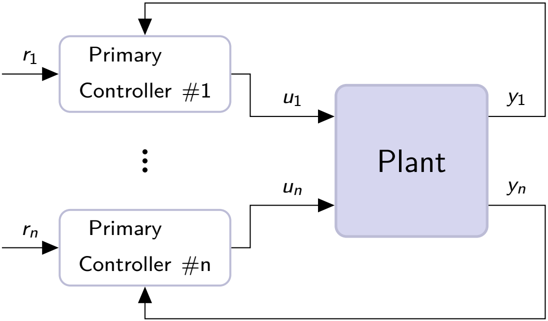
Real-Time Implementation of a Reference Governor on the Arduino Microcontroller

Martin Kalúz, **Martin Klaučo**, Michal Kvasnica

STU FCHPT

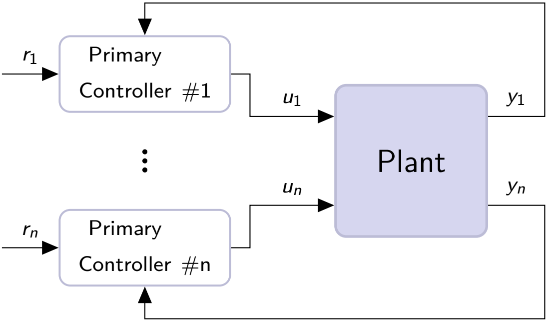
Slovak University of Technology in Bratislava, Slovakia

Standard Control Strategy



Standard Control Strategy

1. PID
2. ON/OFF
3. Rule based



Standard Control Strategy

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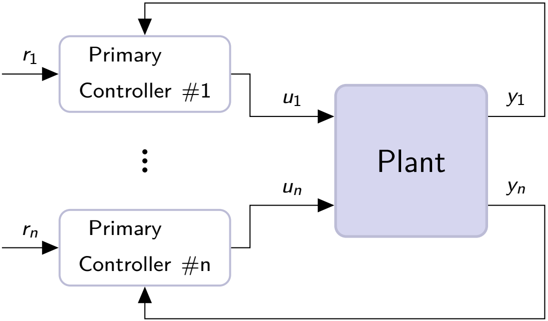
- × Constraints
- × Overall performance



Reference Governor Control Strategy

1. PID
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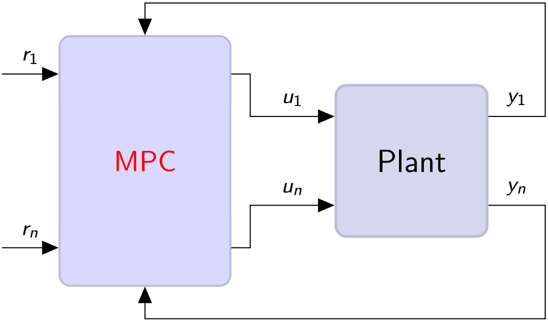
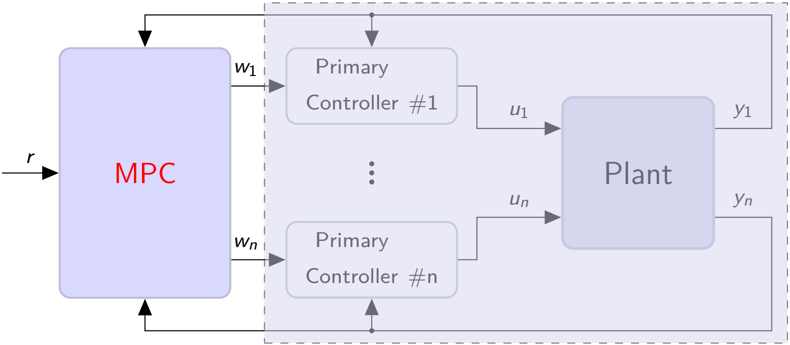
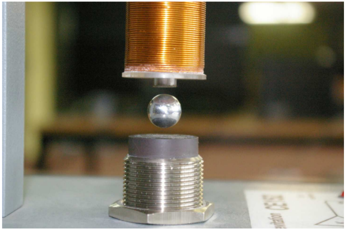
- ✓ Overall performance

