Predictive Control of Complex Systems

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Complex Systems

Thermal Comfort

Complex Systems

Thermal Comfort

Vehicle Routing

Complex Systems

Thermal Comfort

Vehicle Routing

Predictive Control

Thermal Comfort

Thermal Comfort: Motivation

Indoor Temperature

Thermal Comfort: Motivation

Indoor Temperature

Radiant Temperature

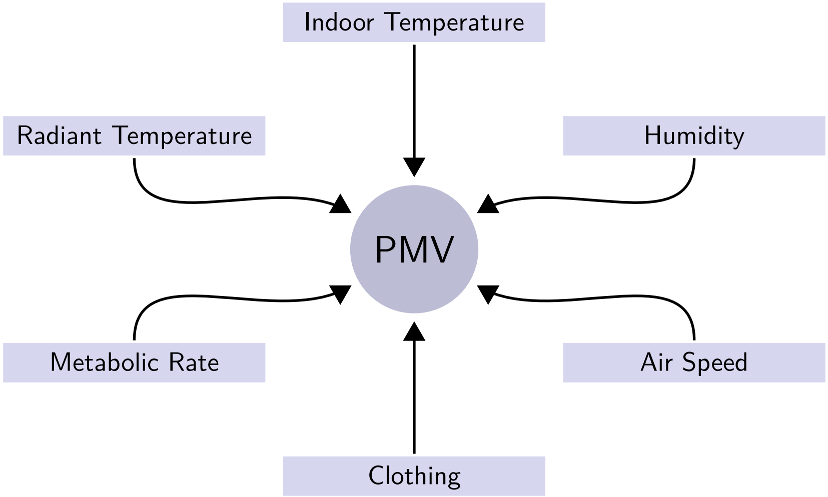
Humidity

Metabolic Rate

Air Speed

Clothing

Thermal Comfort: Predicted Mean Vote Index

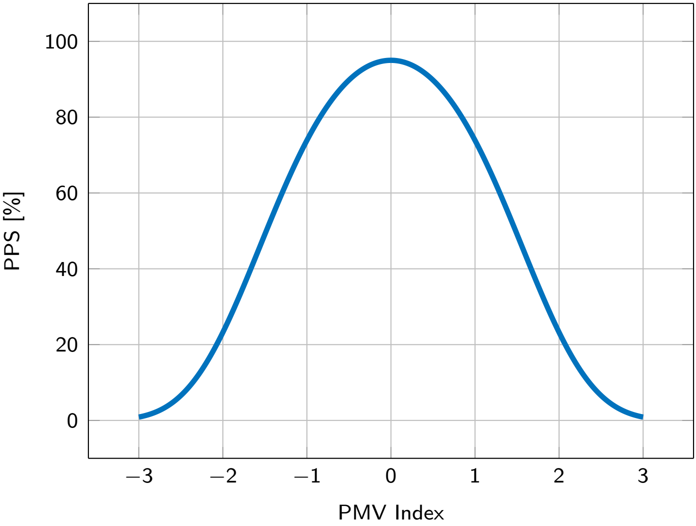


Thermal Comfort: Predicted Mean Vote Index

-3	-2	-1	0	1	2	3

* EN ISO 7730:2006 Ergonomics of Thermal Environment

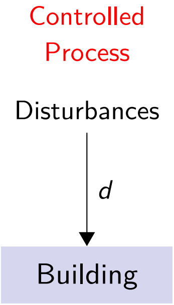
Thermal Comfort: Predicted Mean Vote Index



