Building Temperature Control by Simple MPC-like Feedback Laws Learned from Closed-Loop Data

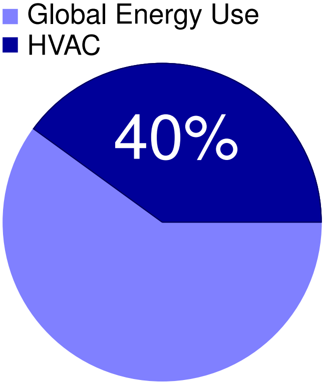
M. Klaučo,
J. Drgoňa, M. Kvasnica, S. D. Cairano

Slovak University of Technology in Bratislava, Slovakia

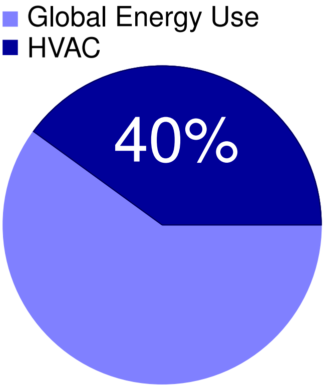
August 25, 2014

STU
FCHPT

Motivation

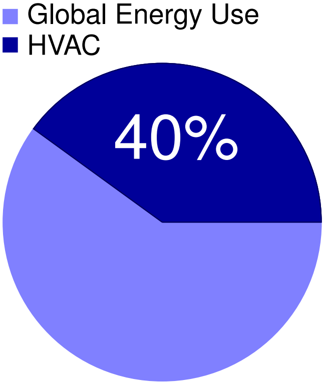


Motivation



Efficient temperature control

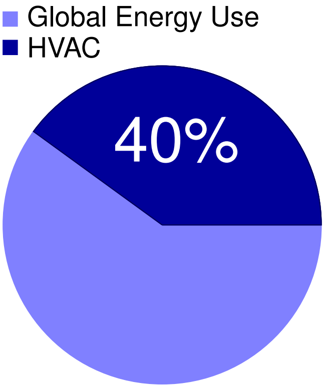
Motivation



Efficient temperature control

Model Predictive Control

Motivation

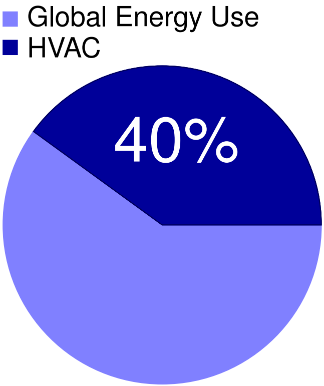


Efficient temperature control

Model Predictive Control

Constraints

Motivation



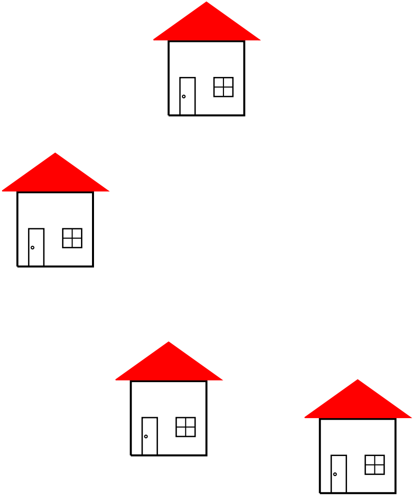
Efficient temperature control

Model Predictive Control

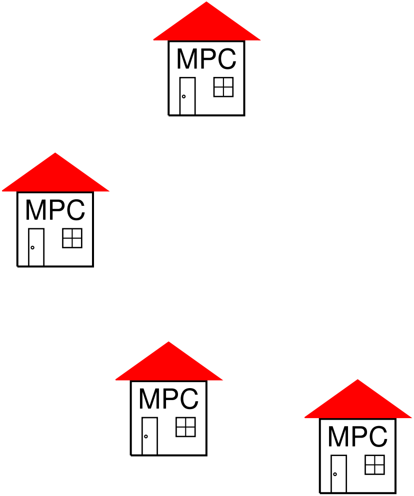
Constraints

Energy minimization

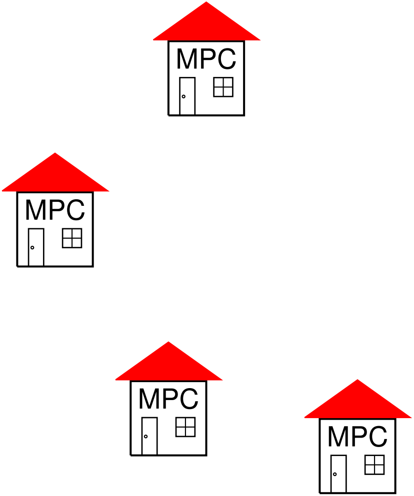
Motivation



Motivation



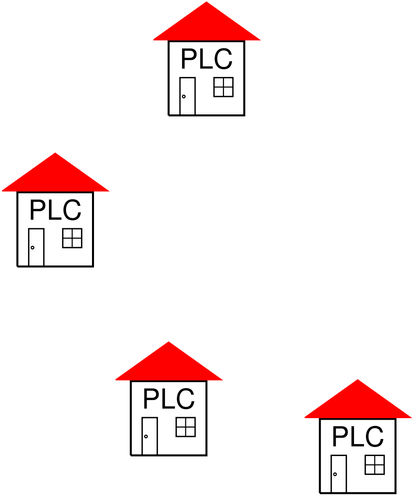
Motivation



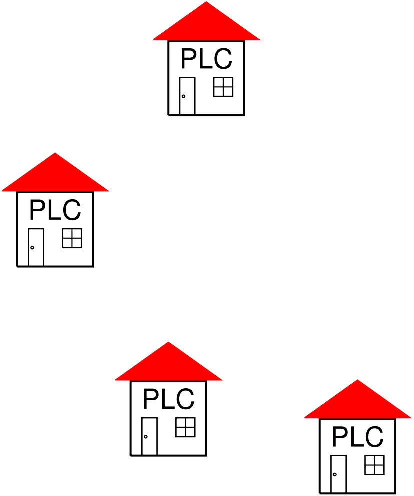
Online MPC

Computational demand

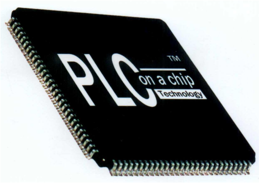
Motivation



Motivation



Explicit MPC



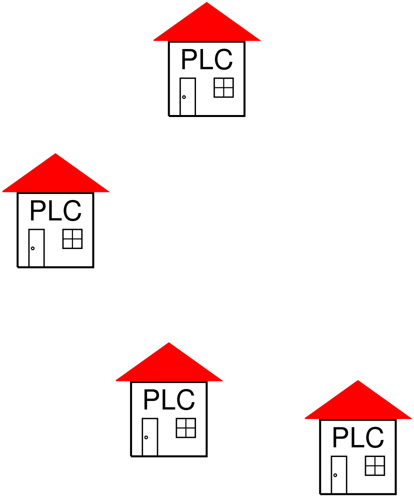
Memory demand

Explicit MPC

$$\begin{aligned} \min \quad & \sum ||u_k|| \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\ & T_{\min} \leq Cx_k \leq T_{\max} \\ & u_{\min} \leq u_k \leq u_{\max} \end{aligned}$$