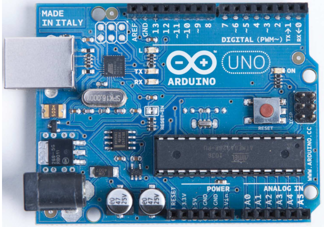


Reference Governor Control Strategy

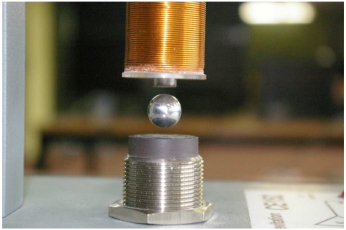


Experimental Setup





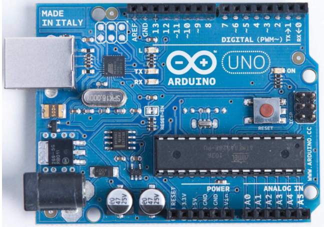
Maglev System



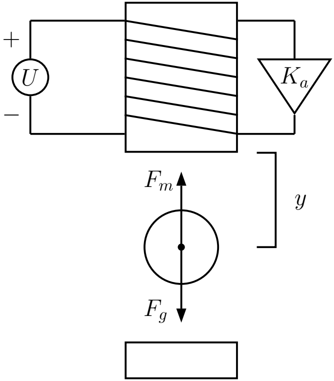
1. Fast sampling ($T_s = 2\,\text{ms}$)
2. Unstable dynamics

Arduino Microcontroller

1. 8-bit 16 MHz
2. 120 FLOPs per 2 ms
3. 2 kB RAM



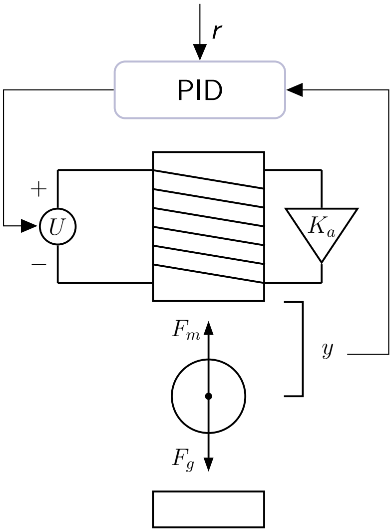
Maglev System Scheme



MV: Voltage to the coil

PV: Position of the ball

Maglev System Scheme



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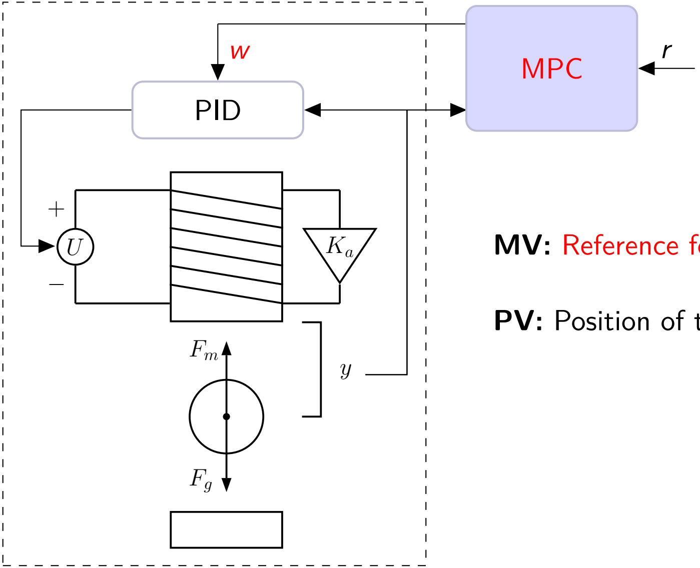