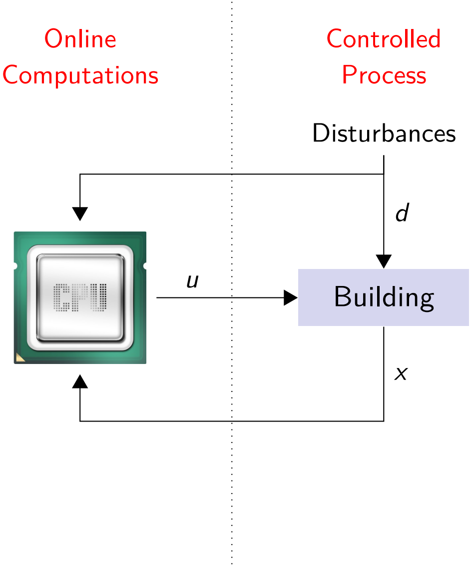
Maintain PMV index within -0.2 to 0.2⋆

⋆EN ISO 7730:2006 Ergonomics of Thermal Environment

Thermal Comfort: Closed-Loop System



Thermal Comfort: Closed-Loop System

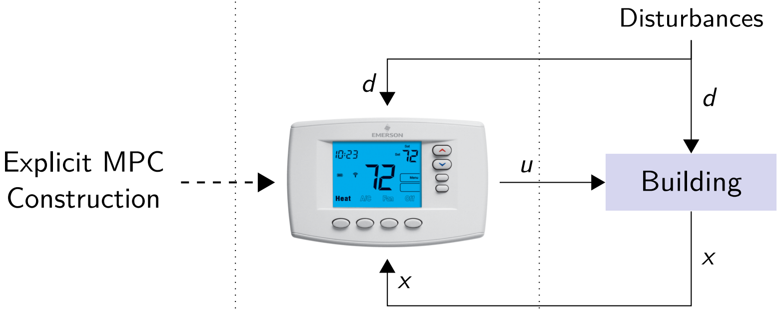


Thermal Comfort: Closed-Loop System

Offline Computations

Online Computations

Controlled Process



Thermal Comfort: MPC Implementation

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& p_k = \mathrm{PMV}(x_k) \\
& p_{\min} \leq p_k \leq p_{\max} \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Explicit MPC

$$
u^{\star}(\theta) = \begin{cases} F_1\theta + g_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ F_L\theta + g_L & \text{if } \theta \in \mathcal{R}_M \end{cases}
$$

Thermal Comfort: MPC Implementation

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& p_k = \mathrm{PMV}(x_k) \\
& p_{\min} \leq p_k \leq p_{\max} \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Explicit MPC

$$
u^\star(\theta) = \begin{cases} F_1\theta + g_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ F_L\theta + g_L & \text{if } \theta \in \mathcal{R}_M \end{cases}
$$

MPC with PMV Index

$$\text{PMV} = \left(0.303 e^{-0.036 M} + 0.028\right) \cdot L$$

$$\begin{aligned}
L =& (M - W) - 3.05 \cdot 10^{-3} (5733 - 6.99 (M - W) - p_{\mathrm{a}}) - \\
&- 0.42 ((M - W) - 58.15) - 1.7 \cdot 10^{-5} M (5867 - p_{\mathrm{a}}) - \\
&- 0.0014 M (34 - T_{\mathrm{in}}) - 3.96 \cdot 10^{-8} f_{\mathrm{cl}} (K_{\mathrm{tcl}} - K_{\mathrm{tr}}) - \\
&- f_{\mathrm{cl}} h_{\mathrm{c}} (T_{\mathrm{cl}} - T_{\mathrm{in}})
\end{aligned}$$

$$\begin{aligned}
T_{\mathrm{cl}} =& - 0.028 (M - W) - I_{\mathrm{cl}} \Big(3.96 \cdot 10^{-8} (f_{\mathrm{cl}} K_{\mathrm{tcl}} - K_{\mathrm{tr}}) + \\
&+ f_{\mathrm{cl}} h_{\mathrm{c}} (T_{\mathrm{cl}} - T_{\mathrm{in}}) \Big) + 35.7
\end{aligned}$$

$$K_{\mathrm{tcl}} = (T_{\mathrm{cl}} + 273.16)^4$$

$$K_{\mathrm{tr}} = (T_{\mathrm{r}} + 273.16)^4$$

Thermal Comfort: MPC Implementation

Online MPC

$$\begin{aligned}\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\ & u_{\min} \le u_k \le u_{\max} \\ & p_k = \mathrm{PMV}(x_k) \\ & p_{\min} \le p_k \le p_{\max} \\ & x_0 = x(t),\ d_0 = d(t)\end{aligned}$$

Explicit MPC

$$u^\star(\theta) = \begin{cases} F_1\theta + g_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ F_L\theta + g_L & \text{if } \theta \in \mathcal{R}_M \end{cases}$$

Klaučo M., Kvasnica M., Explicit MPC Approach to PMV-Based Thermal Comfort Control, IEEE CDC 2014 in Los Angeles, California

Thermal Comfort: MPC Implementation

Online MPC

$$\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 \\
& u_{\min} \leq u_k \leq u_{\max} \\
& p_k = \ell_1^T a(x_0) + \ell_2^T x_{k-1} + \ell_3^T u_{k-1} + \ell_4 \, b(x_0) \\
& p_{\min} \leq p_k \leq p_{\max} \\
& x_0 = x(t), \; d_0 = d(t)
\end{aligned}$$

Explicit MPC

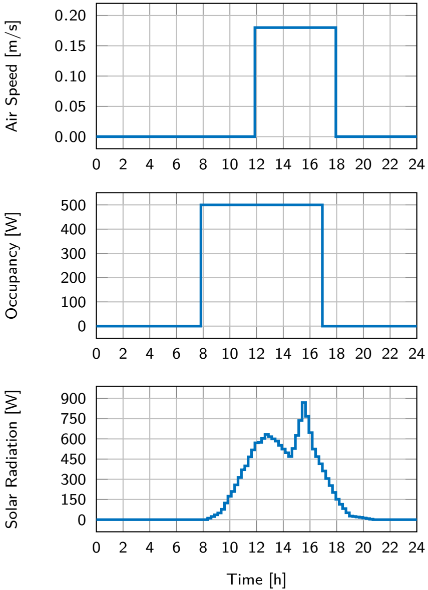
$$u^\star(\theta) = \begin{cases} F_1 \theta + g_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ F_L \theta + g_L & \text{if } \theta \in \mathcal{R}_M \end{cases}$$