Obtained off-line

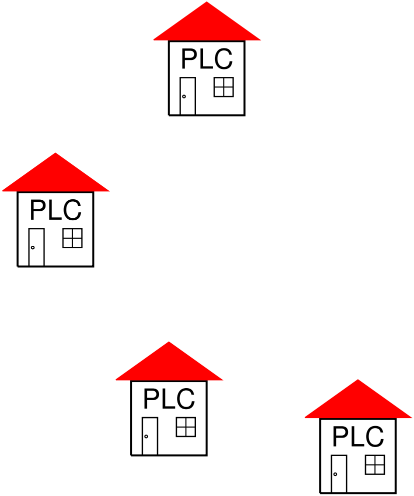
$$u^\star(x) = \begin{cases} F_1 x + g_1 & \text{if } x \in \mathcal{R}_1 \\ & \vdots \\ F_M x + g_M & \text{if } x \in \mathcal{R}_M \end{cases}$$



Explicit MPC

$$u^\star(x) = \begin{cases} F_1 x + g_1 & \text{if } x \in \mathcal{R}_1 \\ & \vdots \\ F_M x + g_M & \text{if } x \in \mathcal{R}_M \end{cases}$$

Memory demand



Explicit MPC

$$u^{\star}(x) = \begin{cases} F_1 x + g_1 & \text{if } x \in \mathcal{R}_1 \\ \vdots & \\ F_M x + g_M & \text{if } x \in \mathcal{R}_M \end{cases}$$

Memory demand

MPC-like Controller

Online MPC

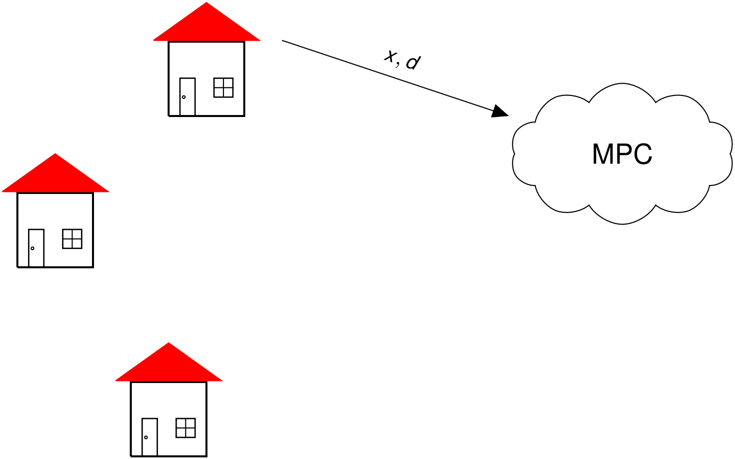
$$u^\star(x)$$

Suboptimal Control Law

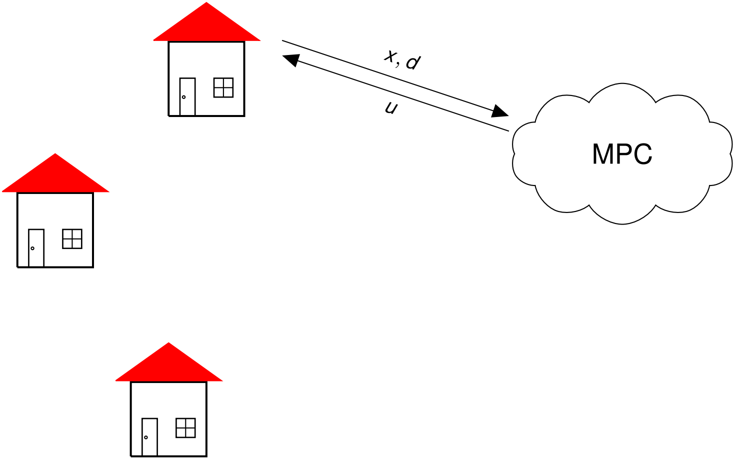
$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ & \vdots \\ \tilde{F}_L x + \tilde{g}_L & \text{if } x \in \mathcal{H}_L \end{cases}$$