MV: Reference for PID

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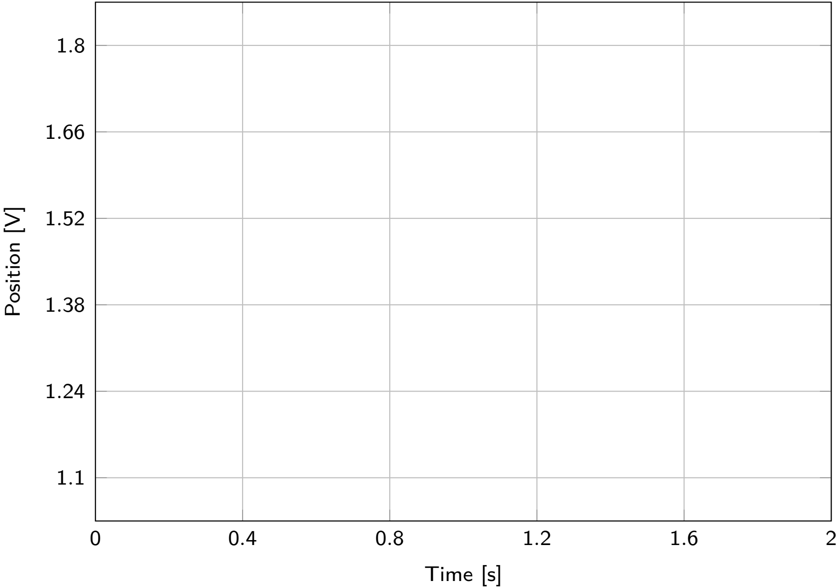
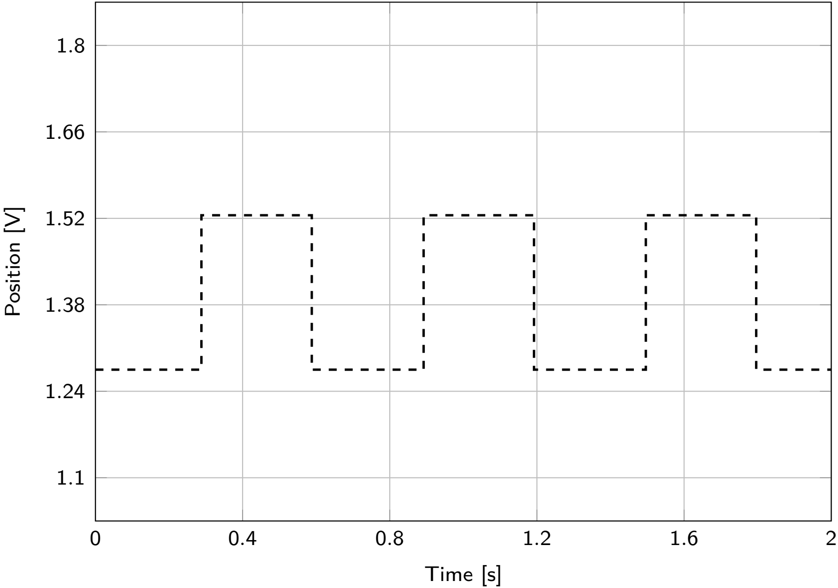
PID Controller