Klaučo M., Kvasnica M., Explicit MPC Approach to PMV-Based Thermal Comfort Control, IEEE CDC 2014 in Los Angeles, California

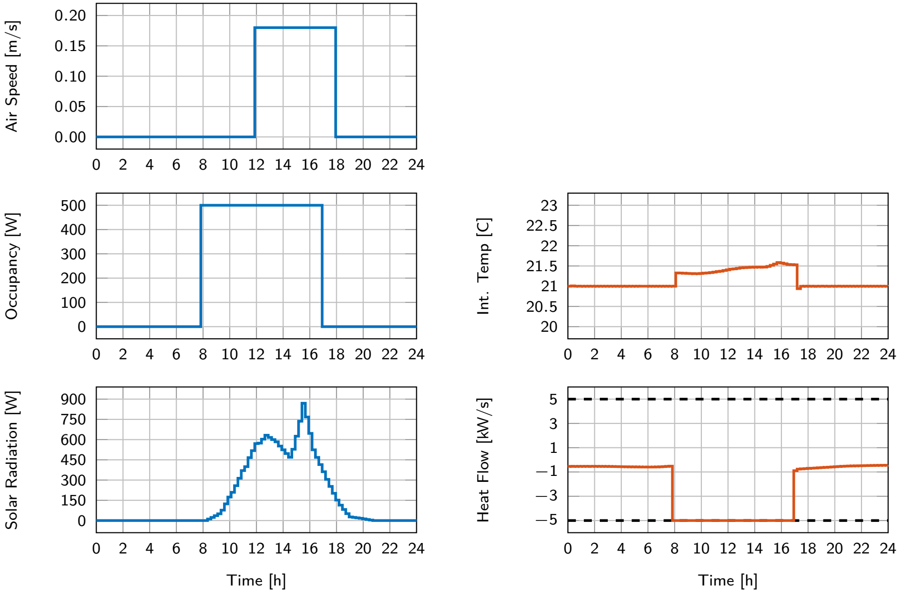
Thermal Comfort: Simulation Test Scenarios

1. Temperature-based control
2. PMV-based control

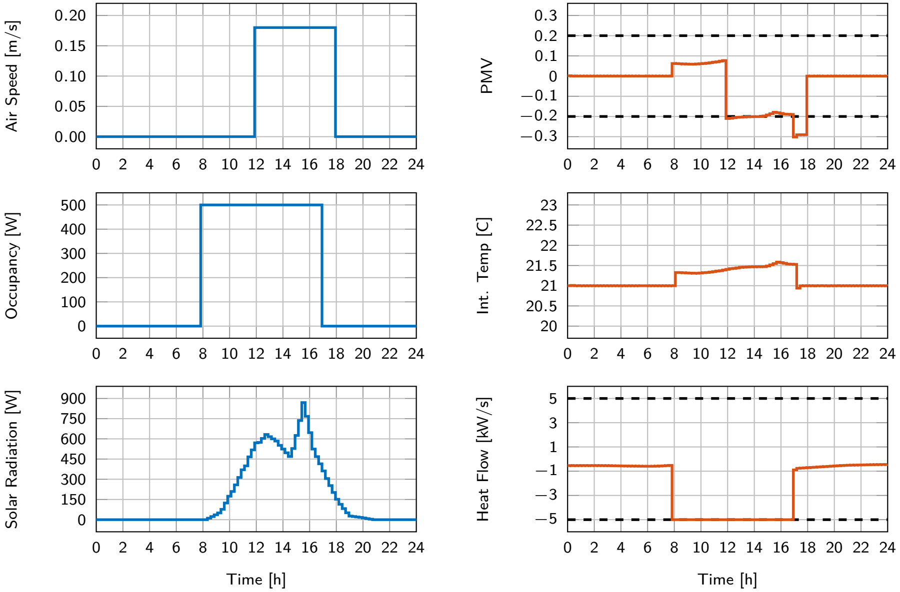
Thermal Comfort: Simulation Test Scenarios