$$||u^\star(x) - \tilde{u}(x)|| < \epsilon$$

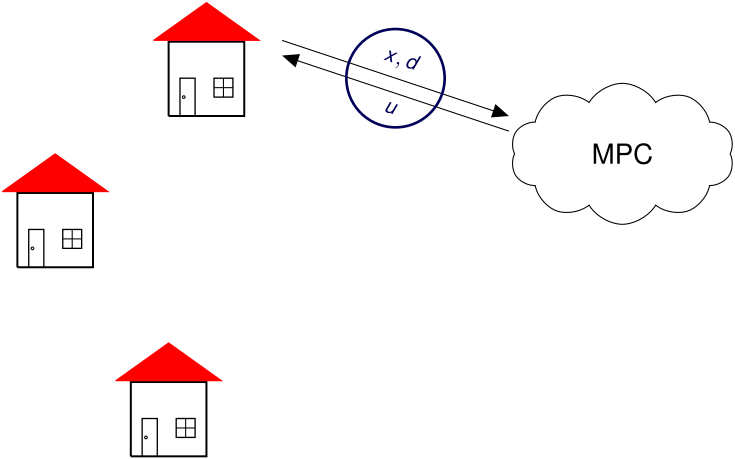
Proposed Solution: Use Machine Learning



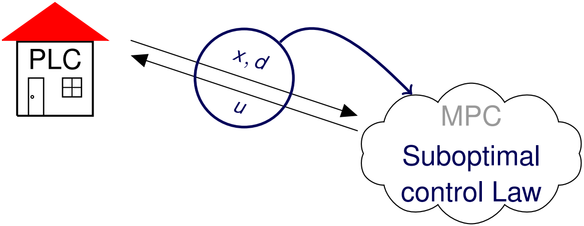
Proposed Solution: Use Machine Learning



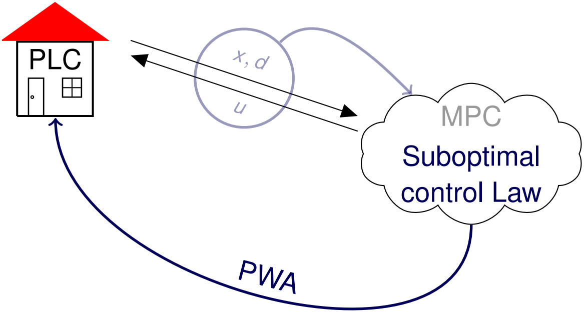
Proposed Solution: Use Machine Learning



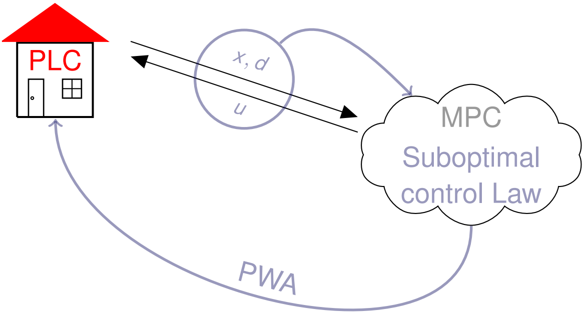
Proposed Solution: Use Machine Learning



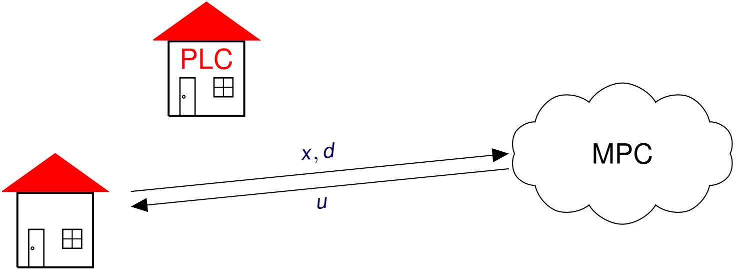
Proposed Solution: Use Machine Learning



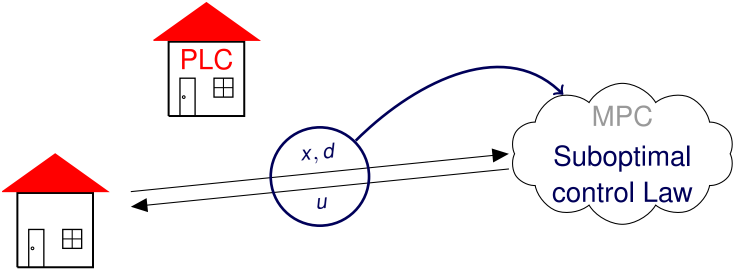
Proposed Solution: Use Machine Learning



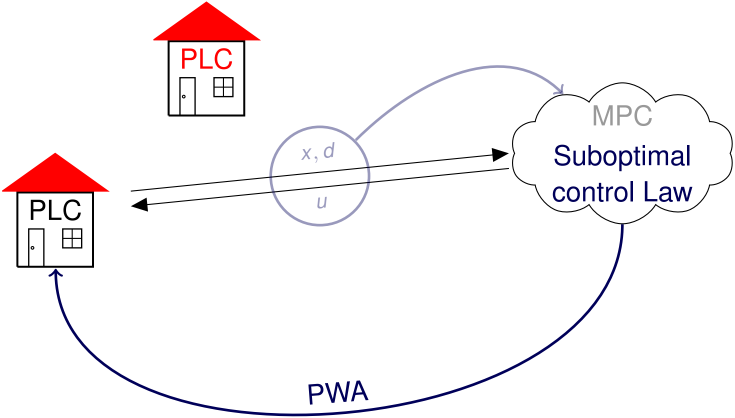
Proposed Solution: Use Machine Learning



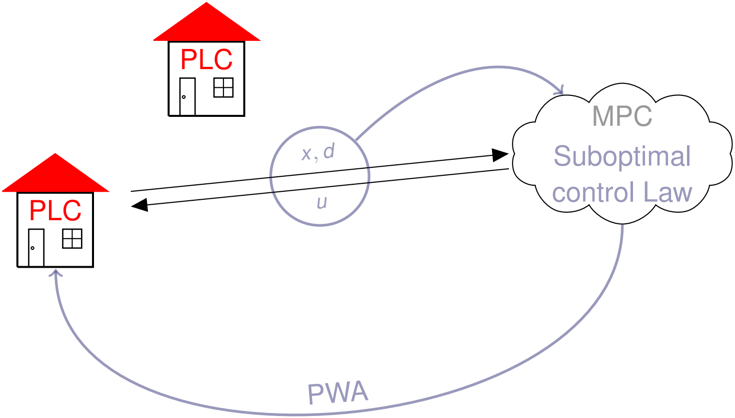
Proposed Solution: Use Machine Learning



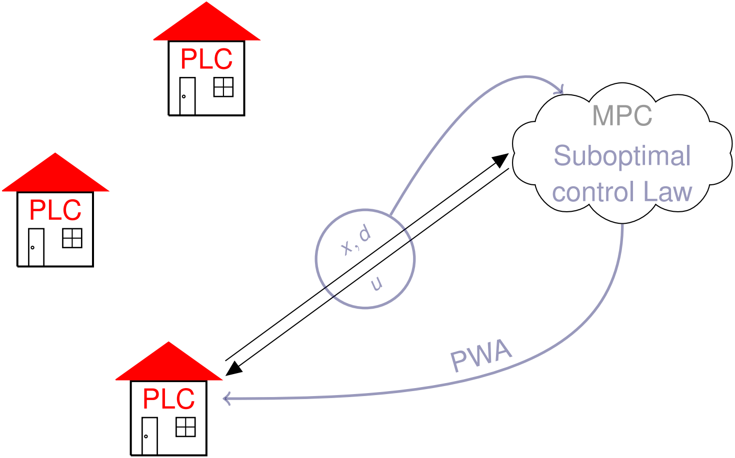
Proposed Solution: Use Machine Learning



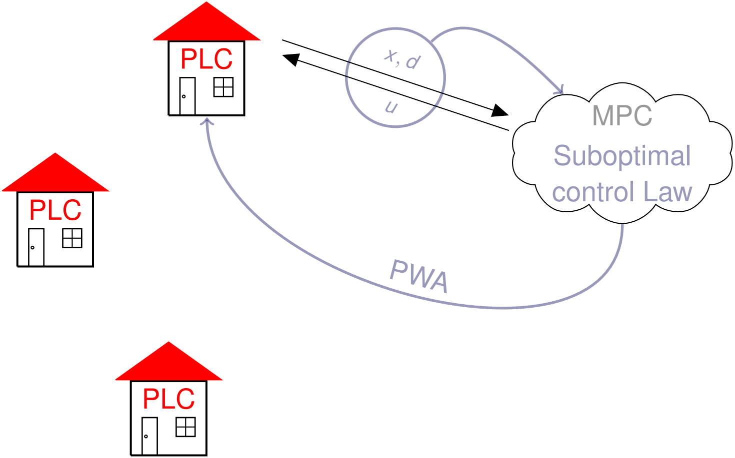
Proposed Solution: Use Machine Learning



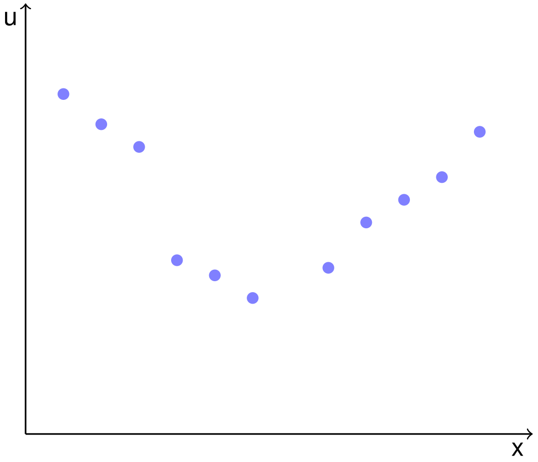
Proposed Solution: Use Machine Learning



Proposed Solution: Use Machine Learning

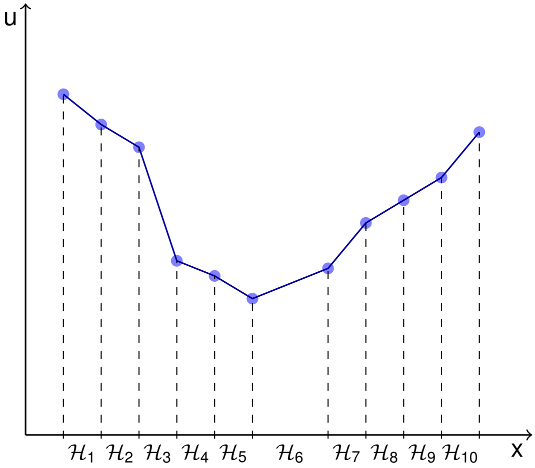


Machine Learning



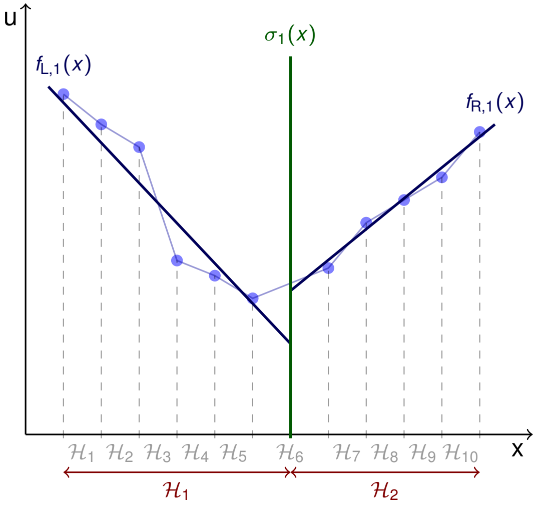
$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ & \vdots \\ \tilde{F}_L x + \tilde{g}_L & \text{if } x \in \mathcal{H}_L \end{cases}$$

Machine Learning



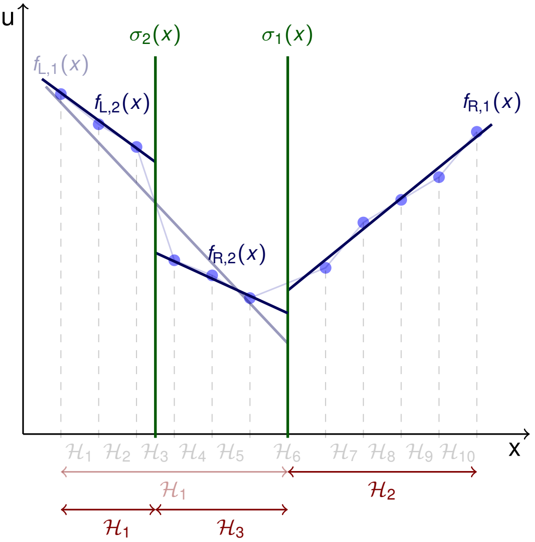