PID Controller



PID Controller



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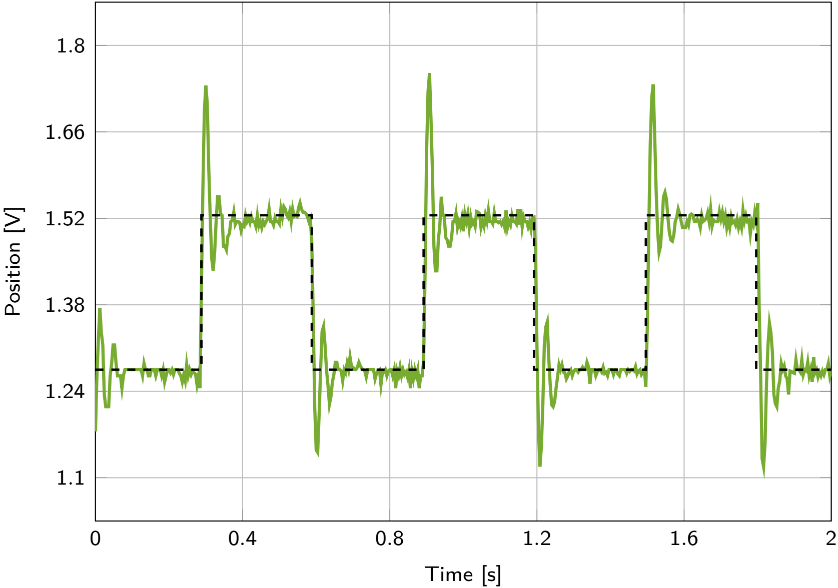
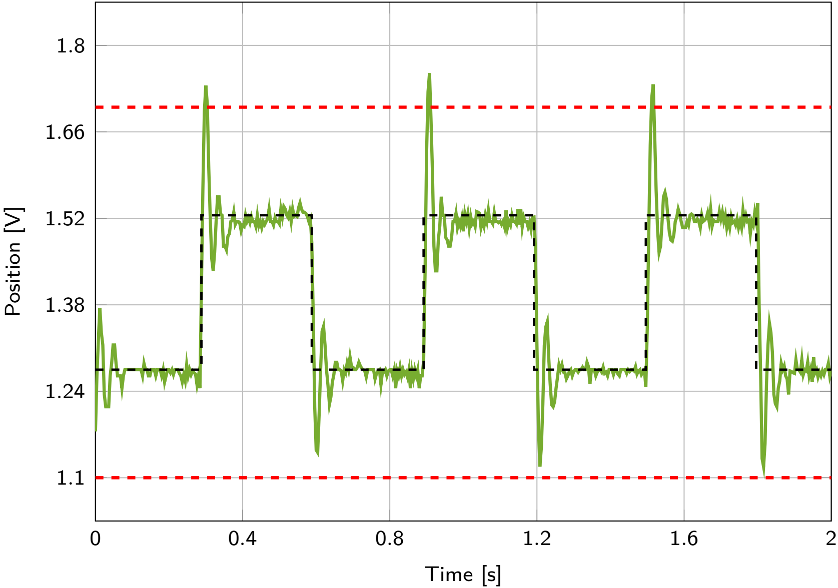
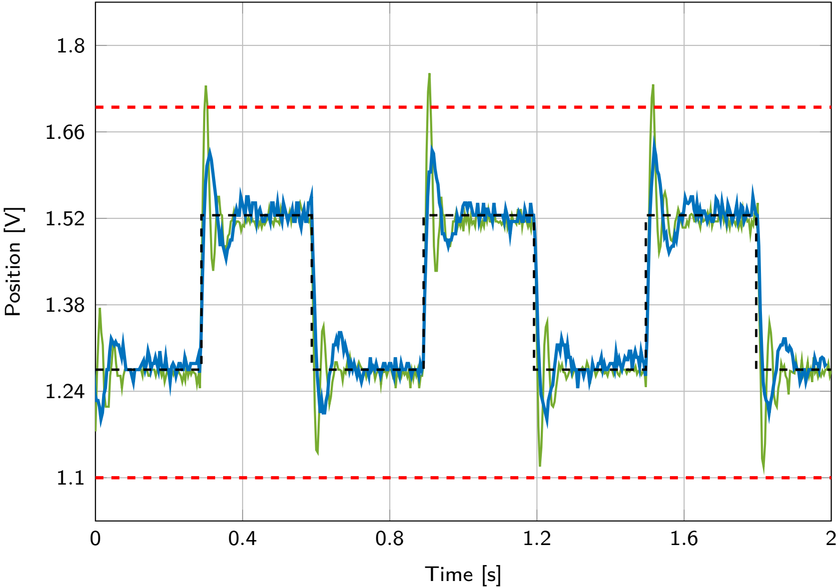


PID Controller



Control Performance Improvement



Ways to Improve

1. Re-tune PID controller

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2. Optimal Control Strategy (MPC)

Model Predictive Control

- Constraints handling
- Model knowledge
- State estimator and disturbance modeling
- Computational complexity

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Model Predictive Control

- Constraints handling
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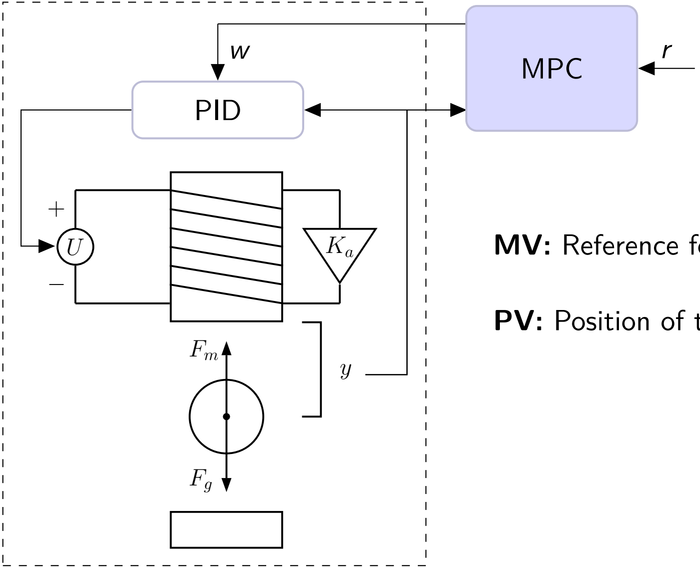
- Computational complexity → Explicit MPC