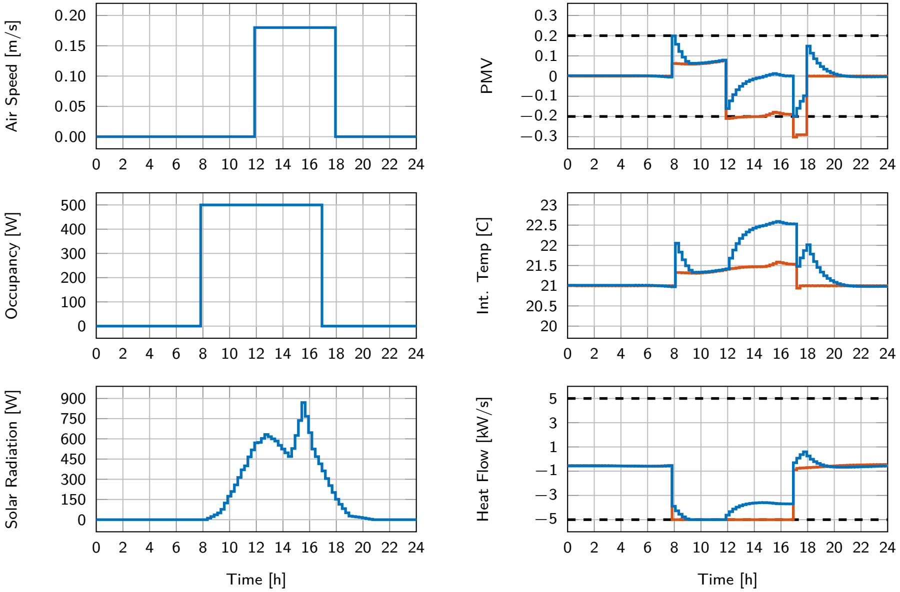
Thermal Comfort: Simulation Test Scenarios



Thermal Comfort: Simulation Test Scenarios



Thermal Comfort: Simulation Test Scenarios

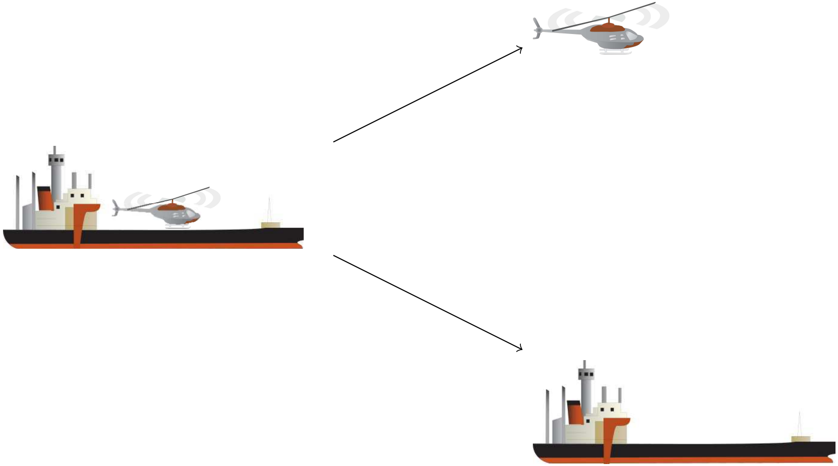


Thermal Comfort: Key Points

1. Thermal comfort model (PMV index)
2. Explicit MPC implementable on thermostat-like device
3. Mathematical framework for eMPC with quadratic constraints