$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ & \vdots \\ \tilde{F}_{10} x + \tilde{g}_{10} & \text{if } x \in \mathcal{H}_{10} \end{cases}$$

Machine Learning: Regression Trees



$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ \tilde{F}_2 x + \tilde{g}_2 & \text{if } x \in \mathcal{H}_2 \end{cases}$$

Machine Learning: Regression Trees



$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ \tilde{F}_2 x + \tilde{g}_2 & \text{if } x \in \mathcal{H}_2 \\ \tilde{F}_3 x + \tilde{g}_3 & \text{if } x \in \mathcal{H}_3 \end{cases}$$

Regression-Based Control Law

$$\min_{\sigma, f_{\mathrm{L}}, f_{\mathrm{R}}} \left(\sum_{x_i \in \mathcal{P}_{\mathrm{L}}} \| u_i - f_{\mathrm{L}}(x_i) \| + \sum_{x_j \in \mathcal{P}_{\mathrm{R}}} \| u_j - f_{\mathrm{R}}(x_j) \| \right)$$

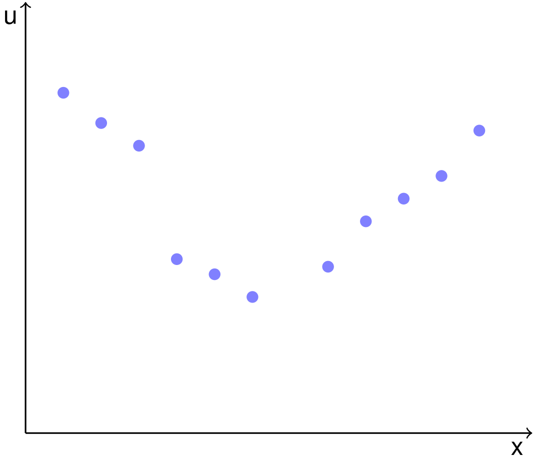
$$\begin{aligned} \mathcal{P}_{\mathrm{L}} &= \{x \mid \sigma(x) \leq 0\} \\ \mathcal{P}_{\mathrm{R}} &= \{x \mid \sigma(x) > 0\} \end{aligned}$$

nonlinear
nonconvex

Regression-Based Control Law

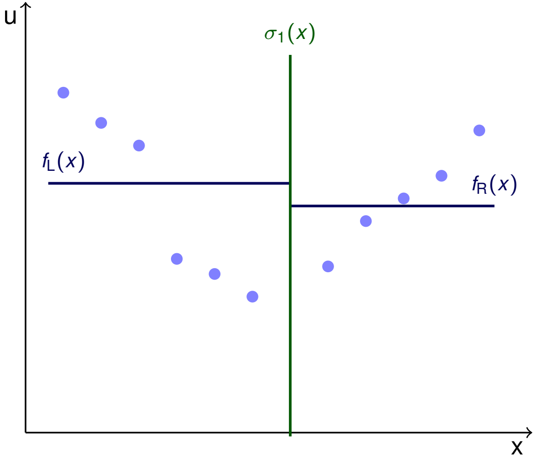
$$\min_{\sigma, f_L, f_R} \left(\sum_{x_i \in \mathcal{P}_L} \|u_i - f_L(x_i)\| + \sum_{x_j \in \mathcal{P}_R} \|u_j - f_R(x_j)\| \right)$$

$$\mathcal{P}_L = \{x \mid \sigma(x) \leq 0\}$$
$$\mathcal{P}_R = \{x \mid \sigma(x) > 0\}$$