Maglev System Scheme with Reference Governor



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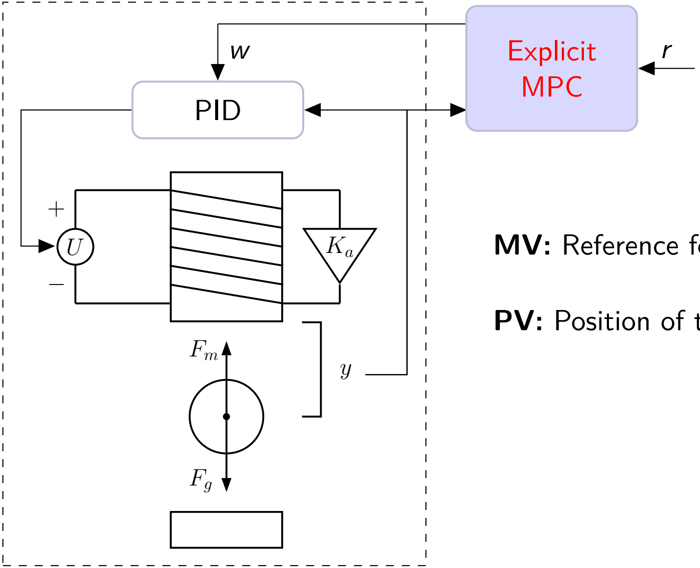
Maglev System Scheme with Reference Governor



MV: Reference for PID

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MPC as Reference Governor

$$\min \quad \sum_{k=1}^{N} \left(\| Q_{\mathrm{dw}} \left(w_k - w_{k-1} \right) \|_2^2 + \| Q_{\mathrm{w}} \left(w_k - r \right) \|_2^2 + \| Q_{\mathrm{yr}} \left(y_k - w_k \right) \|_2^2 \right)$$

$$\text{s.t.} \quad y(k+1) = \tfrac{1}{a_0} \left(- \sum_{i=1}^{n} a_i y(k-i+1) + \sum_{j=0}^{m} b_j w(k-j+1) \right)$$

$$y_{\min} \leq y_{k+1} \leq y_{\max}$$

MPC as Reference Governor

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$$y_{\min} \leq y_{k+1} \leq y_{\max}$$

shaping the reference

MPC as Reference Governor

$$\min \quad \sum_{k=1}^{N} \left(\| Q_{\mathrm{dw}} (w_k - w_{k-1}) \|_2^2 + \| Q_{\mathrm{w}} (w_k - r) \|_2^2 + \| Q_{\mathrm{yr}} (y_k - w_k) \|_2^2 \right)$$

$$\text{s.t.} \quad y(k+1) = \frac{1}{a_0} \left(- \sum_{i=1}^{n} a_i y(k-i+1) + \sum_{j=0}^{m} b_j w(k-j+1) \right)$$

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input-output model

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$$y_{\min} \leq y_{k+1} \leq y_{\max}$$

shaping the reference

input-output model

limits on ball's position

Reference Governor Parameters

$$T_s = 2\,\text{ms}, \quad N = 3, \quad \text{ord}(G_{\text{cl}}) = 4$$

Explicit MPC

$$\min \quad \sum_{k=1}^{N} \left(\left\| Q_{\mathrm{dw}} \left(w_k - w_{k-1} \right) \right\|_2^2 + \left\| Q_{\mathrm{w}} \left(w_k - r \right) \right\|_2^2 + \left\| Q_{\mathrm{yr}} \left(y_k - w_k \right) \right\|_2^2 \right)$$

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$$y_{\min} \leq y_{k+1} \leq y_{\max}$$