Vehicle Routing

Vehicle Routing: Heterogeneous Multi-Vehicle Systems



Vehicle Routing: Heterogeneous Multi-Vehicle Systems

Operation range: $t_{\max}$

Maximum velocity: $v_h > v_c$

Operation range: ∞

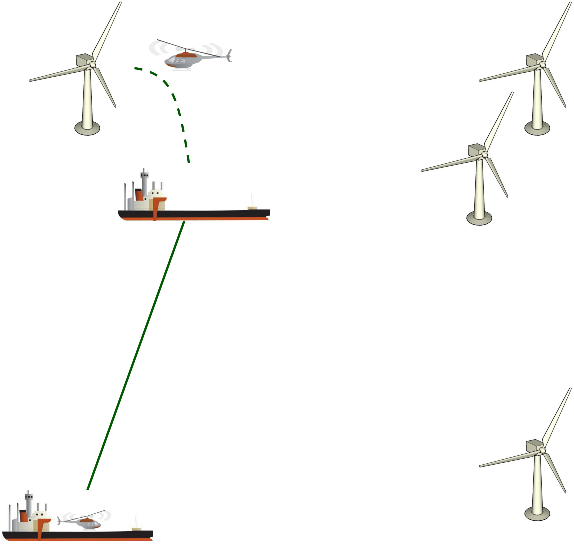
Maximum velocity: v_c

Operation range: ∞

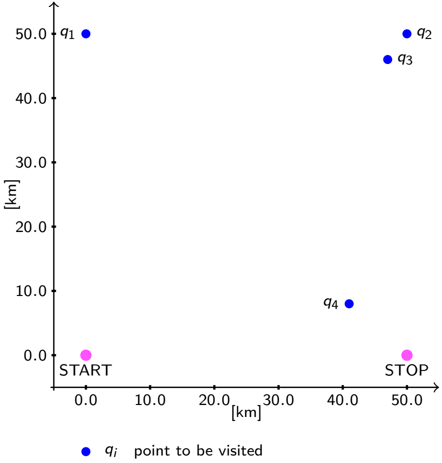
Maximum velocity: v_c

Vehicle Routing: Motivation

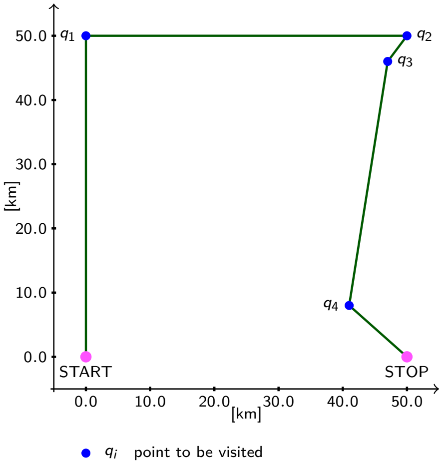
Vehicle Routing: Motivation



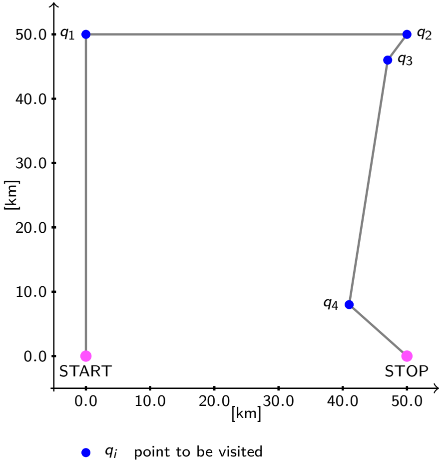
Vehicle Routing: Single Vehicle



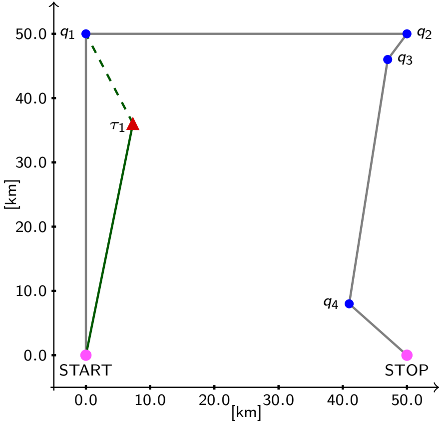
Vehicle Routing: Single Vehicle



Vehicle Routing: Multi-vehicle System

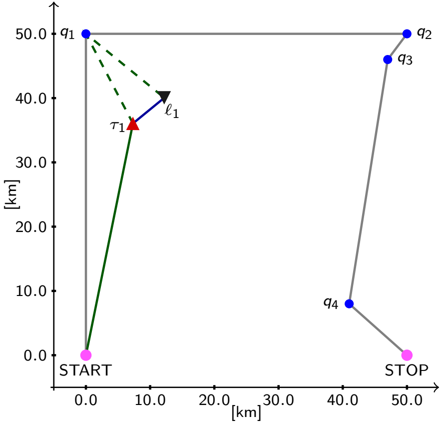


Vehicle Routing: Multi-vehicle System



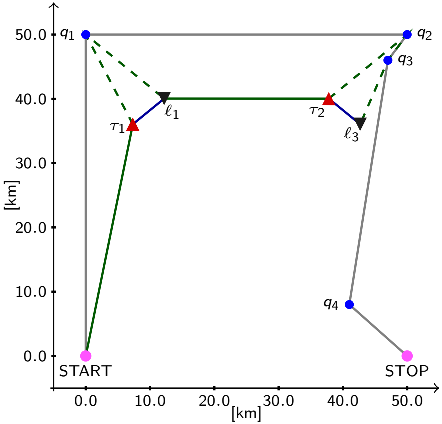
- q_i point to be visited
- τ_i take-off point

Vehicle Routing: Multi-vehicle System



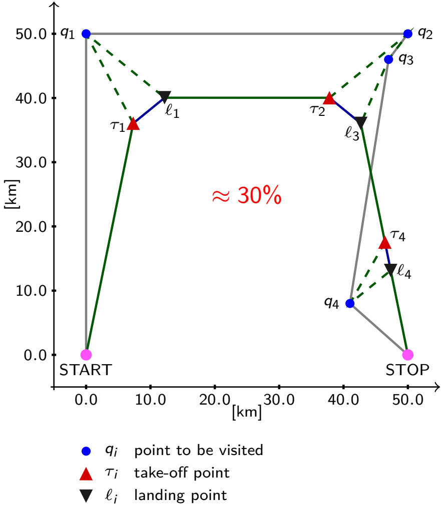
- q_i point to be visited
- τ_i take-off point
- ℓ_i landing point