Breiman, L. 1993; Classification and regression trees

Regression-Based Control Law



$$\min_{\sigma, f_L, f_R} \left(\sum_{x_i \in \mathcal{P}_L} \| u_i - f_L(x_i) \| + \sum_{x_j \in \mathcal{P}_R} \| u_j - f_R(x_j) \| \right)$$

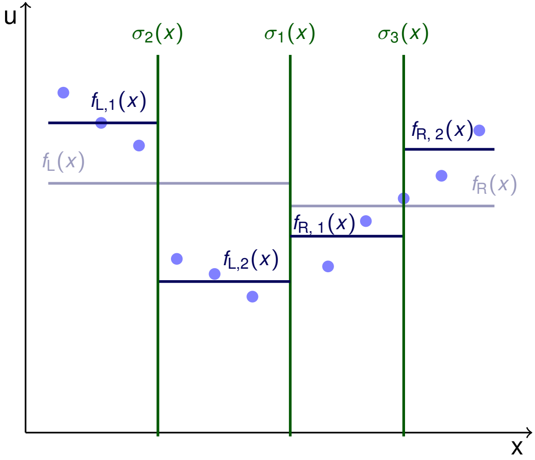
$$\mathcal{P}_L = \{x \mid \sigma(x) \leq 0\}$$
$$\mathcal{P}_R = \{x \mid \sigma(x) > 0\}$$

$$\sigma(x) := e_i$$
$$f_L(x) := g_L$$
$$f_R(x) := g_R$$

Breiman, L. 1993; Classification and regression trees

Regression-Based Control Law

$$\min_{\sigma, f_L, f_R} \left(\sum_{x_i \in \mathcal{P}_L} \|u_i - f_L(x_i)\| + \sum_{x_j \in \mathcal{P}_R} \|u_j - f_R(x_j)\| \right)$$



$$\mathcal{P}_L = \{x \mid \sigma(x) \le 0\}$$
$$\mathcal{P}_R = \{x \mid \sigma(x) > 0\}$$

$$\sigma(x) := e_i$$
$$f_L(x) := g_L$$
$$f_R(x) := g_R$$

Breiman, L. 1993; Classification and regression trees

Regression-Based MPC-Like Policy

$$\min_{\sigma, f_{\mathrm{L}}, f_{\mathrm{R}}} \left(\sum_{x_i \in \mathcal{P}_{\mathrm{L}}} \|u_i - f_{\mathrm{L}}(x_i)\| + \sum_{x_j \in \mathcal{P}_{\mathrm{R}}} \|u_j - f_{\mathrm{R}}(x_j)\| \right)$$

$$\begin{aligned} \mathcal{P}_{\mathrm{L}} &= \{x \mid \sigma(x) \leq 0\} \\ \mathcal{P}_{\mathrm{R}} &= \{x \mid \sigma(x) > 0\} \end{aligned}$$

$$\begin{aligned} \sigma(x) &:= \alpha^T x - \beta \\ f_{\mathrm{L}}(x) &:= F_{\mathrm{L}} x + g_{\mathrm{L}} \\ f_{\mathrm{R}}(x) &:= F_{\mathrm{R}} x + g_{\mathrm{R}} \end{aligned}$$

Regression-Based MPC-Like Policy

$$\min_{\sigma, f_{\mathrm{L}}, f_{\mathrm{R}}} \left(\sum_{x_i \in \mathcal{P}_{\mathrm{L}}} \|u_i - f_{\mathrm{L}}(x_i)\| + \sum_{x_j \in \mathcal{P}_{\mathrm{R}}} \|u_j - f_{\mathrm{R}}(x_j)\| \right)$$

$$\mathcal{P}_{\mathrm{L}} = \{x \mid \sigma(x) \leq 0\}$$
$$\mathcal{P}_{\mathrm{R}} = \{x \mid \sigma(x) > 0\}$$

$$\sigma(x) := \alpha^T x - \beta$$

MPC-like →

$$\boxed{\begin{aligned} f_{\mathrm{L}}(x) &:= F_{\mathrm{L}} x + g_{\mathrm{L}} \\ f_{\mathrm{R}}(x) &:= F_{\mathrm{R}} x + g_{\mathrm{R}} \end{aligned}}$$

Regression-Based MPC-Like Policy

$$\min_{\sigma, f_\mathrm{L}, f_\mathrm{R}} \left(\sum_{x_i \in \mathcal{P}_\mathrm{L}} \|u_i - f_\mathrm{L}(x_i)\| + \sum_{x_j \in \mathcal{P}_\mathrm{R}} \|u_j - f_\mathrm{R}(x_j)\| \right)$$

$$\mathcal{P}_\mathrm{L} = \{x \mid \sigma(x) \leq 0\}$$
$$\mathcal{P}_\mathrm{R} = \{x \mid \sigma(x) > 0\}$$

$$\sigma(x) := \alpha^T x - \beta$$
$$f_\mathrm{L}(x) := F_\mathrm{L} x + g_\mathrm{L}$$
$$f_\mathrm{R}(x) := F_\mathrm{R} x + g_\mathrm{R}$$

NLP $\rightarrow$ MIQP

Process Description

State (Measured) Variables

x_1 − floor temperature
x_2 − internal facade temperature
x_3 − external facade temperature
x_4 − internal temperature

Measured Disturbances

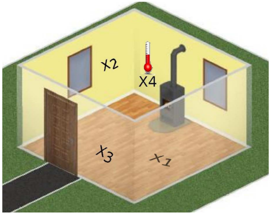
d_1 − external temperature
d_2 − occupancy
d_3 − solar radiation

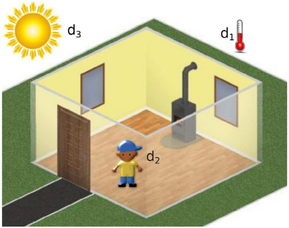
Controlled Variable

$y = x_4$

Manipulated Variable

u − heat flow





MPC for Obtaining Training Data

$$\begin{aligned}
\min_{u_0,\ldots,u_{N-1}} \quad & \sum_{k=0}^{N-1} \left(q_s s_k + |u_k|\right) \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& T_{\text{ref}} - \epsilon - s_k \leq Cx_k \leq T_{\text{ref}} + \epsilon + s_k \\
& s_k \geq 0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& x_0 = x(t),\ d_0 = d(t) \\
& N = 12\text{h}\ (T_s = 15\ \text{min})
\end{aligned}$$

