC 2012

Wrap Up

- Path planing for multi-vehicle system
- SOCP mixed integer formulation
- Large scale applications