Vehicle Routing: Multi-vehicle System



- q_i point to be visited
- τ_i take-off point
- ℓ_i landing point

Vehicle Routing: Multi-vehicle System



Vehicle Routing: Control Objective

Calculate coordinates of take-off and landing points

- minimize mission time

Vehicle Routing: Control Objective

Calculate coordinates of take-off and landing points

- minimize mission time

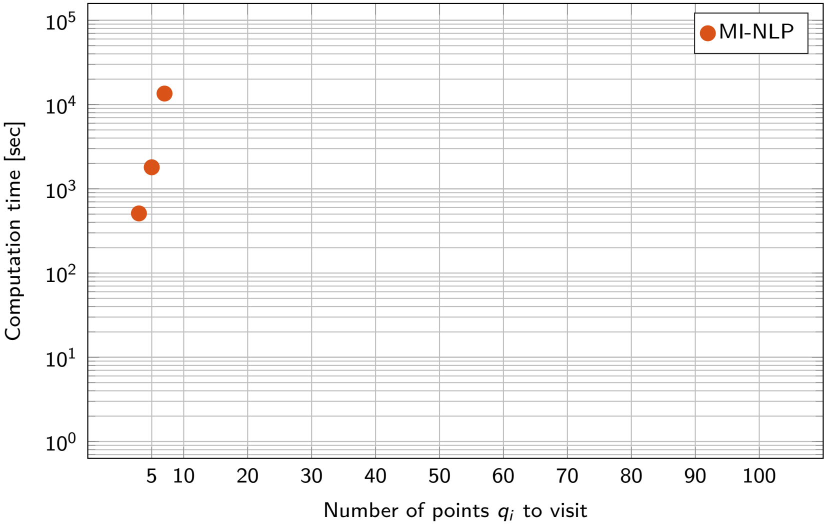
- visit all way-points
- bounded helicopter flyover
- visit multiple way-points during one flyover

Vehicle Routing: Solution

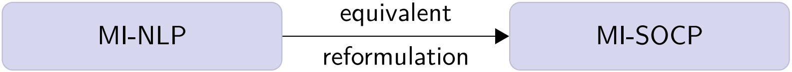
MI-NLP

Garone, Determe, Naldi, CDC 2012

Vehicle Routing: Solution



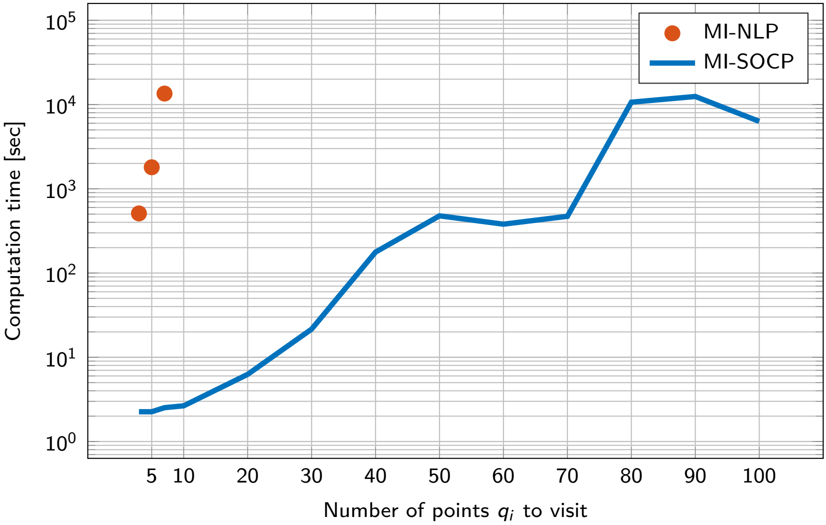
Vehicle Routing: Solution



Garone, Determe, Naldi, CDC 2012

Klaučo, Blažek, Kvasnica, ECC 2014

Vehicle Routing: Solution



Vehicle Routing: Key Points

1. Optimal path planing for multi-vehicle systems
2. Mixed-integer SOCP formulation
3. Applicable to larger number of waypoints

Conclusions

Thermal Comfort

Vehicle Routing

Conclusions

Thermal Comfort

Vehicle Routing

Predictive Control

Conclusions

Thermal Comfort

Vehicle Routing

Predictive Control

Efficient problem formulations

Aims of Thesis

1. Development of computationally efficient optimal control algorithms
2. Verification and implementation of optimal control algorithms on laboratory devices
3. Application of optimization and optimal control to path planning problems

Publications

Publications - A-class

1 Klaučo, Drgoňa, Kvasnica, Cairano. **Building Temperature Control by Simple MPC-like Feedback Laws Learned from Closed-Loop Data.** IFAC World Congress 2014