MPC for Obtaining Training Data

energy consumption minimization

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} (q_s s_k + |u_k|)$$

$$\begin{aligned} \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\ & T_{\text{ref}} - \epsilon - s_k \leq Cx_k \leq T_{\text{ref}} + \epsilon + s_k \\ & s_k \geq 0 \\ & u_{\min} \leq u_k \leq u_{\max} \\ & x_0 = x(t),\ d_0 = d(t) \\ & N = 12\text{h}\ (T_s = 15\ \text{min}) \end{aligned}$$

MPC for Obtaining Training Data

energy consumption minimization

building model

$$\min_{u_0,\dots,u_{N-1}} \quad \sum_{k=0}^{N-1} \left(q_s s_k + |u_k|\right)$$

$$\begin{aligned}
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& T_{\text{ref}} - \epsilon - s_k \leq Cx_k \leq T_{\text{ref}} + \epsilon + s_k \\
& s_k \geq 0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& x_0 = x(t),\ d_0 = d(t) \\
& N = 12\text{h}\ (T_s = 15\ \text{min})
\end{aligned}$$

MPC for Obtaining Training Data

energy consumption minimization

building model

thermal comfort zone

$$
\begin{aligned}
\min_{u_0,\ldots,u_{N-1}} \quad & \sum_{k=0}^{N-1} \left(q_s s_k + |u_k|\right) \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& T_{\text{ref}} - \epsilon - s_k \leq Cx_k \leq T_{\text{ref}} + \epsilon + s_k \\
& s_k \geq 0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& x_0 = x(t),\ d_0 = d(t) \\
& N = 12\text{h}\ (T_s = 15\ \text{min})
\end{aligned}
$$

MPC for Obtaining Training Data

energy consumption minimization

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} (q_s s_k + |u_k|)$$

building model

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$

thermal comfort zone

$$T_{\text{ref}} - \epsilon - s_k \le Cx_k \le T_{\text{ref}} + \epsilon + s_k$$

$$s_k \ge 0$$

limited control authority

$$u_{\min} \le u_k \le u_{\max}$$

$$x_0 = x(t),\ d_0 = d(t)$$

$$N = 12\text{h}\ (T_s = 15\ \text{min})$$

MPC for Obtaining Training Data

energy consumption minimization

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} (q_s s_k + |u_k|)$$

building model

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_0$$

thermal comfort zone

$$T_{\text{ref}} - \epsilon - s_k \leq Cx_k \leq T_{\text{ref}} + \epsilon + s_k$$

$$s_k \geq 0$$

limited control authority

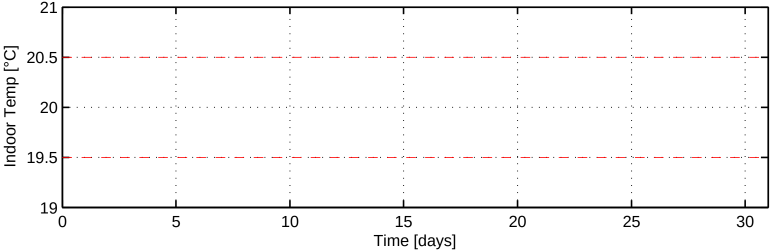
$$u_{\min} \leq u_k \leq u_{\max}$$

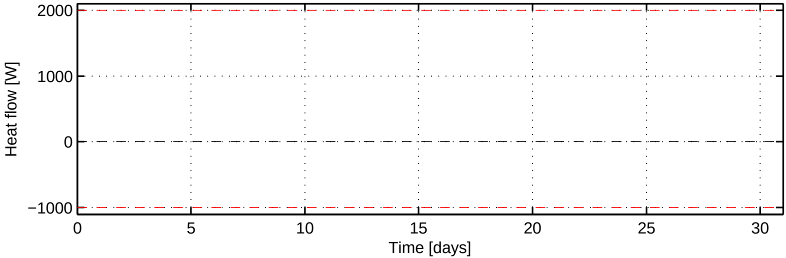
measurements

$$x_0 = x(t),\ d_0 = d(t)$$

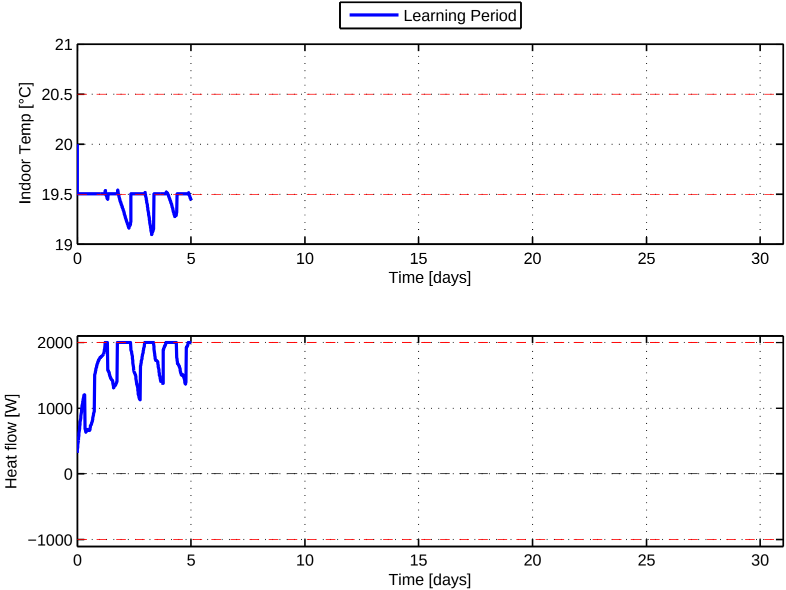
$$N = 12\text{h}\ (T_s = 15\ \text{min})$$