Explicit MPC

$$\min \quad W^T H W + \theta^T F W$$

$$\text{s.t.} \quad G W \leq g + S\theta$$

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$$\min \quad W^T H W + \theta^T F W$$

$$\text{s.t.} \quad GW \leq g + S\theta$$

$$\theta = \begin{bmatrix} y_k \\ y_{k-1} \\ y_{k-2} \\ y_{k-3} \\ w_{k-1} \\ w_{k-2} \\ w_{k-3} \\ r_k \end{bmatrix} \qquad W = \begin{bmatrix} w_k \\ w_{k+1} \\ w_{k+2} \end{bmatrix}$$

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$$W^\star(\theta) = \begin{cases} \alpha_1 \theta + \beta_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ \alpha_L \theta + \beta_L & \text{if } \theta \in \mathcal{R}_L \end{cases}$$

Explicit MPC

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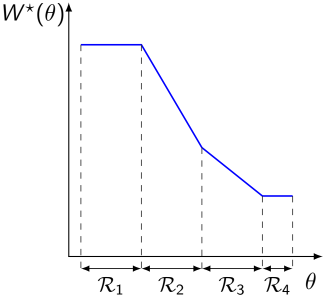

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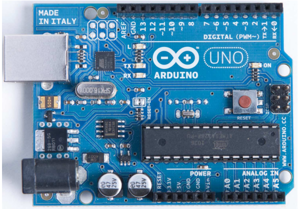

Control Algorithm

Offline:

1. Construct explicit MPC - 8 regions (619 bytes)

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Online:

1. Measure the current position of the ball

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2. Construct vector θ

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3. Calculate optimal setpoint for PID

Control Algorithm

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Online:

1. Measure the current position of the ball
2. Construct vector θ
3. Calculate optimal setpoint for PID
4. Calculate control action by PID

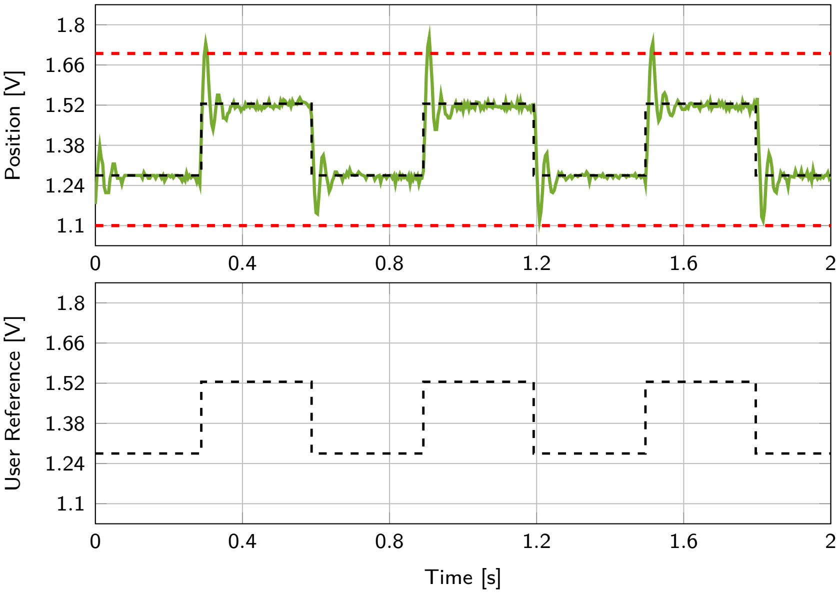
Offline:

1. Construct explicit MPC - 8 regions (619 bytes)

Online: (loop repeated every T_s)

1. Measure the current position of the ball
2. Construct vector θ
3. Calculate optimal setpoint for PID
4. Calculate control action by PID
5. Apply control action

MPC as a Reference Governor



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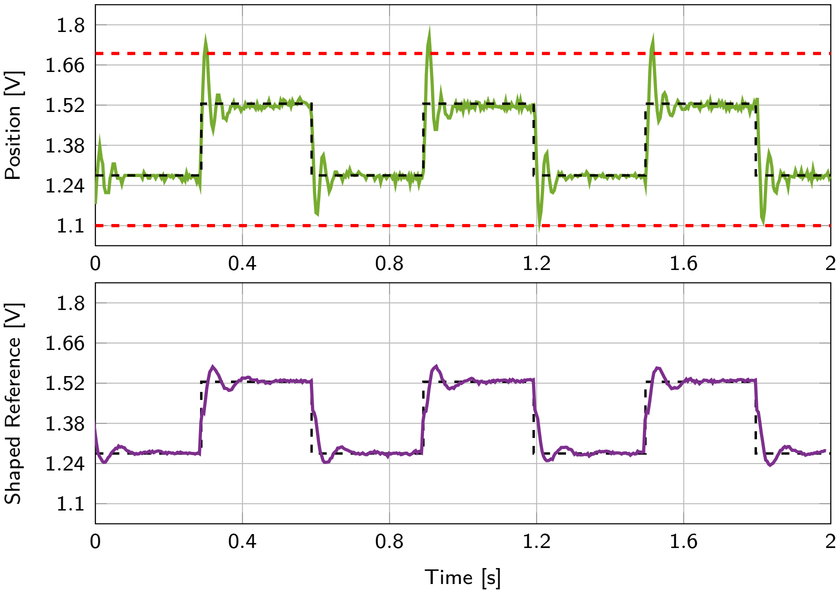