Publications - IEEE Conferences

2 Klaučo, Kvasnica. **Explicit MPC Approach to PMV-Based Thermal Comfort Control.** CDC 2014

3 Klaučo, Blažek, Kvasnica, Fikar. **Mixed-Integer SOCP Formulation of the Path Planning Problem for Heterogeneous Multi-Vehicle Systems.** ECC 2014

4 Drgoňa, Kvasnica, Klaučo, Fikar. **Explicit Stochastic MPC Approach to Building Temperature Control.** CDC 2013

5 Klaučo, Poulsen, Mirzaei, Niemann. **Frequency Weighted Model Predictive Control of Wind Turbine.** Process Control 2013

Publications - In Progress

Accepted:

6 Oravec, Klaučo, Kvasnica, Löfberg. **Vehicle Routing with Interception of Targets' Neighbourhoods.** ECC 2015

In preparation:

7 Drgoňa, Klaučo, Fikar. **Model Identification and Predictive Control of a Laboratory Binary Distillation Column.** Process Control 2015

8 Kalúz, Klaučo, Kvasnica. **Explicit MPC-based Reference Governor Control of Magnetic Levitation via Arduino Microcontroller.** Process Control 2015

9 Klaučo, Blažek, Kvasnica. **Optimal Path Planning Problem for Heterogeneous Multi-Vehicle System.** CC Journal

Opponents Questions

Question 1

Ako súvisí linearita so zhodou modelu a riadeného procesu? (str. 17)

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} (x_k - x^s)^T Q_\mathrm{x} (x_k - x^s) + \sum_{k=0}^{N-1} (u_k - u^s)^T Q_\mathrm{u} (u_k - u^s), \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k \quad k = 0, \ldots, N-1, \\
& x_0 = x(t) \\
& x_k \in \mathbb{X} \quad k = 1, \ldots, N-1 \\
& u_k \in \mathbb{U} \quad k = 1, \ldots, N-1
\end{aligned}
$$

Question 2

Je vektor X v (2.19) uvedený správne? (str. 21)

$$X = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \longrightarrow X = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{bmatrix}$$

Question 3

Čo musia spĺňať východiskové podmienky pre (3.9), aby bol problém riešiteľný? (str. 35)

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u(u_k - u^s)^2 + q_p p_k^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& p_k = \text{PMV}(x_k) \\
& -5000 \leq u_k \leq 5000 \\
& -0.2 \leq p_k \leq 0.2 \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Question 3

Čo musia spĺňať východiskové podmienky pre (3.9), aby bol problém riešiteľný? (str. 35)

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u(u_k - u^s)^2 + q_p p_k^2 + q_s s_k \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 \\
& p_k = \text{PMV}(x_k) \\
& -5000 \leq u_k \leq 5000 \\
& -s_k - 0.2 \leq p_k \leq 0.2 + s_k \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Question 4

Aký je vzťah medzi „k“ a „t“ pri riešení (3.11)? Aká je realistická perióda vzorkovania pre ktorú možno očakávať riešiteľnosť problému (zaručiť PMV v predpísaných hraniciach)? (str. 36)

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u(u_{t+k} - u^s)^2 + q_p p_{t+k}^2 \\
\text{s.t.} \quad & x_{k+t+1} = Ax_{t+k} + Bu_{t+k} + Ed_0 \\
& p_{t+k} = a^T x_{t+k} + b \\
& -5000 \leq u_{t+k} \leq 5000 \\
& -0.2 \leq p_{t+k} \leq 0.2 \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Question 5

Prečo sa robila lineárna interpolácia PMV z kvadratickej aproximácie a nie z pôvodnej funkcie? (str. 40)

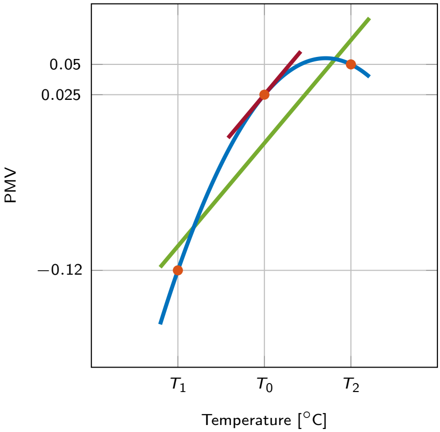
$$\text{PMV} = \left(0.303e^{-0.036M} + 0.028\right) \cdot L$$

$$\begin{aligned} L =& (M - W) - 3.05 \cdot 10^{-3}\left(5733 - 6.99(M - W) - p_a\right) - \\ & - 0.42\left((M - W) - 58.15\right) - 1.7 \cdot 10^{-5} M(5867 - p_a) - \\ & - 0.0014M(34 - T_{in}) - 3.96 \cdot 10^{-8} f_{cl}\left(K_{tcl} - K_{tr}\right) - \\ & - f_{cl} h_c (T_{cl} - T_{in}) \end{aligned}$$

$$\begin{aligned} T_{cl} =& - 0.028(M - W) - I_{cl}\left(3.96 \cdot 10^{-8}\left(f_{cl} K_{tcl} - K_{tr}\right) + \right. \\ & \left. + f_{cl} h_c (T_{cl} - T_{in})\right) + 35.7 \end{aligned}$$

$$\begin{aligned} K_{tcl} &= (T_{cl} + 273.16)^4 \\ K_{tr} &= (T_r + 273.16)^4 \end{aligned}$$