Case Study: Learning Period

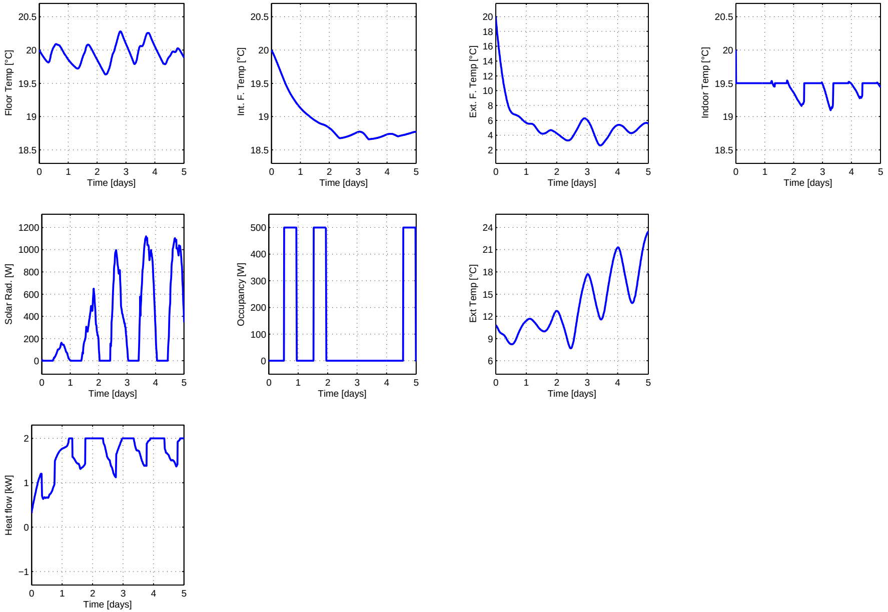




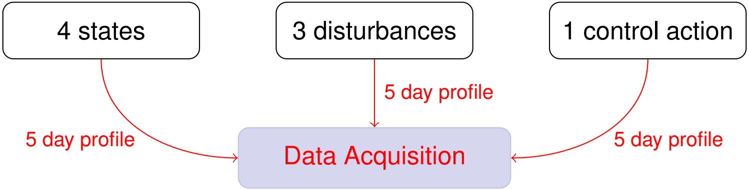
Case Study: Learning Period



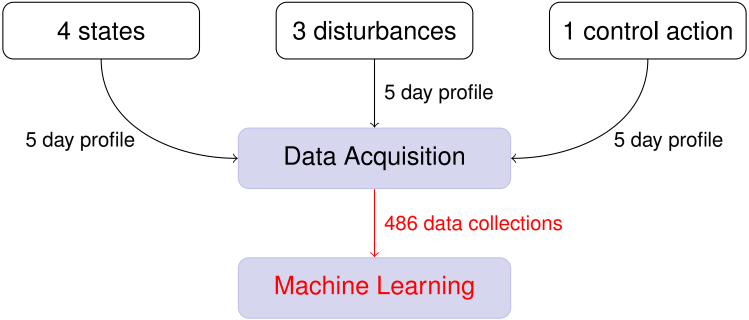
Case Study: Learning Period



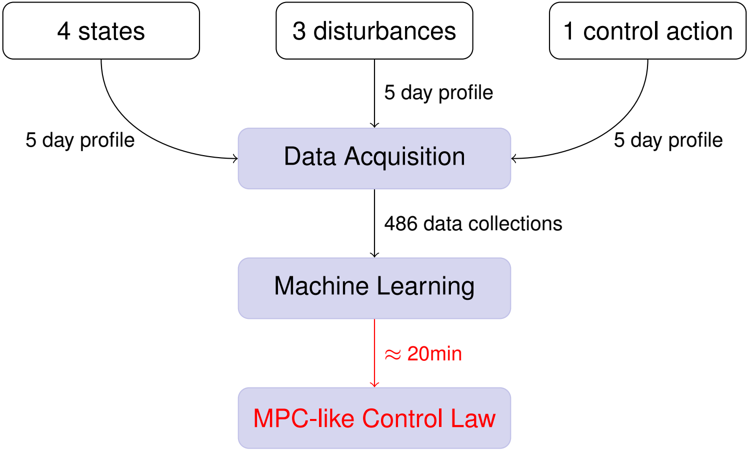
Case Study: Learning Period



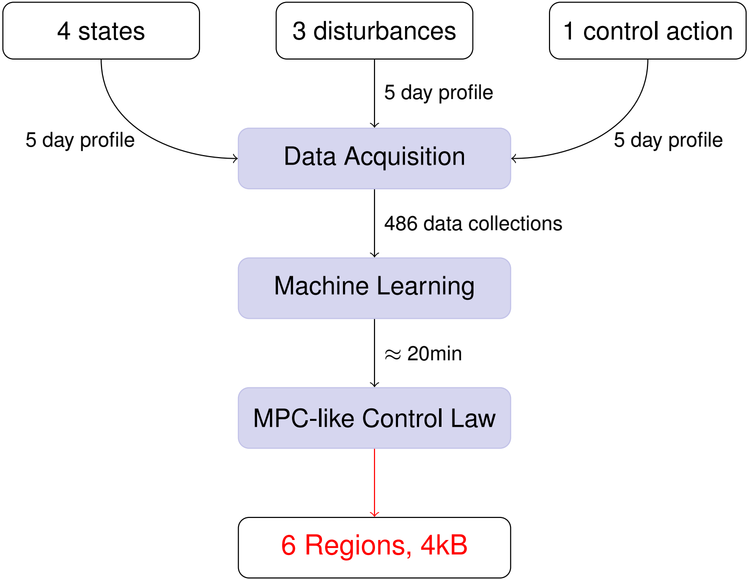
Case Study: Learning Period



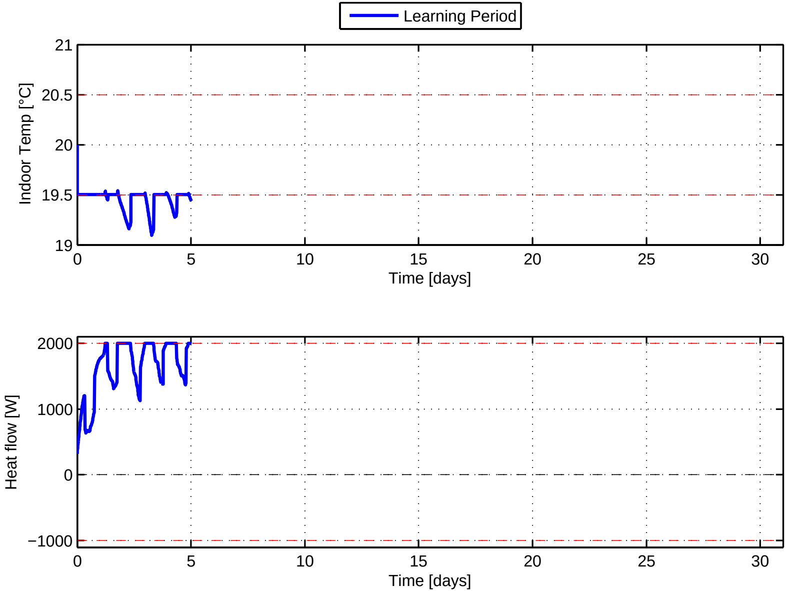
Case Study: Learning Period



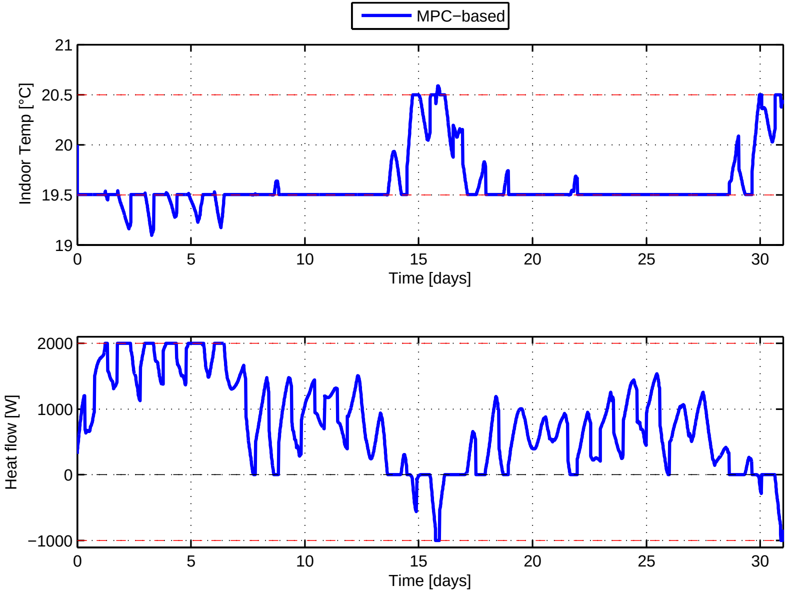
Case Study: Learning Period



Case Study

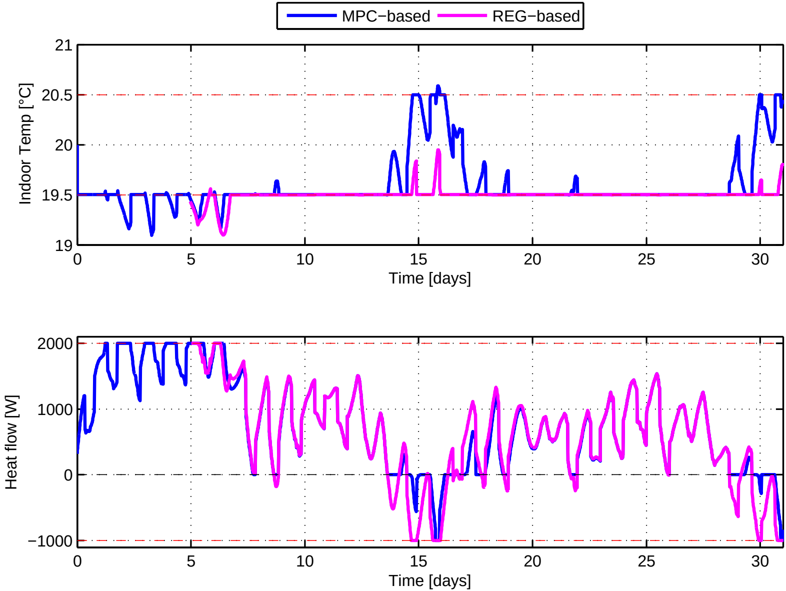

Learning Period
21
20.5
20
19.5
19
Indoor Temp [°C]
0
5
10
15
20
25
30
Time [days]
2000
1000
0
−1000
Heat flow [W]
0
5
10
15
20
25
30
Time [days]

Case Study



MPC−based
21
20.5
20
19.5
19
Indoor Temp [°C]
0
5
10
15
20
25
30
Time [days]
2000
1000
0
−1000
Heat flow [W]
0
5
10
15
20
25
30
Time [days]

Case Study



MPC−based
REG−based
21
20.5
20
19.5
19
Indoor Temp [°C]
0
5
10
15
20
25
30
Time [days]
2000
1000
0
−1000
Heat flow [W]
0
5
10
15
20
25
30
Time [days]

Case Study: Comparison

Online MPC

MPC-like Control Law

Case Study: Comparison

Online MPC	MPC-like Control Law
592 kWh	611 kWh

Case Study: Comparison

Online MPC	MPC-like Control Law
592 kWh	611 kWh

+3%

Case Study: Comparison

Online MPC

MPC-like Control Law

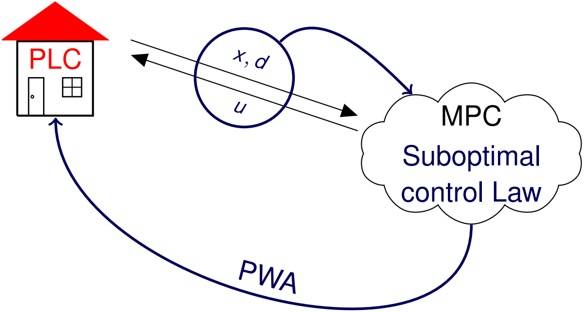
592 kWh

611 kWh

+3%

4 kB

Conclusion



Cloud MPC

Machine Learning

Cheap Implementation

Backup Slide 1: MIQP Formulation

$$
\begin{aligned}