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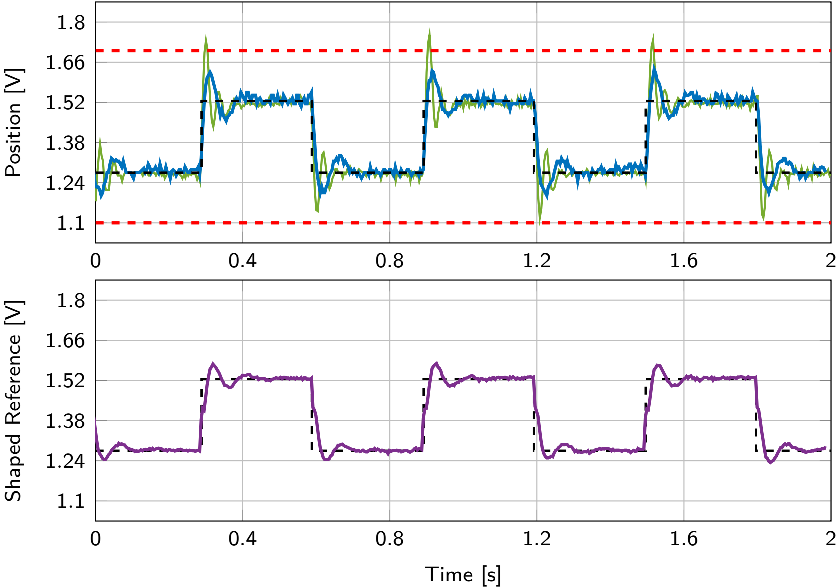
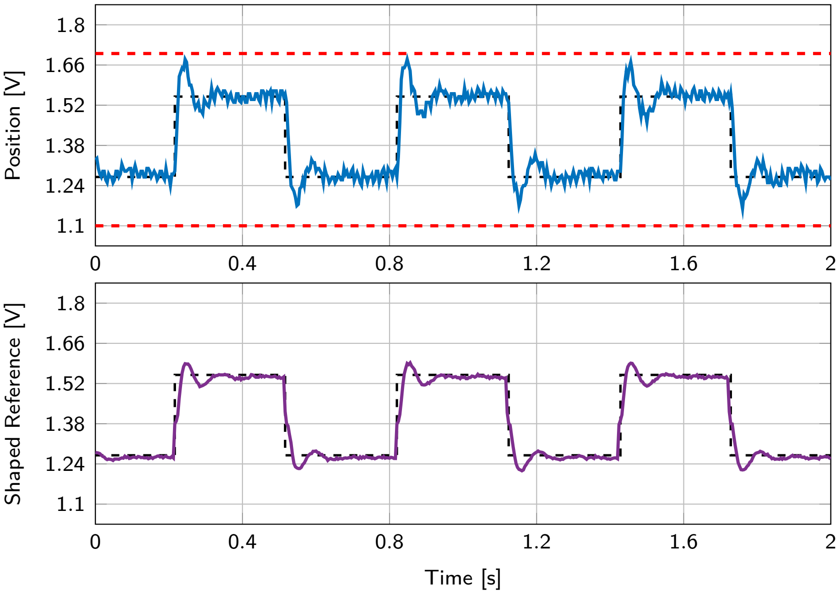


MPC as a Reference Governor with Hard Constraints



Wrap up

1. Improved performance by RG
2. Embedded System (Arduino with 2kB RAM, $T_s = 2\,\text{ms}$)
3. Input-Output models