Prečo sa robila lineárna interpolácia PMV z kvadratickej aproximácie a nie z pôvodnej funkcie? (str. 40)



Question 6

Testovali ste riešenie problému (3.19) aj pre počiatočné podmienky s nerovnakými teplotami? Aký majú počiatočné podmienky vplyv na riešenie/riešiteľnosť problému? (str. 38)

$$
\begin{aligned}
\min \ & \sum_{k=0}^{N-1} q_u(u_k - u^s)^2 + q_p p_k^2 \\
\text{s.t. } & x_{k+1} = Ax_k + Bu_k + Ed_0 \\
& p_k = \theta^T M_k \theta + \theta^T K_k U_{\text{ol}} + m^T \theta \\
& -5000 \leq u_k \leq 5000 \\
& -0.2 \leq p_k \leq 0.2 \\
& x_0 = x(t), \ d_0 = d(t)
\end{aligned}
$$

Question 7

Je interpretácia m vo formulácii problému (str.47) správna? (V ďalšom riešení sa uvažuje m=n.)

$$m = n$$

Question 8

Ako si navrhnuté prístupy riešenia poradia s neurčitosťami modelov? (Stačí na to pri riadení tepelného komfortu spätná väzba?)

- trvalá regulačná odchýlka
- použitie "disturbance modelling"
- pri SISO zaviesť integrátor pred akčný zásah

Discussion - Backup Slides

Quadratic constraints

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 && \longleftarrow \text{convex quadratic} \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 && \longleftarrow \text{linear} \\
& u_{\min} \leq u_k \leq u_{\max} && \longleftarrow \text{linear} \\
& p_k = \mathrm{PMV}(x_k) && \\
& p_{\min} \leq p_k \leq p_{\max} && \longleftarrow \text{linear} \\
& x_0 = x(t), \; d_0 = d(t) &&
\end{aligned}
$$

Quadratic constraints

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 && \longleftarrow \text{convex quadratic} \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 && \longleftarrow \text{linear} \\
& u_{\min} \leq u_k \leq u_{\max} && \longleftarrow \text{linear} \\
& p_k = a(x_0)^T x_k + b(x_0) \\
& p_{\min} \leq p_k \leq p_{\max} && \longleftarrow \text{linear} \\
& x_0 = x(t), \; d_0 = d(t)
\end{aligned}
$$

Quadratic constraints

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_u u_k^2 + q_p p_k^2 && \leftarrow \text{convex quadratic} \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 && \leftarrow \text{linear} \\
& u_{\min} \leq u_k \leq u_{\max} && \leftarrow \text{linear} \\
& p_k = a(x_0)^T \left(Ax_{k-1} + Bu_{k-1}\right) + b(x_0) && \leftarrow \text{non-linear} \\
& p_{\min} \leq p_k \leq p_{\max} && \leftarrow \text{linear} \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}
$$

Quadratic constraints

Online MPC

$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 && \longleftarrow \text{convex quadratic} \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_0 && \longleftarrow \text{linear} \\
& u_{\min} \leq u_k \leq u_{\max} && \longleftarrow \text{linear} \\
& p_k = \ell_1^T a(x_0) + \ell_2^T x_{k-1} + \ell_3^T u_{k-1} + \ell_4\, b(x_0) && \longleftarrow \text{linear} \\
& p_{\min} \leq p_k \leq p_{\max} && \longleftarrow \text{linear} \\
& x_0 = x(t), \;\; d_0 = d(t)
\end{aligned}
$$

Quadratic constraints

Online MPC

$$\min \quad \sum_{k=0}^{N-1} q_{\mathrm{u}} u_k^2 + q_{\mathrm{p}} p_k^2 \longleftarrow \text{convex quadratic}$$

$$\begin{aligned}
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_0 && \longleftarrow \text{linear} \\
& u_{\min} \le u_k \le u_{\max} && \longleftarrow \text{linear} \\
& p_k = \theta^T M_k \theta + \theta^T K_k U_{\mathrm{ol}} + m^T && \longleftarrow \text{linear} \\
& p_{\min} \le p_k \le p_{\max} && \longleftarrow \text{linear} \\
& x_0 = x(t),\ d_0 = d(t)
\end{aligned}$$