\min\ & \sum_{i=1}^{p} (u_i - z_i)^T (u_i - z_i) \\
\text{s.t.}\ & -M(1-\delta_i) \le z_i - (F_{\mathrm{L}} x_i + g_{\mathrm{L}}) \le M(1-\delta_i) \\
& -M\delta_i \le z_i - (F_{\mathrm{R}} x_i + g_{\mathrm{R}}) \le M\delta_i \\
& \alpha^T x_i \le \beta + M(1-\delta_i) \\
& \alpha^T x \ge \beta + \epsilon - M\delta_i \\
& \|\alpha\|_\infty = 1
\end{aligned}
$$

$$
z_i = \begin{cases} F_L x_i + g_L & \text{if } \sigma(x) \le 0 \\ F_R x_i + g_R & \text{if } \sigma(x) > 0 \end{cases}
$$

$$
(\delta_i = 1) \Leftrightarrow (\sigma(x) \le 0)
$$

Backup Slide 3: Continuous PWA Function

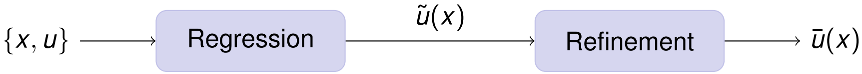
$$\min_{\sigma, f_\mathrm{L}, f_\mathrm{R}} \left(\sum_{x_i \in \mathcal{P}_\mathrm{L}} \|u_i - f_\mathrm{L}(x_i)\| + \sum_{x_j \in \mathcal{P}_\mathrm{R}} \|u_j - f_\mathrm{R}(x_j)\| \right)$$

$$\begin{aligned} \mathcal{P}_\mathrm{L} &= \{x \mid \sigma(x) \leq 0\} \\ \mathcal{P}_\mathrm{R} &= \{x \mid \sigma(x) > 0\} \end{aligned}$$

$$f_\mathrm{L}(x) = f_\mathrm{R}(x) \;\; \forall\, x \in \{x | \sigma(x) = 0\}$$

Mixed Integer Nonlinear Program

Backup Slide 3: Refinement



$$\tilde{u}(x) = \begin{cases} \tilde{F}_1 x + \tilde{g}_1 & \text{if } x \in \mathcal{H}_1 \\ & \vdots \\ \tilde{F}_L x + \tilde{g}_L & \text{if } x \in \mathcal{H}_L \end{cases} \qquad \bar{u}(x) = \begin{cases} \bar{F}_1 x + \bar{g}_1 & \text{if } x \in \mathcal{H}_1 \\ & \vdots \\ \bar{F}_L x + \bar{g}_L & \text{if } x \in \mathcal{H}_L \end{cases}$$

$$\bar{u}(x) \in \mathcal{U} \ \forall x$$